



Research Article

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The research of the knee joint's motion model based on the kinetic equation of multi-body system

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ABSTRACT

During physical exercise, the knee is vulnerable to injuries. In order to prevent such injuries, many people have analyzed knee pads and sports biomechanics of knee. This paper studies the simulation model of knee joint motion. On the basis of analysis on the geometric model of the knee joint during exercise, it establishes the dynamic equations of knee joint motion and obtains the mechanics response function of knee joint muscle, discusses the change trends of the angle and the angular velocity over time by biomechanical simulation data of knee joints and related systems in the starting process, verifies the reliability of the model through the simulation results, and provides a theoretical basis for medicine and sports to prevent damage.

Key words: Human physiology, Kinetic equation, Response function

INTRODUCTION

Organizations around the knee are more complex, and the functions between these structures are mutually interacted. During physical exercise the knee is susceptible to injury, and this injury is often not easy to recover, so excellent potential athletes have to retire. The current research for knee injury mostly stays in level of experience prevention and injury treatment [1, 2]. If we can accurately determine the biomechanical parameters of the knee joint and each structure during movement and the material mechanical parameters of substance itself, which may have better results to prevent knee injury [3, 4]. For exploring the movement of the knee joint, this paper thinks that the use of computer simulation can receive better benefits. In the calculation simulation process of body structure it requires not only the accuracy of simulation software and hardware reliability, but also need to select the appropriate dynamic model. This paper studies the simulation model for knee motion [5, 6].

Model research is generally divided into physical and mathematical modeling process. The physical model is based on experience and theoretical derivation results, which is easy to understand in the operation and realization and has the advantages of high accuracy; while mathematical modeling is the nature mean of the physical model derivation [7, 8]. When the physical model can not meet the need for simulation, you should use the most essential mathematical methods for its modeling; the ultimate goal and method of theoretical description in mathematical modeling is essentially taking the complex problems as simple questions, and then resolve the problem using mathematical method [9].

This paper discusses the dynamics model of computer multi-body system, the dynamic model of the knee joint and the analysis situation of the simulation data to provide reliable model for the computer simulation of the knee joint motion. In this paper, on the basis of previous efforts, it expounds the motion simulation model for knee, and verifies the reasonability of the model through simulation data.

1. The dynamics model principle of computer multi-body system

The dynamics modeling method of computer multi-body system is mainly divided into physical models and mathematical models, wherein physical models refer to build a physical model by the geometry model, and then increase the technical constraints of the kinematics for the geometric model, finally form the physical model that expresses mechanical properties; meanwhile the building form and solving process of the model is also an important symbol of mathematical modeling.

When determining the movement of any point in the multi-body system, you need to transform a point on the study object from the body-fixed coordinate system to the global coordinate system; the coordinate transformation in two-dimensional space is illustrated in Figure 1:

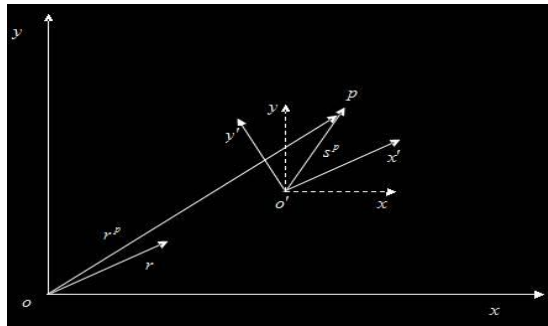


Figure 1: The coordinate transformation chart in two-dimensional space

If the expression of the vector \vec{s} in the xoy coordinates and the $x'o'y'$ coordinate system is shown in the formula (1), then point P in Figure 1 has the transformation relation in the formula (2).

$$\begin{cases} \vec{s} = [s_x, s_y]^T \\ \vec{s}' = [s_{x'}, s_{y'}]^T \end{cases} \quad (1)$$

$$\vec{r}^P = \vec{r} + \vec{s}^P = \vec{r} + \mathbf{A} \vec{s}'^P \quad (2)$$

In Formula (2) \vec{r}^P represents the coordinates of point P in the global coordinate system, \vec{r} means the coordinates of the origin O' of the body-fixed coordinate system in the global coordinate system, s^P means the coordinates of the vector \vec{s} in the global coordinate system, s'^P means the coordinates of vector \vec{s} in the body-fixed coordinate system, \mathbf{A} means the rotational transformation matrix, wherein the form of the rotational transformation matrix is in the formula (3).

$$\mathbf{A} = \mathbf{A}(\phi) = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \quad (3)$$

If the time derivative of the matrix \mathbf{A} is as the formula (4), the global coordinates of any point in the body-fixed coordinate system can be obtained, and the transformation formula is in the formula (5).

$$\dot{\mathbf{A}} = \dot{\phi} \frac{d}{d\phi} \mathbf{A} = \dot{\phi} \begin{bmatrix} -\sin \phi & -\cos \phi \\ \cos \phi & -\sin \phi \end{bmatrix} = \dot{\phi} \mathbf{B} \quad (4)$$

$$\dot{r}^P = \dot{r} + \dot{\mathbf{A}} s'^P = \dot{r} + \dot{\phi} \mathbf{B} s'^P \quad (5)$$

Then solve the time derivative of the formula (5), acceleration conversion formula at any point can be derived, as shown in the formula (6).

$$\ddot{r}^P = \ddot{r} + \ddot{\phi} \mathbf{B} s'^P + \dot{\phi} \dot{\mathbf{B}} s'^P = \ddot{r} + \ddot{\phi} \mathbf{B} s'^P - \dot{\phi}^2 \mathbf{A} s'^P \quad (6)$$

2. The kinetic model of knee joint

Dynamics simulation of the knee joint has great significance for the study of knee joint dynamic response, and

affects the coordination between muscles of the knee joint movement. The following respectively expounds the geometric model, static and quasi-static model and kinetic equations of the knee joint.

