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**Research Article**

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# The transformation relations between the image relative orientation and the fundamental matrix

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**ABSTRACT**

*According to the internal consistency of the relative orientation and the epipolar geometry, this paper deeply analysis and discuss the relationship between relative orientation and the fundamental matrix based on the number of the relative orientation, degrees of freedom of the fundamental matrix and the corresponding relations between the parameter of them. Experimental results of images show that the proposed approach is practicable, gives great inspiration to the understanding of the mathematical principle about them, and the result of the Fundamental Matrix can convert to the result of relative orientation. After get rid of the error matching points, method of calculating the relative orientation through Fundamental Matrix can get higher precision results.*

**Key words:** Computer vision; Relative orientation; Fundamental matrix

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## INTRODUCTION

Photogrammetry is an important branch of survey, and it is the science of making measurement from images taken by camera so that object's location, shape, size, and its attitude and movement, as well as other geometrical information in three dimensions shall be determined. The development of photogrammetry has passed through three phases: from analog photogrammetry, analytical photogrammetry to digital photogrammetry. Computer vision is the science of study how to make the computer perceiving, that is to say camera and computer are used to recognize, track and measure objectives instead of human vision. Furtherly, computer vision can process the graphics which will be easily perceived by human vision or instruments. Academician Zhang Zuxun has pointed out that though there are difference between digital photogrammetry and computer vision, the intersection between photogrammetry and computer vision is getting more and more important along with the development of digital close range photogrammetry and the demand of computer vision of digital close range photogrammetry pattern. It seems that a new branch, computer vision of photogrammetry[1], shall be formed in the field of computer vision under the intersection between photogrammetry and computer vision.

The conventional process of Photogrammetry in area topographic surveying and mapping include: recognizing relative orientation of images, model connection, route line connection, aerial triangulation and block adjustment to determine the exterior orientation elements of each image and the three-dimensional information of the measuring points[2,3]. Therefore, recognizing the relative orientation of images is the key step of all processes. Based on stereopair constituted by two images, computer vision usually could simulate human eyes to do relevant study. Fundamental matrix is theory of algebras of epipolar geometry between stereo points, and it is the foundation of image matching, camera calibration, 3D reconstruction, as well as other related study[4-7]. Essentially, it is completely equivalent to the relative orientation between the two images. Mr. Shan and others have deduced and proved relationship between them[8], Mr. Zhang and others have proposed a method of Direct Relative Orientation (RLT) based on and using four independent constraint conditions of fundamental matrix[9]. Mr. Pan and others have proposed a method for solving the equation of the closed relative orientation [10].

Based on these comprehensive studies, we can see there is a close relationship between relative orientation of photogrammetry and fundamental matrix of computer vision. This paper has made further study based on these foundations, and this paper has deeply analyzed and discussed the internal relation between numbers of relative orientation and degree of fundamental matrix freedom, as well as transition relation of their parameters between them. Through the experimental verification of real images, the content of this paper has been proved. And, in this paper, Robust method has been used to solve relative orientation of aerial images, which can preferably remove abnormal data in matching point set and can effectively restrain accidental errors. The high precision of calculation result in the paper has illustrated the meaning of study.

