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The ruin problem of dependent risk model based on copula function

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ABSTRACT

An extension to classical risk model with a dependence structure between the claim amounts and the interclaim times, which is embedded though an FGM copula, is considered. In this so-called Erlang(2) dependent risk model, the generalized Lundberg's equation and the integro-differential equation that the Gerber-Shiu discounted penalty function satisfies are derived.

Keywords: risk model, copula, Lundberg's equation, discounted penalty function

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INTRODUCTION

Recently, within the actuarial world, modern risk management techniques play a central role[8]. One of the famous problems of nonlife insurance field is the collective risk theory, which is studied through various models of the risk business of an insurance company. By introducing an insurance risk model, we could study the ruin probability, i.e., the probability that the risk business ever will below some specified value, thus take risk control of nonlife business.

In the past decade, the analysis of the well-known Sparre Andersen risk model, as an extension to the compound Poisson risk model with the interclaim time exponentially distributed, has received considerable attention. Following the introduction of the so-called Gerber-Shiu discounted penalty function by Gerber and Shiu[7] in the framework of the compound Poisson risk model, various authors have contributed to the analysis of the Sparre Andersen risk model via the study of this analytic tool. Two special cases of Sparre Andersen risk models have received extensive attention in the literature. One is the compound Poisson model, the other is the Erlang(2) risk model where the interclaim time has an Erlang(2) distribution. The Erlang(2) risk model has been investigated by Dickson and Hipp[5][6], Cheng and Tang[3], etc. They have enlarged the class of Sparre Andersen risk models for which an explicit expression for the Gerber-Shiu discounted penalty function can be found.

Although traditionally, insurance has been built on the assumption of independence and the law of large numbers has governed the determination of premiums, the increasing complexity of insurance and reinsurance products has led to increased actuarial interest in the modeling of dependent risk[4]. Classic risk models rely on an assumption of independence between the claim amounts and the interclaim times. This hypothesis simplifies the study of many quantities under such a framework, however it has proven to be inadequate and too restrictive in many cases. For example, in Nikoloulopoulos and Karlis[9], they point out that on the occurrence of a catastrophe, the total claim amount and the time elapsed since the previous catastrophe are dependent. Albrecher and Teugels[1] relax the independence assumptions by introducing an arbitrary dependence structure based on a copula for the interclaim time and the subsequent claim size. Boudreault et al.[2] consider the compound Poisson risk model with a dependence structure where the distribution of the next claim amount is defined in terms of the time elapsed since the last claim.

In this paper, we apply a dependence structure between the claim amounts and the interclaim times defined with the Farlie-Gumbel-Morgenstein (FGM) copula to the Erlang(2) risk model. In our dependent model, we derive the generalized Lundberg's equation, which is an important tool in the analysis of the ruin measures. Finally we study the Gerber-Shiu discounted penalty function and obtain the integro-differential equation for it.

ERLANG(2) RISK MODEL

Let $(\Omega, \mathcal{F}, P)$ be a complete probability space containing all objects defined in the following. The Erlang(2) risk model is defined by

$$U(t) = u + ct - \sum_{i=1}^{N(t)} X_i, t \geq 0 \quad (1)$$

where $\{N(t), t \geq 0\}$ represents the number of claims up to time t. The interclaim times $\{\theta_i, i \in \mathbb{N}^+\}$ constitute a sequence of i.i.d. non-negative random variables with probability density function (p.d.f.) $f_\theta(t)$, cumulative distribution function (c.d.f.) $F_\theta(t)$ and Laplace transform (L.T.) $f_\theta^*(s)$.

