



The triple-parameters stochastic dynamics of ecological tourism system

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ABSTRACT

The visitor education programmes are the most effective and light handed management strategies to address the visitation-related resource and culture impacts in the protected areas. The triple-parameters stochastic dynamics of ecological tourism system is proposed to study the efficacy of the visitor education intervention. The model assumes that the intrinsic increasing rate of stakeholders, the immigration rate of stakeholders outside the system, the emigration rate of stakeholders from the inner system are both stochastically perturbed by the visitor education intervention. The simulations result shows that the ecological tourism system develops steadily and sustainably, when employing effective visitor education intervention programmes.

Keywords: Visitor Education Intervention, Ecological Tourism System, Recreation Impacts

INTRODUCTION

The fast expanding visitation to some protected areas in China severely challenges the protection of the environment and precious natural resource, cultural resource and social conditions. In the peak of tourist season, a lot of scenic spots are in a state of overload reception. The rapid increase of scenic tourist causes congestion, management and service quality drop, potential safety hazard are outstanding, a sharp increase in carbon dioxide emissions serious environmental damage [1]. United state was also facing the same situation in 1970s, and the land managers of protected area managers, tourism providers and other stakeholders commonly employ regulations, site management to address resource and social impacts by fast expanding visitation [2]. Visitor education is designed to persuade visitors to adopt low-impact practices. And it is considered a more appropriate, light-handed and indirect management response to reduce resource impacts or improve visitor experiences [3]. And many visitor education programmes such as Leave No Trace, Code of conduct, Environmental Guidelines for Tourists have already been applying in many countries [4]. Many researches has being done in US and Australia, Canada, including discussions of education messages content, delivery, audience characteristics and theoretical grounding, the efficacy of educational efforts and so on [5-6].

However, the most of researches in references focus on experiential results based on some visitor education interventional experiment data. Few researchers tried to construct mathematical model to describe the dynamics of the tourism system under visitor education. The ecological tourism system contains all the natural resource including air, rocks, mountains, soils, trees, wildlife, etc. and all the stakeholders in visitation destination. The stakeholders in ecotourism destinations can't survive without the support from one another, and they must unite to a population in a certain space [7]. It is valuable to analysis the dynamic property of the ecological tourism system under the visitor educational intervention. Wei, D. and Wen, S. proposed an ordinary differential equation and stochastic Dynamics to study the efficacy of the visitor education intervention [8]. Their model only considered one stochastic parameter perturbed by visitor education intervention. In fact, the simulation results of their model show the system is unstable or

chaos at some small interval time. It is valuable try to propose a model to study whether the system can avoid the chaos situation or not. This paper assumes triple-parameters of the system are perturbed by visitor education intervention.

The Triple-Parameters Stochastic Dynamics of Ecological Tourism System

$x(t)$ is defined as stakeholders in an ecological tourism system at time t . Stakeholders in the ecological system can move into the system freely. And stakeholders can leave the system freely when they finished their activities or business. Firstly, it is necessary to propose the ordinary differential equation of ecological tourism without visitor educational intervention

$$\frac{dx}{dt} = x \left(\alpha + \beta - \gamma - \frac{1}{K}x \right) \quad (1)$$

where the parameters α is the intrinsic increasing rate of stakeholders within the ecological tourism system. The parameter β is the immigration rate of stakeholders from outside of the system. The parameter γ is the emigration rate of stakeholders from the inner system. The parameter K is the maximum carrying capacity of the ecological tourism system.

