



## The solving method of generalized weng model parameters based on curve fitting

Cao Jing<sup>1,2</sup>, Yu Gaoming<sup>1</sup> and Xie Yunhong<sup>1</sup>

<sup>1</sup>College of Petroleum Engineering, Yangtze University, Wuhan, Hubei, China

<sup>2</sup>School of Information and Mathematics, Yangtze University, Jingzhou, Hubei, China

### ABSTRACT

The generalized Weng model is widely used in oil production prediction. At present, there are linear iterative trial and error method, regression method and genetic algorithm, etc. Linear iterative trial and error method and regression method have uncertainty influenced by artificial factors, which likely cause bigger errors. Genetic algorithm is more complex. In this paper, we solve the generalized Weng model parameter by making use of the nonlinear curve fitting and Levenberg-Marquard iterative method, and establish iterative initial value by the method of multiple regression. Through an example, this method effectively enhances the precision of parameter calculation, and is simple in calculation. It is suitable for parameters calculation of all kinds of time quantum oil and gas reservoir.

**Keywords** generalized Weng model; linear iteration trial and error method; multiple regression; nonlinear curve fitting; Levenberg-Marquard iteration method

### INTRODUCTION

Study on the decline of oil, to development index calculation of oil field, prediction of recoverable reserves, planning and deployment of oil and gas field, and the reservoir parameter evaluation, has play a decisive role. Since Arps[1] proposed the production decline theory in 1945, domestic and foreign scholars have put forward a lot of production decline models, such as the Logistic[2] model, Weibull model[3,4], Rayleigh model[5] and HCZ model[6], etc. In China, famous geophysical experts Mr. Weng Wenbo in 1984 first proposed the Poisson cycle model (referred to as Weng model) [7] in the 《Prediction Theory Basis》 monographs. It laid a solid foundation for the prediction of oil production in China, and caused attention of domestic petroleum engineer. In 1996, Chen Yuanqian deduced the model[8], extended it to the generalized Weng model, and put forward the linear iterative trial and error method. Using this method to solve the model, because the parameter B value and the selected regression period are uncertainty, it will lead to bigger error. Therefore, this paper proposes using the nonlinear curve fitting method with higher accuracy to solve the model parameters.

### 2 Generalized Weng Model [8]

Equation of the model :

$$Q = At^B e^{-\frac{t}{C}}, \quad (1)$$

with  $Q$ —year yield,  $10^4 t/a$  (oil),  $10^8 t/a$  (gas);

$t$ —relative development time,  $a$  (year);

$A, B, C$ —Predictive model constant.

### 3 Nonlinear Curve Fitting

Assuming there are experimental data  $(x_i, y_i)$  ( $i = 1, 2, \dots, n$ ), we search function  $f(x)$  which makes the squares sum of the deviation of function value in the point  $x_i$  ( $i = 1, 2, \dots, n$ ) and the observed data minimize. Also is to search function  $f(x)$ , it makes  $\sum_{i=1}^n (f(x_i) - y_i)^2$  minimize.

Making use of nonlinear curve fitting to solve the generalized Weng model parameters, the function  $f(x)$  equal to Eq. 1:

$$f(t) = Q(t) = At^B e^{-\frac{t}{C}}. \quad (2)$$

Then we look for the optimal estimation  $\hat{A}, \hat{B}, \hat{C}$  of the parameters  $A, B, C$ , it may make the following objective function minimize:

$$F(X) = \sum_{t=1}^n g_t^2(X) = \sum_{t=1}^n (Q(t) - q_t)^2, \quad (3)$$

With  $X = [A, B, C]^T$ ,  $q_t$  is the actual yield of annual oil,  $t$  is the relative development time. This is a nonlinear multivariable optimization problem, we commonly use Newton iterative method for solving. The iterative formula of the Newton iterative method for [9]:

$$X^{(k+1)} = X^{(k)} - \nabla^2 F^{-1}(X^{(k)}) \nabla F(X^{(k)}). \quad (4)$$

In the formula:  $X^{(k)}$  is the current approximate point of optimal solution;  $X^{(k+1)}$  is the following approximate point of optimal solution;  $\nabla F(X^{(k)})$  is the gradient of  $F(X)$  in the point  $X^{(k)}$ ;

$\nabla^2 F(X^{(k)})$  is the Hesse matrix.

The Hesse matrix calculation is relatively complex, we may make use of the first derivative replacing the second derivative, make  $G(X) = [g_1(X), g_2(X), \dots, g_n(X)]^T$ , can get the iterative formula of Gauss-Newton [9]:

$$X^{(k+1)} = X^{(k)} - [J(X^{(k)})^T J(X^{(k)})]^{-1} J(X^{(k)})^T G(X^{(k)}), \quad (5)$$

with  $J(X)$  is Jacobian matrix of  $G(X)$  in the point  $X^{(k)}$ .

In the Gauss-Newton iterative method, matrix  $J(X)^T J(X)$  is sometimes ill conditioned (singular or near singular), then finding inverse matrix is difficult, even not to come out; In addition, sometimes the search direction  $p^{(k)} = -J(X^{(k)})^T G(X^{(k)})$  in the formula and gradient  $\nabla F(X^{(k)})$  of the point  $X^{(k)}$  are near orthogonal, then it can result in slow progress or false convergence. So, in this paper, we adopts Levenberg-Marquard [9] iterative method. The iterative formula for concrete:

$$X^{(k+1)} = X^{(k)} - [J(X^{(k)})^T J(X^{(k)}) + \lambda_k I]^{-1} J(X^{(k)})^T G(X^{(k)}), \quad (6)$$

with  $I$  is the unit matrix;  $\lambda_k$  is a positive real number, controls the direction of search.

