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Two-sided matching decision under incomplete score environment

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ABSTRACT

A novel method is proposed for solving the two-sided matching problem under incomplete score environment. The two-sided matching problem with incomplete scores is firstly described. Then the formula of satisfaction degree is given. To maximize the satisfaction degree of each agent, a multi-objective optimization model is set up. Considering equal priority of each agent of one side, the multi-objective optimization model is converted into a bi-objective optimization model. The linear weighted method is used to convert the multi-objective optimization model into a single-objective optimization model. The matching alternative can be obtained by solving the model. Finally, an example is given to illustrate the potential application of the proposed method.

Keywords: two-sided matching; incomplete score; satisfaction degree; optimization model

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INTRODUCTION

Two-sided matching markets are markets where participants on each side have preferences over the participants on the other side. It is a common problem widely existing in real life. Examples include stable marriage problem [1], CEOs selection [2], college admissions [3], personnel assignment [4], etc. Therefore two-sided matching is a research topic with extensive practical application backgrounds.

Gale and Shapley study initially the famous marriage and college admission models half a century ago [5]. Following that, various methods, techniques and algorithms have been proposed for solving the two-sided matching problem from different point of view. For example, Manlove et al. give a 2-approximation algorithm for the stable marriage problem with incomplete lists and ties of finding a stable matching of maximum or minimum size [6]. Ehlers studies truncation strategies in matching markets using the deferred acceptance algorithm, and show that truncation strategies are also applicable to all priority mechanisms and all linear programming mechanisms [7]. Gale investigates the origin, development and current issues of the two-sided matching problem [8].

The existing studies enrich the theories and methods for solving the two-sided matching problems, and expand the practical application background. In spite of the very large literature on matching markets, one sort of matching market has received very little attention. Namely, in some practical problem, preferences provided by agents are maybe incomplete scores, and the existing studies seldom consider solving this kind problem. Therefore, how to solve the two-sided matching problem with incomplete scores is a valuable research topic. This is the motivation of this study.

The remainder of this paper is arranged as follows: Section 2 formulates the two-sided matching problem under incomplete score environment. Section 3 proposes a method to solve the two-sided matching problem with incomplete scores. Section 4 gives an example to show the use of the proposed method. Section 5 summarizes the main features of the proposed method.

2. The Problem

This paper considers the two-sided matching problems where preferences given by agents are in the format of incomplete scores. The notation description is given as follows.

Let $P=\{P_1,P_2,\ldots,P_m\}$ $(m\geq 2)$ be the set of agents of side P, where P_i denotes the i th agent of side P; Let $Q=\{Q_1,Q_2,\ldots,Q_n\}$ $(m\leq n)$ be the set of agents of side Q, where Q_j denotes the j th agent of side Q. Let $S_E=\{s_1, s_2, \ldots, s_h, \varphi\}$ be the extended set of scores. Let $S_P=[s_{ij}^P]_{m\times n}$ be the incomplete score matrix from side P to Q, where s_{ij}^P denotes the score preference for agent P_i towards Q_j, $s_{ij}^P\in S_E$. Here, $s_{ij}^P=\varphi$ denote that the preference of agent P_i towards Q_j doesn't exist. Let $S_Q=[s_{ij}^Q]_{m\times n}$ be the incomplete score matrix from side Q to P, where s_{ij}^Q denotes the score preference of agent Q_j towards P_i, $s_{ij}^Q\in S_E$. Here, $s_{ij}^Q=\varphi$ denote that the preference of agent Q_j towards P_i doesn't exist.

Remark 1. In different actual problems, the expressions of set S_E may be different. For example, S_E ={s_1=1(very unsatisfied), s_2=3(unsatisfied), s_3=5(moderate), s_4=7(satisfied), s_5=9(very satisfied), φ }, and S_E ={s_1=1(absolute poor), s_2=3(very poor), s_3=4(poor), s_4=5(moderate), s_5=6(good), s_6=7(very good), s_7=9(complete good), φ }, etc.

Remark 2. A two-sided matching is a one-to-one mapping $\mu: P\bigcup Q\to P\bigcup Q$ [9, 10] such that (i) $\mu(P_i)\in Q$, (ii) $\mu(Q_j)\in P\bigcup\{Q_j\}$, (iii) $\mu(P_i)=Q_k$ iff $\mu(Q_k)=P_i$. Here $\mu(P_i)=Q_k$ denotes that P_i and Q_k are matched with each other. $\mu(Q_j)=Q_j$ denotes that Q_j is not matched.

The problem concerned in this paper is how to determine the reasonable matching alternative based on incomplete score matrixes S_P and S_Q .