Throughout the paper, it is assumed that θ has an Erlang(2) distribution with parameter λ so that

$$f_\theta(t) = \lambda^2 t \mathrm{e}^{-\lambda t} \quad (2)$$

$$F_\theta(t) = 1 - \mathrm{e}^{-\lambda t} - \lambda t \mathrm{e}^{-\lambda t} \quad (3)$$

$$f_\theta^*(s) = E[\mathrm{e}^{-s\theta}] = (\frac{\lambda}{\lambda + s})^2 \quad (4)$$

It's easy to verify that p.d.f. of the corresponding n-th claim occurrence time $T_n = \sum_{i=1}^{n} \theta_i$ and the renewal process $\{N_t, t \geq 0\}$ are

$$f_{T_n}(t) = \frac{\lambda^{2n} t^{2n-1} \mathrm{e}^{-\lambda t}}{(2n-1)!} \quad (5)$$

and

$$P(N_t = n) = \frac{(\lambda t)^{2n}}{(2n)!} \mathrm{e}^{-\lambda t} + \frac{(\lambda t)^{2n+1}}{(2n+1)!} \mathrm{e}^{-\lambda t} \quad (6)$$

respectively. Further we could do some calculations and obtain that

$$E(N_t) = \frac{1}{2}\lambda t + \frac{1}{4}(\mathrm{e}^{-2\lambda t} - 1) \quad (7)$$

By the discussion above, it's easy to verify that the Erlang(2) process has the properties of independent increments but non-stationary increments, so does the Erlang(2) risk process (1). That is the major difference between the compound Poisson risk process and Erlang(2) risk process.

The claim amount variables $\{X_i, i \in \mathbb{N}^+\}$ are assumed to be a sequence of positive i.i.d. with p.d.f. $f_X(x)$, c.d.f. $F_X(x)$ and L.T.$f_X^*(s)$. Denote $\mu = E(X_i)$. Further more, we assume that $\{(X_i, \theta_i), i \in \mathbb{N}^+\}$ are i.i.d. random vectors with the joint p.d.f. $f_{X,\theta}(x, t)$, and the associated bivariate L.T. is

$$\begin{aligned} f_{X,\theta}^*(s_1, s_2) &= E[\mathrm{e}^{-s_1 X} \mathrm{e}^{-s_2 \theta}] \\ &= \int_0^\infty \int_0^\infty \mathrm{e}^{-s_1 x} \mathrm{e}^{-s_2 t} f_{X,\theta}(x, t) \mathrm{d}x \mathrm{d}t \end{aligned}$$

Let T denote the time to ruin, then the probability of ultimate ruin with initial surplus u is defined by

$$\psi(u) = P(T < \infty | U(0) = u) \quad (8)$$

The deficit at ruin and the surplus just prior to ruin are denoted by $|U(T)|$ and $U(T^-)$ respectively. In recent research, the expected value of the discounted penalty function introduced by Gerber and Shiu has been a central tool of ruin theory[7]. This function is given by

$$\phi_\delta(u) = E[\mathrm{e}^{-\delta T} w(U(T^-), |U(T)|) I(T < \infty) | U(0) = u] \quad (9)$$

where $w(x,y)$ is the penalty function at time of ruin for the surplus prior to ruin and the deficit at ruin, δ is a non-negative parameter and I is the indicator function. The probability of ultimate ruin $\psi(u)$ is a special case of the Gerber-Shiu penalty function with $\delta=0$ and $w(x,y)=1$ for all $x,y\in\mathbb{R}$.

DEPENDENCE STRUCTURE BASED ON COPULA FUNCTION

The dependence between the real-valued random variables X,Y is completely described by their joint c.d.f.

$$F_{X,Y}(x,y)=P(X\le x, Y\le y) \tag{10}$$

A bivariate copula is the distribution function of a random vector in $\mathbb{R}^2$ with uniform-(0,1) marginals. Alternatively a copula is any function $C:[0,1]^2\to[0,t]$ which has the following three properties:

1. $C(x,y)$ is increasing in each component x,y.
2. $C(x,1)=x, C(1,y)=y, \forall x,y\in[0,1]$.
3. For all $(x_1,y_1),(x_2,y_2)\in[0,1]^2$ with $x_1\le x_2, y_1\le y_2$ we have

$$\sum_{i=1}^{2}\sum_{j=1}^{2}(-1)^{i+j}C(x_i,y_j)\ge 0 \tag{11}$$

For any continuous bivariate distribution the representation

$$\begin{aligned}F_{X,Y}(x,y)&=P(F_X(X)\le x, F_Y(Y)\le y)\\&=C(F_X(x),F_Y(y))\end{aligned}$$

holds for a unique copula C.