Levenberg-Marquard algorithm is sensitive to the initial value of parameters, and initial value determines the accuracy of the estimated parameters. The Binary regression method for solving the generalized Weng model is put forward in the [10]. Disadvantages of this method is the regression period selected in three-dimensional space. So, if evaluators have different observation angles, the data length observed good linear relationship is different. The human factors will

impact on the precision of the results. In this paper, all the data is regressed using the binary regression method. Although the approximate value of parameters obtained has more poor accuracy, it is better than arbitrary value as the initial value of the Levenberg-Marquard algorithm.

#### 4 Application Examples

##### Example 1

Select example in literature [11]. Romanshkinian oil fields is one of the large oil fields of the former Soviet Union, in 1952 put into development. In the table 1, the actual oil production is oil production data of this oil field from 1952 to 1979.

**Table 1 Romanshkinian Oil Fields Production Data**

Year	Time [a]	Actual [ $10^4$ t/a]	Prediction		Relative error/%	Year	Time [a]	Actual [ $10^4$ t/a]	Prediction		Relative error/%
			Genetic	L-M					Genetic	L-M	
1952	1	200	25	27	86.5	1966	15	6800	6988	6967	2.45
1953	2	300	147	157	47.66	1967	16	7000	7321	7296	4.22
1954	3	500	395	414	17.2	1968	17	7600	7587	7561	0.51
1955	4	1000	769	795	20.5	1969	18	7900	7787	7762	1.74
1956	5	1400	1250	1281	8.5	1970	19	8150	7925	7901	3.05
1957	6	1900	1814	1847	2.78	1971	20	8000	8003	7983	0.21
1958	7	2400	2437	2468	2.83	1972	21	8000	8027	8011	0.13
1959	8	3050	3091	3117	2.19	1973	22	8000	8000	7989	0.13
1960	9	3800	3753	3773	0.71	1974	23	8000	7927	7923	0.96
1961	10	4400	4402	4415	0.34	1975	24	8000	7814	7817	2.28
1962	11	5000	5022	5027	0.54	1976	25	7775	7666	7677	2.03
1963	12	5600	5600	5597	0.05	1977	26	7500	7487	7506	0.08
1964	13	6040	6124	6114	1.22	1978	27	7230	7282	7309	1.09
1965	14	6600	6588	6572	0.42	1979	28	6800	7056	7091	4.27

Using genetic algorithm to solve the model parameters in literature [11]. In this paper, we firstly make use of the binary regression method in reference [10] for all data to get the approximate parameters value as the initial value of iteration method:  $A_0 = 127.75$ ,  $B_0 = 1.78$ ,  $C_0 = 16.07$ . The above initial values are brought into Levenberg-Marquard iterative method, and through programming to calculate approximate three parameter value. Fitting results are shown in table 1 and table 2. By comparison, using this method, in addition to the relative error of initial 5 data points are large, the relative error of the rest each point is controlled within 5%, and the residual squares sum than genetic algorithm is smaller, simpler calculation. The solution developed in the paper is more precise, and simpler and easier for calculation.

**Table 2 Comparison of Genetic and L-M Results**

Parameters	Genetic	L-M
$A$	28.003	30.846
$B$	2.771	2.7162
$C$	7.559	7.7494
Residuals squared sum	484164	472416

#### CONCLUSION

In this paper, we solve the parameters of the generalized Weng model by using nonlinear curve fitting and Levenberg-Marquard iterative method, and determine the initial value of iteration by using binary regression method. After example validating, the method effectively improves the precision of parameter calculation, reduces man-made factors, and the calculation is simple. This method is suitable for the model parameters of oil gas reservoir of various time periods.

#### Acknowledgements

I am very grateful for the advice and guidance of my tutor Professor Yu Gaoming during writing this article, and thank my classmates to help me.

#### REFERENCES

- [1] Arps J J. *Trans. AIME*. **1945**, 160, 228-247.
- [2] Chen Yuanqian; Hu Jianguo; Zhang Dongjie. *Xinjiang Petroleum Geology*, **1996**, 17(2), 150-155.

- 
- [3] Chen Yuanqian. *Xinjiang Petroleum Geology*,**1995**,16(3),250-255.
- [4] Chen Yuanqian;Zhao Qingfei. *Xinjiang Petroleum Geology*,**2000**,21(5),405-407.
- [5] Chen Yuanqian. *Oil and Gas Geology and Recovery Efficiency*,**2004**,1(4),39-41.
- [6] Hu Jianguo;Chen Yuanqian;Zhang Shengzong. *Acta Petrolei Sinica*,**1995**,16(1),79-86.
- [7] Weng Wenbo. Prediction theory basis, 1st Edition, Petroleum Industry Press, Beijing, **1984**;24-26.
- [8] Chen Yuanqian. *Natural Gas Industry*,**1996**,16(2),22-26.
- [9] Xue Lvzhong. Engineering optimization technology, 1st Edition, Tianjin University Press, Tianjin, **1988**;112-141.
- [10] Zhao Lin;Feng Lianyong et al. *Xinjiang Petroleum Geology*,**2009**,30(5),658-660.
- [11] Hou Jian;Qu Changxue. *Journal of the University of Petroleum (Natural Science Edition)*,**2002**,26 (3), 55-59.