3. The Method

According to the meaning of the extended score set S_E, we know that the greater the score is, the higher the satisfaction degree is. For simplicity, the satisfaction degrees of one agent over another are in interval [0, 1]. Let α_{ij}^P be satisfaction degree of agent P_i over Q_j, α_{ij}^Q be the satisfaction degree of agent Q_j over P_i, then α_{ij}^P and α_{ij}^Q are calculated by

$$\alpha_{ij}^P=\begin{cases}1\big/\left(s_h+s_1-s_{ij}^P\right), & s_{ij}^P\neq\varphi,\\ \varphi, & s_{ij}^P=\varphi,\end{cases}\quad i\in M\ ,\ j\in N \tag{1}$$

$$\alpha_{ij}^Q=\begin{cases}1\big/\left(s_h+s_1-s_{ij}^Q\right), & s_{ij}^Q\neq\varphi,\\ \varphi, & s_{ij}^Q=\varphi,\end{cases}\quad i\in M\ ,\ j\in N \tag{2}$$

Here $M = \{1, 2, \ldots, m\}$, $N = \{1, 2, \ldots, n\}$, and $\alpha_{ij}^{P} = \varphi$ and $\alpha_{ij}^{Q} = \varphi$ denote that satisfaction degrees α_{ij}^{P} and α_{ij}^{Q} don't exist. By Eqs. (1) and (2), incomplete score matrixes $S_P = [s_{ij}^{P}]_{m\times n}$ and $S_Q = [s_{ij}^{Q}]_{m\times n}$ can be transformed into incomplete satisfaction degree matrixes $\Phi_P = [\alpha_{ij}^{P}]_{m\times n}$ and $\Phi_Q = [\alpha_{ij}^{Q}]_{m\times n}$.

Let x_{ij} be an 0-1 variable, where $x_{ij} = \begin{cases} 1, & \mu(A_i) = B_j \\ 0, & \mu(A_i) \neq B_j \end{cases}$. To maximize the satisfaction degree of each agent, the following multi-objective optimization model (3) considering the matching constraints can be established:

$$\max \quad Z_{P_i} = \sum_{j=1}^{n} \alpha_{ij}^{P} x_{ij}, \ i \in M \tag{3a}$$

$$\max \quad Z_{Q_j} = \sum_{i=1}^{m} \alpha_{ij}^{Q} x_{ij}, \ j \in N \tag{3b}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} x_{ij} \leq 1, \ i \in M \tag{3c}$$

$$\sum_{i=1}^{m} x_{ij} \leq 1, \ j \in N \tag{3d}$$

$$x_{ij} \in \{0, 1\}, \ i \in M, \ j \in N \tag{3e}$$

In general, each agent of one side has equal priority. In this case, model (3) can be further transformed into the following bi-objective optimization model (4):

$$\max \quad Z_P = \sum_{i=1}^{m}\sum_{j=1}^{n} \alpha_{ij}^{P} x_{ij} \tag{4a}$$

$$\max \quad Z_Q = \sum_{i=1}^{m}\sum_{j=1}^{n} \alpha_{ij}^{Q} x_{ij} \tag{4b}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} x_{ij} \leq 1, \ i \in M \tag{4c}$$

$$\sum_{i=1}^{m} x_{ij} \leq 1, \ j \in N \tag{4d}$$

$$x_{ij} \in \{0, 1\}, \ i \in M, \ j \in N \tag{4e}$$

In order to solve model (4), the linear weighted method is used. Let w_P and w_Q be the weight of objective functions Z_P and Z_Q respectively, such that $0 < w_P, w_Q < 1$, $w_P + w_Q = 1$, then model (4) is transformed into the single-objective optimization model (5):

$$\max \quad Z = \sum_{i=1}^{m}\sum_{j=1}^{n} \alpha_{ij} x_{ij} \tag{5a}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} x_{ij} \leq 1, \ i \in M \tag{5b}$$

$$\sum_{i=1}^{m} x_{ij} \leq 1, \ j \in N \tag{5c}$$

$$x_{ij} \in \{0,1\},\ i \in M,\ j \in N \tag{5d}$$

where $\alpha_{ij} = w_P \alpha_{ij}^P + w_Q \alpha_{ij}^Q$.

Remark 3. In the process of matching, if the statuses of two-sided agents are the same, then $w_P = w_Q$, otherwise $w_P \neq w_Q$.

Remark 4. In the process of solving model (5), if $\alpha_{i_1 j_1}^P = \varphi$ or $\alpha_{i_1 j_1}^Q = \varphi$, then $\varphi = -K$, where K is a sufficiently large positive number.