It is not difficult to prove that the solution of Equation (1) cannot avoid explosion when giving large initial value x_0 . The population of stakeholder $x(t)$ must be positive value on time $t \geq 0$. Wei,D. and Wen,S. in 2013 obtained the solution of Equation (1) as following

$$x(t) = \frac{\alpha + \beta - \gamma}{-\frac{1}{K} + e^{-(\alpha+\beta-\gamma)t}(\alpha + \beta - \gamma - \frac{1}{K}x_0)/x_0}$$

where the parameters of $\alpha, \beta, \gamma \in (0,1), K \in R^+$. But it is a local the solution, because $x(t)$ explodes to infinity when $t \rightarrow T$

$$T = -\frac{1}{\alpha + \beta - \gamma} \log \frac{-\frac{1}{K}x_0}{\left(\alpha + \beta - \gamma - \frac{1}{K}x_0\right)}$$

In another words, the solution of Equation (1) $x(t)$ explodes to infinity in a finite time T . It means that the ecological tourism system is unstable and collapse easily. The local solution of Equation (1) $x(t)$ shows that the ecological tourism system without visitor education cannot develop sustainably. The following simulation can show the result clearly. Fig.1 shows that the ordinary differential equation of ecological tourism system without visitor educational intervention explodes at different time with different parameters. Table.1 shows the explosion time of the solutions of Equation (1) when the parameters are different.

Table.1 The solution explosion time t with different parameters

x_0	α	β	γ	K	Explosion time t
30	0.15	0.12	0.08	1000	8.90
40	0.12	0.05	0.07	1500	9.95
50	0.14	0.08	0.06	1200	7.89

Because the ecological tourism system without any intervention is unstable, it is necessary to find the most effective way to suppress the explosion of the system. It is assumed that the ecological tourism system intervened by visitor education. The intrinsic increasing rate of stakeholders within the ecological tourism system is stochastically perturbed with

$$\alpha \rightarrow \alpha + \varepsilon\omega(t)$$

Hence, the stochastic dynamics of ecological tourism system with one stochastic parameter can be denoted as following

$$dx = x \left(\alpha + \beta - \gamma - \frac{1}{K}x \right) dt + \varepsilon x^2 d\omega(t) \quad (2)$$

Wei,D. and Wen,S. in 2013 proved that the stochastic dynamics of ecological tourism system can effectively suppress the explosion of the solution of Equation (1) in a finite time with any initial value x_0 . So the ecological tourism system visitor education programme can be more stable, safe and profitable. Model (2) never explodes at any time with same

parameters shown in Fig.1.