Two particular copulas are comonotonic copula with $C_u(x,y)=\min(x,y)$ and countermonotonic copula with $C_l(x,y)=\max(x+y-1,0)$. For every copula the well-known Fréchet bounds apply

$$C_l(x,y)\le C(x,y)\le C_u(x,y) \tag{12}$$

In our paper, we consider the Farlie-Gumbel- Morgenstern copula, FGM copula for short,

$$C_\alpha(x,y)=xy+\alpha xy(1-x)(1-y) \tag{13}$$

where $\alpha\in[-1,1]$. The p.d.f. associate to (13) is

$$c_\alpha(x,y)=1+\alpha(1-2x)(1-2y) \tag{14}$$

When $\alpha=0$, it reduces to the independent case, where the copula trivially takes the form $C^{\perp}(x,y)=xy$.

We assume that the joint distribution of (X,θ) is defined with the FGM copula C_α. Given (13), the joint c.d.f. $F_{X,\theta}$ is defined by

$$\begin{aligned}&F_{X,\theta}(x,t)=C_\alpha(F_X(x),F_\theta(t))\\&=F_X(x)F_\theta(t)+\alpha F_X(x)F_\theta(t)(1-F_X(x))(1-F_\theta(t))\end{aligned}$$

consequently, the joint p.d.f. $f_{X,\theta}$ is

$$\begin{aligned}&f_{X,\theta}(x,t)=c_\alpha(F_X(x),F_\theta(t))f_X(x)f_\theta(t)\\&=f_X(x)f_\theta(t)+\alpha f_X(x)f_\theta(t)(1-2F_X(x))(1-2F_\theta(t))\end{aligned}$$

LUNDBERG’S FUNDAMENTAL EQUATION

In order to derive Lundberg's fundamental equation in our model, we consider the discrete-time process imbedded in the continuous-time process $U(t)$. Precisely, denote the surplus immediately after j-th claim,

$$\widetilde{U}_j=U(T_j)=u+\sum_{i=1}^{j}(c\theta_i-X_i) \tag{15}$$

We seek a number s such that the process

$$\widetilde{V} = \{\exp(-\delta\sum_{i=1}^{j}\theta_i + s\widetilde{U}_j), k = 0, 1, \ldots\} \quad (16)$$

will form a martingale. Here this martingale condition is equivalent to

$$E[\mathrm{e}^{-\delta\theta}\mathrm{e}^{s(c\theta - X)}] = 1 \quad (17)$$

which is the so-called Lundberg's fundamental equation. For notion simplicity, we denote $h(x) = x(1-x)$, $g(y) = y(1-y)$, thus $C_\alpha(x, y) = xy + \alpha h(x)g(y)$, and $c_\alpha(x, y) = 1 + \alpha h^{'}(x)g^{'}(y)$.

By the expression of $f_{X,\theta}(x, t)$, the left-hand side of (17) can be written as

$$E[\mathrm{e}^{-\delta\theta}\mathrm{e}^{s(c\theta - X)}] = \int_0^\infty \mathrm{e}^{t(sc-\delta)}\mathrm{e}^{-sx} f_{X,\theta}(x, t)\mathrm{d}x\mathrm{d}t$$
$$= \int_0^\infty \mathrm{e}^{t(sc-\delta)}\mathrm{e}^{-sx} f_X(x) f_\theta(t)\mathrm{d}x\mathrm{d}t$$
$$+ \alpha\int_0^\infty \mathrm{e}^{t(sc-\delta)}\mathrm{e}^{-sx} f_X(x) f_\theta(t) h^{'}(F_X(x)) g^{'}(F_\theta(t))\mathrm{d}x\mathrm{d}t$$

Now we define another r.v. Z with p.d.f. given by

$$f_Z(x) = f_X(x) - f_X(x)h^{'}(F_X(x)) = 2f_X(x)F_X(x) \quad (18)$$

and denote a function

$$k(t) = f_\theta(t) g^{'}(F_\theta(t)) = f_\theta(t)(1 - 2F_\theta(t)) \quad (19)$$

so that we can rewrite the expression as

$$E[\mathrm{e}^{-\delta\theta}\mathrm{e}^{s(c\theta - X)}]$$
$$= f_X^*(s) f_\theta^*(\delta - sc) + \alpha k^*(\delta - sc)(f_X^*(s) - f_Z^*(s))$$