## CORRESPONDING CONVERSION AND RELATION DERIVATION BETWEEN MATHEMATIC MODELS

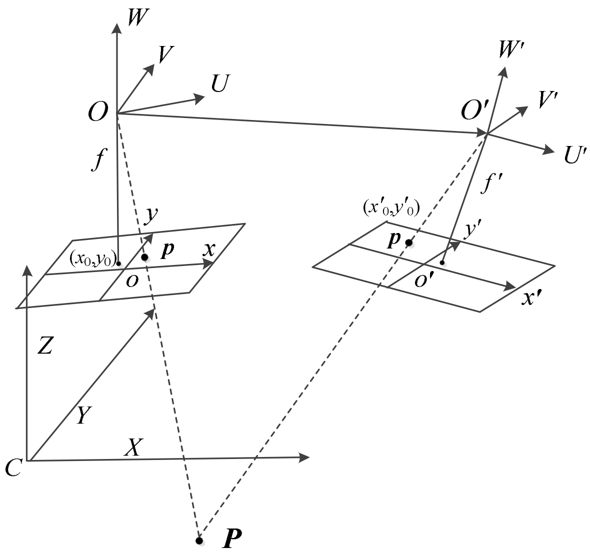


**Figure1: Geometry relationship between stereo images**

As shown in Figure 1, the world coordinate system is ***C-XYZ***, the photo plane-coordinate system of left photo image is ***o-xy***, and image space coordinate system is ***O-UVW***. Then, the photo plane-coordinate of ***P***, the three dimensional point is ***m***, and its image space coordinate is $\tilde{\boldsymbol{m}}$, as well as its space auxiliary coordinate is $\bar{\boldsymbol{m}}$, And we can get following relationships:

$$\bar{\boldsymbol{m}} = \boldsymbol{R}\tilde{\boldsymbol{m}} = \boldsymbol{R}\boldsymbol{\Omega}\boldsymbol{m} \tag{1}$$

$$\boldsymbol{m} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}, \boldsymbol{\Omega} = \begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & -f \end{bmatrix}, \boldsymbol{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \tag{2}$$

In the formula, $\boldsymbol{\Omega}$ is the matrix about elements of interior orientation in the three-parameter, and $(x_0,y_0,f)$ are interior orientation elements of camera, and ***R*** is the deflection matrix between image space coordinate system and space auxiliary coordinate, and the deflection angle between the axes can be set as $\alpha, \beta,\gamma$ (There are a lot of Angle conversion systems in Photogrammetry, and here we just use one of them. And the systems can be converted to each other[4] ), so ***R*** can be decomposed into following formula:

$$\boldsymbol{R} = \boldsymbol{R}_\alpha \boldsymbol{R}_\beta \boldsymbol{R}_\gamma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

Similarly, the photo plane-coordinate on the right image of ***P*** is $\boldsymbol{m}'$, and its image space coordinate is $\tilde{\boldsymbol{m}}'$, as well as its space auxiliary coordinate is $\bar{\boldsymbol{m}}'$, then we can get following formula.

$$\bar{m}' = R'\tilde{m}' = R'\Omega' m' \tag{4}$$

$$m' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}, \Omega' = \begin{bmatrix} 1 & 0 & -x'_0 \\ 0 & 1 & -y'_0 \\ 0 & 0 & -f' \end{bmatrix}, R' = \begin{bmatrix} r'_{11} & r'_{12} & r'_{13} \\ r'_{21} & r'_{22} & r'_{23} \\ r'_{31} & r'_{32} & r'_{33} \end{bmatrix} \tag{5}$$

Given: The director vector of two photo sites is ***B***, on the basis of two photo sites, two plane-image sites and one object point, then we can get following formula[8]:

$$\bar{m}^{\mathrm{T}}(B \times \bar{m}') = \bar{m}^{\mathrm{T}}[B]_\times \bar{m}' = 0 \tag{6}$$

Here, we plug formula (1) and formula (4) into formula (6), we can get following formula.

$$m^{\mathrm{T}}\Omega^{\mathrm{T}}R^{\mathrm{T}}[B]_\times R'\Omega' m' = 0 \tag{7}$$

So the fundamental matrix ***F*** is equal to $\Omega^{\mathrm{T}}R^{\mathrm{T}}[B]_\times R'\Omega'$ in the case of lacking one scaling factor, and also we overlap the world coordinate system and image space coordinate system together( which means ***R***=***I***), then we can get following formula:

$$F = \begin{bmatrix} f_1 & f_4 & f_7 \\ f_2 & f_5 & f_8 \\ f_3 & f_6 & f_9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & -f \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} 0 & -b_z & b_y \\ b_z & 0 & -b_x \\ -b_y & b_x & 0 \end{bmatrix} \begin{bmatrix} r'_{11} & r'_{12} & r'_{13} \\ r'_{21} & r'_{22} & r'_{23} \\ r'_{31} & r'_{32} & r'_{33} \end{bmatrix} \begin{bmatrix} 1 & 0 & -x'_0 \\ 0 & 1 & -y'_0 \\ 0 & 0 & -f' \end{bmatrix} \tag{8}$$