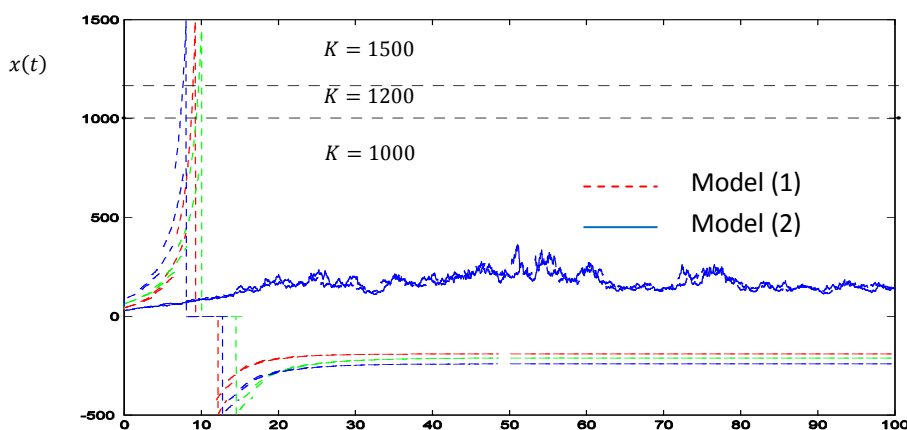


Fig.1 The simulations of Model (1) and Model (2) with different parameters

It is clearly to see that Model (2) can effectively suppress the explosion of the system (1), but the system (2) is still unstable or chaos at some small interval time. The peak value of stakeholders $x(t)$ is larger than the maximum capacity. Although the peak value returns to normal in a short time, it is obvious to see that system (2) is improper to describe the system at sometime. Hence it is valuable to improve Model (2) to avoid the chaos situation. The stochastic dynamics of ecological tourism system (2) only assumed the intrinsic increasing rate of stakeholders is stochastically perturbed by the visitor education intervention. In fact, the immigration rate of stakeholders from outside of the system and the emigration rate of stakeholders from the inner system are also stochastically perturbed by the visitor education intervention. Hence, the triple-parameters stochastically dynamics of ecological tourism system is proposed to study the efficacy of the visitor educational programmes. The immigration rate of stakeholders β may be larger than the acceptable level without visitor education. And β stochastically fluctuates from time to time. β can be considered as a Brownian Movement. Meanwhile, the emigration rate of stakeholders γ may be lower than acceptable level without visitor education. The immigration rate of stakeholders γ also fluctuates according to the effects of visitor education intervention. γ can be also considered as a Brownian Movement.

Hence, it can suppose that the parameters α , β , γ are both stochastically perturbed with

$$\alpha \rightarrow \alpha + \varepsilon_1 \omega_1(t), \beta \rightarrow \beta + \varepsilon_2 \omega_2(t), \gamma \rightarrow \gamma + \varepsilon_3 \omega_3(t).$$

$\varepsilon_1, \varepsilon_2, \varepsilon_3$ are the intervention level of the intrinsic increasing rate of stakeholders, the immigration rate of stakeholders from outside of the system, the emigration rate of stakeholders from the inner system induced by visitor education respectively. $\omega_1, \omega_2, \omega_3$ are the Brownian movements. These Brownian movement can cause the intrinsic increasing rate of stakeholders, the immigration rate of stakeholders from outside of the system, the emigration rate of stakeholders from the inner system change stochastically.

Then the triple-parameters stochastic dynamics of ecological tourism system perturbed by visitor educational intervention can be described as following

$$dx = x \left(\alpha + \beta - \gamma - \frac{1}{K} x \right) dt + x^2 (\varepsilon_1 d\omega_1(t) + \varepsilon_2 d\omega_2(t) + \varepsilon_3 d\omega_3(t)) \quad (3)$$

RESULTS AND DISCUSSION

It is necessary to define a probability space. With the same definition of the reference paper, $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathcal{P})$ is a complete probability space with a filtration $\{\mathcal{F}_t\}_{t \geq 0}$. $\{\mathcal{F}_t\}_{t \geq 0}$ satisfies the usual condition that it is right continuous. \mathcal{F}_0 contains all p-null sets [9]. $\omega_i(t) \neq 0, (i = 1, \dots, 3)$ is a multi-dimensional Brownian motion defined on the proceeding probability space. The following two results can be obtained by stochastic differential theory and Doob's martingale convergence theorem [10].

Result 1: For any system parameters $\alpha, \beta, \gamma, K \in R^+$ and any given initial value $x_0 \in R^+$, if $\varepsilon_i \neq 0, (i = 1, \dots, 3)$ then there is a unique value $x(t)$ to Equation (3) on $t \geq 0$ and the solution is positive with probability one, namely $x(t) > 0$ when $t \geq 0$ almost surely.

It is useful to give the definition of stochastically ultimately bounded. For any initial value $x_0 > 0$, Equation (3) is said to be stochastically ultimately bounded with probability one, when the solution of Equation (3) $x(t, x_0)$ has the property that

$$\lim_{t \rightarrow \infty} \inf x(t, x_0) > 0 \quad (4)$$

The following result shows the condition of Equation (3) when it is stochastically ultimate bounded.

Result 2: If the coefficients of Equation (3) satisfy the conditions as following

$$K < 1/(\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2) \quad (5)$$

$$\frac{4}{K^2} - \frac{6}{K} + 4(\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2) + (\alpha + \beta - \gamma)^2 \leq 0 \quad (6)$$

then the solution of Equation (3) $x(t, x_0)$ is stochastically ultimate bounded with probability one. Hence, given any initial value $x_0 \in R^+$, if the conditions (5) and (6) are satisfied, then the solution of Equation (3) has the property that

$$\lim_{t \rightarrow \infty} \sup x(t, x_0) < \infty \quad (7)$$

Inequality (7) shows that the supremum of the solution of Equation (3) is not infinite.