Since $f_\theta^*(s) = (\frac{\lambda}{\lambda+s})^2$ and

$$k^*(s) = \int_0^\infty \mathrm{e}^{-ts} f_\theta(t) g^{'}(F_\theta(t))\mathrm{d}t$$
$$= \int_0^\infty \mathrm{e}^{-ts}\mathrm{d}g(F_\theta(t))$$
$$= s\int_0^\infty \mathrm{e}^{-ts} g(F_\theta(t))\mathrm{d}t$$
$$= s\int_0^\infty \mathrm{e}^{-ts} F_\theta(t)(1 - F_\theta(t))\mathrm{d}t$$
$$= s\Big[\frac{1}{\lambda+s} - \frac{1}{2\lambda+s} - \frac{2\lambda}{(2\lambda+s)^2} + \frac{\lambda}{(\lambda+s)^2} - \frac{2\lambda^2}{(2\lambda+s)^3}\Big]$$

By inserting f_θ^* and k^* into the expression of $E[\mathrm{e}^{-\delta\theta}\mathrm{e}^{s(c\theta - X)}]$ above, we obtain the generalized Lundberg's equation in our case

$$(2\lambda + \delta - sc)^3(\lambda + \delta - sc)^2 = \lambda^2(2\lambda + \delta - sc)^3 f_X^*(s)$$
$$+ \alpha(f_X^*(s) - f_Z^*(s))\Big[2\lambda^2(2\lambda + \delta - sc)(\lambda + \delta - sc)^2$$
$$+ 4\lambda^3(\lambda + \delta - sc)^2 - \lambda^2(2\lambda + \delta - sc)^3\Big]$$

INTEGRO-DIFFERENTIAL EQUATION FOR ϕ_δ

In this section, we show that $\phi_\delta(u)$ satisfies an integro-differential equation.

By conditioning on the time and the first claim amount, and by (18) and (19), we have

$$\begin{aligned}\phi_\delta(u) &= \int_0^\infty \int_0^\infty \mathrm{e}^{-\delta t}\phi_\delta(u+ct-x)f_{X,\theta}(x,t)\mathrm{d}x\mathrm{d}t \\ &= \int_0^\infty \int_0^\infty \mathrm{e}^{-\delta t}\phi_\delta(u+ct-x)f_X(x)f_\theta(t)\mathrm{d}x\mathrm{d}t \\ &+ \alpha\int_0^\infty \int_0^\infty \mathrm{e}^{-\delta t}\phi_\delta(u+ct-x) \\ &\times f_X(x)f_\theta(t)h'(F_X(x))g'(F_\theta(t))\mathrm{d}x\mathrm{d}t \\ &= \int_0^\infty \int_0^\infty \mathrm{e}^{-\delta t}\phi_\delta(u+ct-x)f_X(x)f_\theta(t)\mathrm{d}x\mathrm{d}t \\ &+ \alpha\int_0^\infty \int_0^\infty \mathrm{e}^{-\delta t}\phi_\delta(u+ct-x) \\ &\times [g'(F_\theta(t))f_\theta(t)f_X(x) - g'(F_\theta(t))f_\theta(t)f_Z(x)]\mathrm{d}x\mathrm{d}t \\ &= \int_0^\infty E[\phi_\delta(u+ct-X)]\mathrm{e}^{-\delta t}f_\theta(t)\mathrm{d}t \\ &+ \alpha\int_0^\infty E[\phi_\delta(u+ct-X)]\mathrm{e}^{-\delta t}k(t)\mathrm{d}t \\ &- \alpha\int_0^\infty E[\phi_\delta(u+ct-Z)]\mathrm{e}^{-\delta t}k(t)\mathrm{d}t\end{aligned}$$