Given the image plane center is the origin of image plane coordinate system, then parameters $x_0$, $y_0$, $x'_0$ and $y'_0$ are all very close to 0. Also during calculation process of fundamental matrix, the waveform normalization for picpointed coordinate is necessary. And these four parameters can be similarly regarded as 0, prior to camera calibration, then parameters in formula (8) are ($b_x$, $b_y$, $b_z$, $\alpha'$, $\beta'$, $\gamma'$, $f$, $f'$), and the number of unknown parameters is 8. In the case of lacking one scaling factor, degrees of freedom are 7 which is the same as the freedom degrees of fundamental matrix.

In those parameters, $f$ and $f'$ are elements of interior orientation of two images, and ($b_x$, $b_y$, $b_z$, $\alpha'$, $\beta'$, $\gamma'$) are parameters of relative positions, in which three of them are range shift scale ,and three of them are rotation scale. Without regarding to scaling factor, then there are five parameters.

Given that $b_x$, is scaling factor parameter, then unknown parameters are ($b_y$, $b_z$, $\alpha'$, $\beta'$, $\gamma'$), which is consistent with the elements of relative orientation of independent relative orientation method. We plug formula (8) into formula (7), and we get following corresponding relationship between unknown parameters and elements of fundamental matrix.

$$f_1 x'x + f_2 x'y + f_3 x' + f_4 y'x + f_5 y'y + f_6 y' + f_7 x + f_8 y + f_9 = 0 \tag{9}$$

Among,

$$\begin{array}{lll} f_1 = b_y r'_{31} - b_z r'_{21} & f_2 = b_z r'_{11} - b_x r'_{31} & f_3 = f(b_y r'_{11} - b_x r'_{21}) \\ f_4 = b_y r'_{32} - b_z r'_{22} & f_5 = b_z r'_{12} - b_x r'_{32} & f_6 = f(b_y r'_{12} - b_x r'_{22}) \\ f_7 = f'(b_z r'_{23} - b_y r'_{33}) & f_8 = f'(b_x r'_{33} - b_z r'_{13}) & f_9 = ff'(b_x r'_{23} - b_y r'_{13}) \end{array} \tag{10}$$

Here, the relationships between fundamental matrix elements and camera orientation elements, relative orientations are formed. Based on formula (10), we can get the value of fundamental matrix is decided by focuses of two images and six orientation parameters of relative position relationships (3 range shifts and 3 rotation scales). In formula (10), the result value of fundamental matrix in computer vision can be converted into the result of related orientation of photogrammetry, so the conversion can be used as a bridge to help them converted into each other. In addition, the freedom degree of fundamental matrix is 7, while by deduction 2 focus, the freedom degree is changed into 5, which is consistent with numbers of relative orientation parameters, also this can approve the correctness of these formulas.

## EXPERIMENTAL VERIFICATION AND ANALYSIS

Based on above results of formulas, which are using Matlab and Visual Studio2010 as tool and images of stereo pair as experimental data, this paper has conducted verification experiment on corresponding conversion between relative orientation elements and fundamental matrix, and the conversion is used to practical experiment of stereo pairs relative orientation.