Result 1 shows that the stochastic dynamics of ecological tourism system (2) never explode with any initial value x_0 . In another words, although there are large amount visitors in public vocation, the visitation negative impacts in protected areas will be minimizing by the high efficient low impact visitor education programmes. Result 2 shows that the solution of Equation (3) $x(t)$ is varying in the interval of $[\lim_{t \rightarrow \infty} \inf x(t), \lim_{t \rightarrow \infty} \sup x(t)]$. Result 2 also shows that the population of stakeholder under visitor educational can be retained in a safety interval intervention.

The numerical solutions of Equation (3) with different parameters can be obtained by Euler-Maruyama method [11]. The following simulation data and figures can present the Model (3) has some improvements comparing with Model (2). Result 1 shows that there is unique positive solution of Equation (3). Result 2 shows that the solution of Equation (3) is not infinite. In another words, the solution of Equation (3) never explode. The simulation data confirms that the two results are true. Fig. 2 shows that Model (3) can also effectively suppress the explosion of Model (1) with same parameters. It is necessary to give definition of the chaos situation as following. If the peak value of visitors and other stakeholders is over the maximum capacity of ecological tourism system, the system is under chaos situation. In fact, the stochastic dynamics of ecological tourism system (2) cannot avoid some chaos situations. For example, Figure 2 shows that Model (2) has five times of chaos situation when $\alpha = 0.25, \beta = 0.09, \gamma = 0.06, K = 1000, x_0 = 900$. Table.2 shows that the five chaos situations appear at $t = 12, 13, 14, 4728, 4729$ and peak value of stakeholders are 1054, 1058, 1114, 1075, 1136. Table.2 also shows that Model (2) has 2 times of chaos situation when $x_0 = 30$. And 2 times of chaos situation appear when $x_0 = 800$. The similar results Table.3 can be seen from when $\alpha = 0.15, \beta = 0.07, \gamma = 0.04, K = 1000$. Table.1 and Table.2 show that chaos situations is getting larger when given larger initial value x_0 .

Fortunately, Model (3) can keep the peak value of visitors and other stakeholders under the maximum capacity of ecological tourism system comparing with Model (2). In another words, Model (3) can effectively avoid the chaos situations. For example, it is interesting to see that the chaos situations disappear in model as shown in Fig.2. In fact, Model (3) can reduce the times of chaos situation obviously. Table.2 shows that none chaos situations appear in Model (3) with the same parameters $\alpha = 0.25, \beta = 0.09, \gamma = 0.06, K = 1000, x_0 = 900$. And Table.3 also shows none chaos situations appear in Model (3) with the parameters $\alpha = 0.15, \beta = 0.07, \gamma = 0.04, K = 1000$.