Where

$$\begin{aligned}E[\phi_\delta(u+ct-X)] &= \int_0^{u+ct} \phi_\delta(u+ct-x)f_X(x)\mathrm{d}x \\ &+ \int_{u+ct}^\infty w(u+ct, x-u-ct)f_X(x)\mathrm{d}x \\ E[\phi_\delta(u+ct-Z)] &= \int_0^{u+ct} \phi_\delta(u+ct-x)f_Z(x)\mathrm{d}x \\ &+ \int_{u+ct}^\infty w(u+ct, x-u-ct)f_Z(x)\mathrm{d}x\end{aligned}$$

With $y = u + ct$, we have

$$\begin{aligned}c\phi_\delta(u) &= \int_u^\infty E[\phi_\delta(y-X)]\mathrm{e}^{-\frac{\delta}{c}(y-u)}f_\theta(\frac{y-u}{c})\mathrm{d}y \\ &+ \alpha\int_u^\infty E[\phi_\delta(y-X)]\mathrm{e}^{-\frac{\delta}{c}(y-u)}k(\frac{y-u}{c})\mathrm{d}t \\ &- \alpha\int_u^\infty E[\phi_\delta(y-Z)]\mathrm{e}^{-\frac{\delta}{c}(y-u)}k(\frac{y-u}{c})\mathrm{d}t\end{aligned}$$

Differentiating the two sides of the above equation with respect to u,

$$\begin{aligned}c\phi_\delta'(u) &= -\frac{1}{c}\int_u^\infty E[\phi_\delta(y-X)]\mathrm{e}^{-\frac{\delta}{c}(y-u)}f_\theta'(\frac{y-u}{c})\mathrm{d}y \\ &- \frac{\alpha}{c}\int_u^\infty E[\phi_\delta(y-X)]\mathrm{e}^{-\frac{\delta}{c}(y-u)}k'(\frac{y-u}{c})\mathrm{d}y \\ &+ \frac{\alpha}{c}\int_u^\infty E[\phi_\delta(y-Z)]\mathrm{e}^{-\frac{\delta}{c}(y-u)}k'(\frac{y-u}{c})\mathrm{d}y \\ &+ \frac{\delta}{c}\cdot c\phi_\delta(u)\end{aligned}$$

Since

$$f_\theta^{'}(t) = \lambda^2 \mathrm{e}^{-\lambda t} - \lambda^3 t\mathrm{e}^{-\lambda t}$$
$$= \lambda^2 \mathrm{e}^{-\lambda t} - \lambda f_\theta(t)$$
$$k^{'}(t) = 2\lambda^2 \mathrm{e}^{-2\lambda t} - \lambda^2 \mathrm{e}^{-\lambda t} + \lambda^3 t\mathrm{e}^{-\lambda t} - 4\lambda^4 t^2 \mathrm{e}^{-2\lambda t}$$
$$= \lambda^2 \mathrm{e}^{-\lambda t}(1 - 2F_\theta(t)) - 2f_\theta^2(t) - \lambda k(t)$$

Consequently

$$c\phi_\delta^{'}(u) = (\delta + \lambda)\phi_\delta(u)$$
$$- \frac{\lambda^2}{c}\int_u^\infty E[\phi_\delta(y - X)]\mathrm{e}^{-\frac{\delta+\lambda}{c}(y-u)}\mathrm{d}y$$
$$- \frac{\alpha\lambda^2}{c}\int_u^\infty E[\phi_\delta(y - X)]\mathrm{e}^{-\frac{\delta+\lambda}{c}(y-u)}(1 - 2F_\theta(\frac{y-u}{c}))\mathrm{d}y$$
$$+ \frac{2\alpha}{c}\int_u^\infty E[\phi_\delta(y - X)]\mathrm{e}^{-\frac{\delta}{c}(y-u)}f_\theta^2(\frac{y-u}{c})\mathrm{d}y$$
$$+ \frac{\alpha\lambda^2}{c}\int_u^\infty E[\phi_\delta(y - Z)]\mathrm{e}^{-\frac{\delta+\lambda}{c}(y-u)}(1 - 2F_\theta(\frac{y-u}{c}))\mathrm{d}y$$
$$- \frac{2\alpha}{c}\int_u^\infty E[\phi_\delta(y - Z)]\mathrm{e}^{-\frac{\delta}{c}(y-u)}f_\theta^2(\frac{y-u}{c})\mathrm{d}y$$