**(a) Left image** **(b) Right image**

**Figure 2 The stereo images of building**

**(a) Left Image** **(b) Right Image**

**Figure3 The stereo images of Corridor**

**Table 1 The resulting value of the relative orientation and fundamental matrix**

| | Relative orientation elements | | Fundamental matrix elements | | Fundamental Matrix | | Relative orientation elements | | Fundamental matrix elements | | Fundamental matrix |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Building | $b_x$ | 2.221/m | $f_1$ | 0.0204 | | Corridor | $b_x$ | 0.421/m | $f_1$ | 0.0003 | |
| | $b_y$ | 0.831/m | $f_2$ | 0.4192 | | | $b_y$ | 0.087/m | $f_2$ | 0.0177 | |
| | $b_z$ | 0.462/m | $f_3$ | 36.8064 | | | $b_z$ | 0.019/m | $f_3$ | 0.0019 | |
| | $\alpha'$ | 0.024/rad | $f_4$ | -0.4816 | $\begin{bmatrix} 0 & 0 \\ -1 & 0 \\ -350 & 954 \end{bmatrix}$ | | $\alpha'$ | 0.009/rad | $f_4$ | -0.0198 | $\begin{bmatrix} 149 & -990 \\ 8853 & 1905 \\ 946 & 4611 \end{bmatrix}$ |
| | $\beta'$ | 0.019/rad | $f_5$ | 0.0575 | | | $\beta'$ | 0.003/rad | $f_5$ | 0.0038 | |
| | $\gamma'$ | 0.010/rad | $f_6$ | -95.5563 | | | $\gamma'$ | 0.001/rad | $f_6$ | -0.0092 | |
| | $f$ | 43.2/mm | $f_7$ | -35.0757 | | | $f$ | 21.8/mm | $f_7$ | -0.0019 | |
| | $f'$ | 42.8/mm | $f_8$ | 95.3899 | | | $f'$ | 21.9/mm | $f_8$ | 0.0092 | |
| | | | $f_9$ | 127.7212 | | | | | $f_9$ | 0.000002 | |

## 1. Relative relationship verification

The image size of stereo pair is 3264 pixel×2448pixel, and the pixel is 60μm (shown as below picture 2 and picture 3). We can obtain characteristic points on following images, and then conduct stereo pair relative orientation and fundamental matrix calculation to verify the correctness of formula (10) result.

Firstly, the SSD characteristic point extraction algorithm[11] is used to obtain characteristic points set on stereo pair. In figure 2, there are 15 characteristic points, and 12 in figure 3. And then the independent relative orientation method is used to get the value of relative orientation elements [12]. Finally, the method of 8 point[13] is used to get the value of fundamental matrix. In table 1, the calculation result of two methods are listed.

In table 1, the value of relative orientation elements is calculated by the method of single relative orientation. And then the value of fundamental matrix elements is converted based on formula (10), while fundamental matrix is calculated by eight point method. By comparison, under the situation of lacking one scale factor, fundamental matrix of $f_1 - f_9$ calculated by factors ($b_x, b_y, b_z, \alpha', \beta', \gamma', f, f'$) is roughly equal with fundamental matrix F. Because the data in calculation is not very accurate, the above results are not totally equality, which is in line with actual calculation environment. Hereby, based on above results, the corresponding conversion between relative orientation elements and fundamental matrix elements built in this paper is correct. The conversion can be accurately achieved by formula (10).

## 2. Application and Verification

During relative orientation of aerial images, characteristic point extraction algorithm is generally used to obtain corresponding matching points，and then set the initial value of relative orientation elements. According to coplanarity condition equation and using the coordinate of matching points, the final values are calculated via iterative computation. When matching characteristic points, the points may be affected by noise and abnormal data. The noise is generally caused by accidental error which suits Gaussian error distribution and the effect can be decreased by redundant observation. The abnormal data can be divided into two categories: incorrect location and incorrect matching. Characteristic point incorrect location means the error range is more than three pixels, those points can seriously affect the accuracy of calculation results. Characteristic point incorrect matching means when building corresponding relationships between characteristic points, the incorrect relationship occurred for different kinds of reasons, this also can affect the whole process of relative orientation.

During the process of relative orientation, noise effect can be decreased by characteristic point values as much as possible. And abnormal data must be removed which is usually by the method of pick up gross errors during photogrammetry process. While, it is very complex to pick up gross errors, and there are no very clear boundaries between characteristic point incorrect location, characteristic point incorrect matching and Gaussian noise, so the whole result is often not very good.