In sum, an algorithm and its steps are provided as follows:

Step 1. Transform incomplete score matrixes $S_P = [s_{ij}^P]_{m\times n}$ and $S_Q = [s_{ij}^Q]_{m\times n}$ into incomplete satisfaction degree matrixes $\Phi_P = [\alpha_{ij}^P]_{m\times n}$ and $\Phi_Q = [\alpha_{ij}^Q]_{m\times n}$ by Eqs. (1) and (2).

Step 2. Establish the multiple-objective optimization model (3) according to incomplete satisfaction degree matrices $\Phi_P = [\alpha_{ij}^P]_{m\times n}$ and $\Phi_Q = [\alpha_{ij}^Q]_{m\times n}$.

Step 3. Transform model (3) into model (5) by using the linear weighted method.

Step 4. Determine the matching alternative by solving model (5).

4. The Example

In this section, an example is used to illustrate the potential application of the proposed method. Pinnacle Technology Company is a fast growing software company in Shanghai. Pinnacle Technology Company plans to hire staffs in four positions ($P_1, P_2, \ldots, P_4$). After preliminary screening, six applicants ($Q_1, Q_2, \ldots, Q_6$) enter into the final decision. Each position is held by an applicant, and each applicant is assigned to one position. The experts from four departments evaluate six applicants from five perspectives: personality, creative ability, foreign language, previous experience, and human relationship skill. The applicants evaluate four positions from four perspectives: salary and welfare, development space, work environment, and market prospect. Furthermore, assume the extended score set S_E ={s_1=1(complete unsatisfied), s_2=3(very unsatisfied), s_3=5(moderate), s_4=7(very satisfied), s_5=9(complete satisfied), φ }. The incomplete score matrixes $S_P = [s_{ij}^P]_{4\times 6}$ and $S_Q = [s_{ij}^Q]_{4\times 6}$ are shown as follows.

$$S_P = \begin{bmatrix} s_3 & s_1 & s_4 & s_5 & s_2 & \varphi \\ s_1 & s_4 & \varphi & s_3 & s_5 & s_2 \\ s_5 & \varphi & s_2 & s_4 & s_1 & s_3 \\ \varphi & s_5 & s_3 & s_2 & s_4 & s_1 \end{bmatrix}$$

$$S_Q = \begin{bmatrix} s_2 & s_3 & s_2 & s_1 & s_4 & \varphi \\ s_3 & s_4 & s_3 & s_5 & s_3 & s_4 \\ s_1 & s_1 & \varphi & s_2 & s_1 & s_2 \\ s_5 & s_2 & s_4 & s_4 & s_2 & s_5 \end{bmatrix}$$

To obtain the reasonable matching alternative, a brief decision process is given below.

Firstly, according to incomplete score matrixes S_P and S_Q, incomplete satisfaction degrees matrixes $\Phi_P = [\alpha_{ij}^P]_{4\times 6}$ and $\Phi_Q = [\alpha_{ij}^Q]_{4\times 6}$ are built by Eqs. (1) and (2). Based on incomplete satisfaction degree matrices Φ_P and Φ_Q, model (3) is built. Suppose $w_A = 0.45$, $w_B = 0.55$, then by using linear weighted method, model

(3) is transformed into model (5), where coefficient matrix $\mathrm{A}_C = [\alpha_{ij}]_{4\times 6} = [0.45\alpha_{ij}^P + 0.55\alpha_{ij}^Q]_{4\times 6}$ is given as follows.

$$\mathrm{A}_C = \begin{bmatrix} 0.1686 & 0.16 & 0.2286 & 0.5111 & 0.2476 & \varphi \\ 0.16 & 0.3333 & \varphi & 0.64 & 0.56 & 0.2476 \\ 0.5111 & \varphi & \varphi & 0.2286 & 0.1111 & 0.1686 \\ \varphi & 0.5286 & 0.2733 & 0.2476 & 0.2286 & 0.6 \end{bmatrix}$$

By solving model (5), the unique optimal solution is obtained, i.e.,

$$X^* = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus, the matching alternative $\mu*$ is obtained, i.e., $\mu* = \{(P_1, Q_4), (P_2, Q_5), (P_3, Q_1), (P_4, Q_6), (Q_2, Q_2), (Q_3, Q_3)\}$.

CONCLUSION

This paper presents a new method for solving the two-sided matching problem with incomplete scores. The formula of satisfaction degree is given. Then, a multi-objective optimization model by maximizing the satisfaction degree of each agent is set up. By solving the model, the matching alternative is obtained. Comparing with the existing methods, the proposed method has two distinct characteristics as discussed below. First, the satisfaction degrees of agents are considered. This is sometime absent in the existing methods. Second, the proposed method is theoretically sound and computationally simple which provides a new way to solve the two-sided matching problem with incomplete scores and can be adopted for practical use.

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