Differentiating the two sides of the above equation once again and with some rearrangements, we obtain

$$c^2\phi_\delta^{''}(u) - 2c(\delta+\lambda)\phi_\delta^{'}(u) + (\delta+\lambda)^2\phi_\delta(u)$$
$$=(1+\alpha)\lambda^2\mu_1(u) - \alpha\lambda^2\mu_4(u)$$
$$+(\frac{2\alpha\lambda^4 u}{c^2} - \frac{2\alpha\lambda^5 u^2}{c^3})T_{\frac{\delta+2\lambda}{c}}\mu_1(u) + \frac{4\alpha\lambda^2 u}{c^2}T_{\frac{\delta+\lambda}{c}}\mu_1(u)$$
$$-(\frac{2\alpha\lambda^4}{c^2} - \frac{4\alpha\lambda^5 u}{c^3})T_{\frac{\delta+2\lambda}{c}}\mu_2(u) - \frac{4\alpha\lambda^2}{c^2}T_{\frac{\delta+\lambda}{c}}\mu_2(u)$$
$$-\frac{2\alpha\lambda^5}{c^3}T_{\frac{\delta+2\lambda}{c}}\mu_3(u)$$
$$-(\frac{2\alpha\lambda^4 u}{c^2} - \frac{2\alpha\lambda^5 u^2}{c^3})T_{\frac{\delta+2\lambda}{c}}\mu_4(u) - \frac{4\alpha\lambda^2 u}{c^2}T_{\frac{\delta+\lambda}{c}}\mu_4(u)$$
$$+(\frac{2\alpha\lambda^4}{c^2} - \frac{4\alpha\lambda^5 u}{c^3})T_{\frac{\delta+2\lambda}{c}}\mu_5(u) + \frac{4\alpha\lambda^2}{c^2}T_{\frac{\delta+\lambda}{c}}\mu_5(u)$$
$$+\frac{2\alpha\lambda^5}{c^3}T_{\frac{\delta+2\lambda}{c}}\mu_6(u) \qquad (20)$$

where

$$\mu_1(y) = E[\phi_\delta(y - X)], \quad \mu_4(y) = E[\phi_\delta(y - Z)]$$
$$\mu_2(y) = yE[\phi_\delta(y - X)], \quad \mu_5(y) = yE[\phi_\delta(y - Z)]$$
$$\mu_3(y) = y^2E[\phi_\delta(y - X)], \quad \mu_6(y) = y^2E[\phi_\delta(y - Z)]$$

and the operator $T_r p$ of a given integrable function p for real r, introduced by Dickson and Hipp[6]:

$$T_r p(u) = \int_u^\infty \mathrm{e}^{-r(y-u)}p(y)\mathrm{d}y, \quad r \in \mathbb{R} \qquad (21)$$

Note that when $\alpha = 0$, i.e., the independent model, (20) reduces to Eq.(2.1) of Dickson and Hipp[6]. Further more, by taking the Laplace transform of (20), we may solve this second order differential equation and derive an exact representation for $\phi_\delta(u)$, which is also the solution of a defective renewal equation. But $\phi_\delta(u)$ can be evaluated through this defective renewal function only for several special choices of claim amount distributions such as combinations of exponentials, mixtures of Erlangs, phase-type, etc., since its representation is rather involved. Usually we could only obtain asymptotic for $\phi_\delta(u)$ for general claim amount distributions.

CONCLUSION

In this paper, we have shown that some well-known techniques can be used to solve the Gerber-Shiu penalty function $\phi_\delta(u)$ under the framework of the Erlang(2) risk model based on FGM copula, of which the ultimate ruin probability $\psi(u)$ is a special case.

We derive a system of integro-differential equation of the Gerber-Shiu function. Although we have discussed some properties of the Gerber-Shiu function, it seems that we can't apply neither the probability measure transform technique of Gerber and Shiu[7] nor the Laplace transform technique of Dickson and Hipp[5][6] to determine an expression for $\phi_\delta(u)$ when $\delta > 0$. But we could give an upper bound of the ruin probability here through stochastic optimization technique, that we leave as an open problem for the moment.

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