In order to verify application value of this paper study, and via Robust method for fundamental matrix, relative orientation method has been converted. In below pictures, the image size of stereo images is 7500 pixel×5000 pixel, and the pixel is 90μm. Firstly, in the stereo images of picture 4, twenty, forty or sixty characteristic points are chosen to build relative relationships. And then, general photogrammetry relative orientation method is used to calculate relative orientation elements. Thirdly, classic Robust method for fundamental matrix is used to calculate fundamental matrix elements, the calculation result shall be converted into the result of relative orientation via the formula 10 for comparison. The comparison result is listed in Table 2.

**（a）Left Image** **（b）Right Image**

**Figure4 The stereo images of aviation**

In table 2, when characteristic points number is twenty, the accuracy between the result of relative orientation and the result converted by fundamental matrix is with little difference. When characteristic points number is forty, the result of relative orientation is not improved, yet the result of fundamental matrix is improved to some extent. When characteristic points number is sixty, the result of fundamental matrix is largely accurate than relative orientation. The reason is, during relative orientation calculation, gross error has not been removed. When there are more matching points, incorrect matching shall be induced inevitably, so the accuracy of calculation is affected. However, RANSAC calculation method of fundamental matrix can well remove abnormal data, so good accuracy result is obtained. Robust method for fundamental matrix is a reference to relative orientation calculation, and the cross study between them is very necessary.

**Table 2 The resulting value between the relative orientation and fundamental matrix**

| Classification | Number Of Points | The value of relative orientation | Error value of relative orientation | Fundamental matrix method result value | Error value in the basis matrix method |
|---|---|---|---|---|---|
| $b_y$ | | 0. 011326 | 0. 000 105 | 0. 011 416 | 0. 000 125 |
| $b_z$ | | 0. 036 394 | 0. 000 094 | 0. 036 412 | 0. 000 102 |
| $\alpha'$ | **20** | - 0. 002 108 | 0. 000 089 | - 0. 001708 | 0. 000 099 |
| $\beta'$ | | - 0. 021748 | 0. 000 307 | - 0. 030 919 | 0. 000 401 |
| $\gamma'$ | | 0. 235701 | 0. 000 216 | 0. 235703 | 0. 000 226 |
| $b_y$ | | 0. 019 486 | 0. 000 223 | 0. 010 486 | 0. 000 105 |
| $b_z$ | | 0. 036 702 | 0. 000 196 | 0. 036 202 | 0. 000 074 |
| $\alpha'$ | **40** | - 0. 000 108 | 0. 000 299 | - 0. 002208 | 0. 000 053 |
| $\beta'$ | | -0. 035701 | 0. 000 421 | -0. 016703 | 0. 000 388 |
| $\gamma'$ | | 0. 235702 | 0. 000 526 | 0. 234301 | 0. 000 221 |
| $b_y$ | | 0. 029 448 | 0. 001854 | 0. 009 246 | 0. 000 051 |
| $b_z$ | | 0. 026 662 | 0. 000 398 | 0. 032114 | 0. 000 056 |
| $\alpha'$ | **60** | - 0. 002308 | 0. 000 850 | - 0. 002 198 | 0. 000 049 |
| $\beta'$ | | - 0. 040 919 | 0. 000 917 | - 0. 022 919 | 0. 000 114 |
| $\gamma'$ | | 0. 135702 | 0. 002726 | 0. 221 702 | 0. 000 126 |

## CONCLUSION

According to internal consistency between relative orientation of photogrammetry and epipolar geometry of computer vision, this paper has deeply analyzed the essential relationship between them. From the view of relative orientation numbers and freedom degree of fundamental matrix, the relationship between them has been analyzed, also the corresponding conversion formula between relative orientation and fundamental matrix has been deducted and demonstrated. The experimental result of stereo images has presented the corresponding conversion relationship is practical and achievable. When the photo focus is available, we can use relative orientation to get the fundamental matrix of stereo images. Conversely, we can get relative orientation from fundamental matrix. In addition, according to the result of relative orientation converted by Robust method for fundamental matrix, it is useful for photogrammetry relative orientation and fundamental matrix calculation to learn from each other, and also cross development of digital photogrammetry and computer vision have been promoted.

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