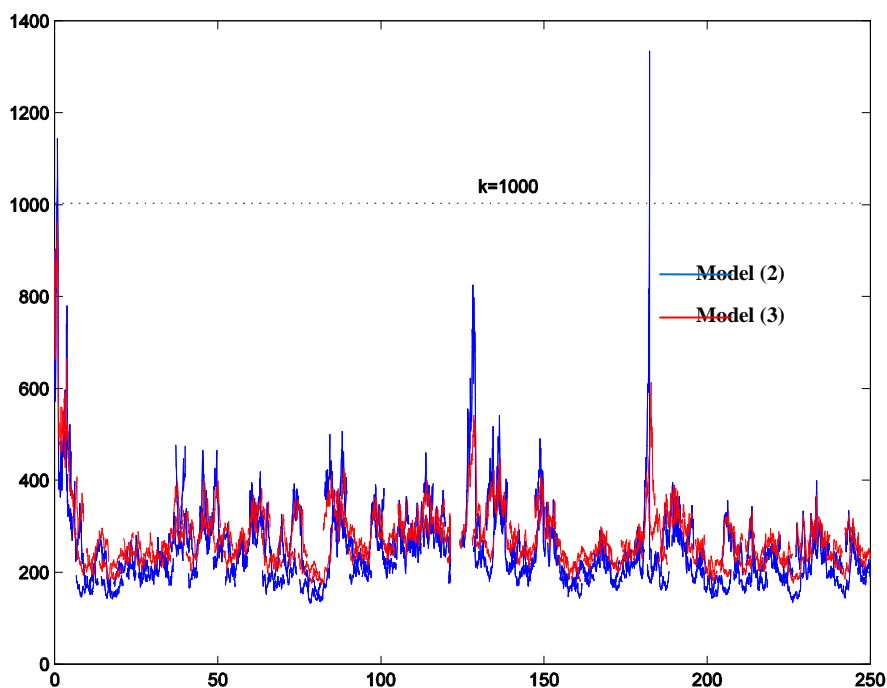


Fig.2 The simulations of Model (2) and Model (3), when $\alpha = 0.25, \beta = 0.09, \gamma = 0.06, K = 1000, x_0 = 900$

It is no difficult to find from the simulations, that Model (2) is not stable in some chaotic situation, so it is necessary to develop a more stable model. Comparing with Model (2), Model (3) assumes that three parameters are perturbed by continuous visitor educational intervention. Model (3) can keep the property of Model (2) that is avoiding the explosion of the solution. Furthermore, Model (3) has much more stable properties as shown in the simulations. Model (3) can catch the properties that effective continuous visitor educational intervention is affecting all the parameters of the ecological tourism system. But Model (2) only consider one parameter is intervened by the visitor education. Hence, in order to keep the ecological tourism system developing sustainably, the governments and parks managers should try to involve all of the stakeholders including the visitation product sellers group and the special interest group, etc., in the visitor education programmes. Because all of stakeholders should finish their visitation related activities normally and orderly. In that case the ecological tourism system can develop sustainably.

Table.2 the over maximum capacity of ecological tourism system times ($x(t) > 1000$)

$\alpha = 0.25, \beta = 0.09, \gamma = 0.06, K = 1000$				
	Model (2)			Model (3)
x_0	t	$x(t)$	Over maximum points	No over maximum capacity points
30	4728	1075	2 points	
	4729	1336		
800	14	1030	3 points	
	4728	1075		
900	4729	1336	5 points	
	12	1054		
	13	1058		
	14	1144		
	4728	1075		
	4729	1336		

Table.3 the over maximum capacity of ecological tourism system times ($x(t) > 1000$)

$\alpha = 0.15, \beta = 0.07, \gamma = 0.04, K = 1000$				
	Model (2)			Model (3)
x_0	t	$x(t)$	Over maximum points	No over maximum capacity points
30			No points	
850	14	1017	1 points	
950	13	1038	2 points	
	14	1117		

CONCLUSION

The stochastic dynamics of ecological tourism system (2) is not stable in some chaotic situation. Hence, the triple-parameters stochastically dynamics of ecological tourism system is proposed to improve the performance of the system. The theoretical results show that the solutions of the triple-parameters stochastic dynamics of ecological tourism system is varying in a finite interval. The simulations of the triple-parameters stochastic dynamics of ecological tourism system shows that the populations of the system retain in an acceptable level. It is easy to find from the simulations that the triple-parameters stochastic dynamics of ecological tourism system can almost get rid of the chaos situations. So the triple-parameters stochastic dynamics can describe the ecological tourism system with visitor education intervention much better than system (2). It can conclude that ecological tourism system with visitor education intervention can develop sustainably. In another words, the visitor education programmes can keep sustainable opportunities for high quality visitor experiences while avoiding or minimizing associated negative impacts to protected area resources although there is very large amount of tourists in a short time.

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