3.1 The geometry model of knee joint

Knee joint has four ligaments. This organizational structure restricts the tibial forward and rearward displacement of the knee joint, which can only rotate in the sagittal plane. The muscles that impact knee joint motion are quadriceps muscles, biceps femora's, and gastronomies muscle. These muscles are the muscles striding the knee joint, has a direct relationship with the knee joint. Meanwhile quadriceps muscles and biceps femora's also cross the hip joint; if the knee joint is simplified as viscoelastic hinge, hinge exist the torque generated by the soft tissue effects. Meanwhile the muscles is simplified to facilitate, through the anatomical knowledge on muscle and the corresponding skeletal, the muscle force direction of muscle maintain a slight change with the skeleton, and this change has little to do with the corresponding angle. So: the muscle forces at the connection point of the two muscles and bone is represented by the unidirectional force, the direction of these forces changes with the spatial orientation of the bone, and the relative direction to connecting skeletal muscle stays unchanged. The simplified force diagram of knee joint is shown in Figure 2.

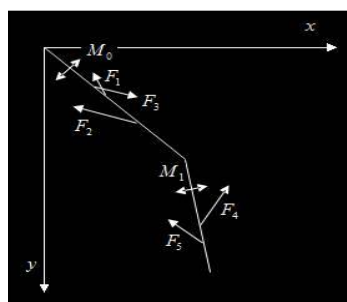


Figure 2: The force analysis diagram of knee joint

The five forces in Figure 2 are the simplified force for human lower limb muscle, and the lower limbs links are divided as shown in Table 1.

Table 1: Lower limb link

Link	Demarcation point between links		Measuring point from the centroid
	Proximal point	Distal point	
Thigh	Bone spines point	Tibial point	Tibial point
Calf	Tibial point	malleolus medialis point	malleolus medialis point

The model parameter of lower limbs is shown in Table 2.

Table 2: The model parameters of human lower limb

Item	Symbolic representation	Unit	Value
Thigh-length	l_1	m	0.48
Calf and foot length	l_2	m	0.362
Thigh mass	m_1	kg	7.727
Calf and foot mass	m_2	kg	3.149
Rotational inertia of thigh	J_1	kg m ²	0.11794
Rotational inertia of calf and foot	J_2	kg m ²	0.0198
Centroid position of the thigh	l_{m1}	m	0.247
Centroid position of calf and foot	l_{m2}	m	0.19

3.2 Knee joint static and quasi-static model

The joint force between the femur and tibia is the resultant force of the ligament tension, various muscle tension and contact force. The model uses the Hefzy and Grood graphical representation as shown in Figure 3

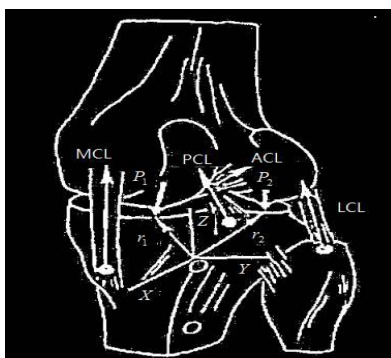


Figure 3: Hefzy and Grood knee joint model

The ACL, PCL, MCL and LCL in Figure 3 respectively denote anterior cruciate ligament, posterior cruciate ligament, median cruciate ligament and lateral cruciate ligaments. The tensions corresponding to the four ligaments are respectively F_1, F_2, F_3, F_4 . The contact force by the intermeshing of femur and tibia is P_1, P_2 . Through the contact force of the four knee joint's ligament tension and engaging to maintain the balance of the knee joint. As shown in Figure 3, the spatial coordinate system $O - XYZ$ is established on the tibia, the equilibrium equations are in formula (7) below.

$$\begin{cases} \sum_n F^n + P_1 + P_2 = 0 \\ \sum_n r_T^n \times F^n + r_1 \times P_1 + r_2 \times P_2 = 0 \end{cases} \tag{7}$$

3.3 The kinetic equation of knee joint

This paper uses the model phenomenon, fixes the pelvis, releases femur and tibia; the hip and knee joint use viscoelastic hinges; add force couple on the hinge to describe the soft tissue effect of the joint, add obstruct force couple to the knee joint to describe the effect that the knee joint cannot bend forward due to the existence of the patella and the structure of ligaments at the knee joint, the foot and tibia are fixedly connected, it can be seen as a rigid body, which has similarities with Lindbeck model and both of them use double pendulum structure, the structure is shown in Figure 4:

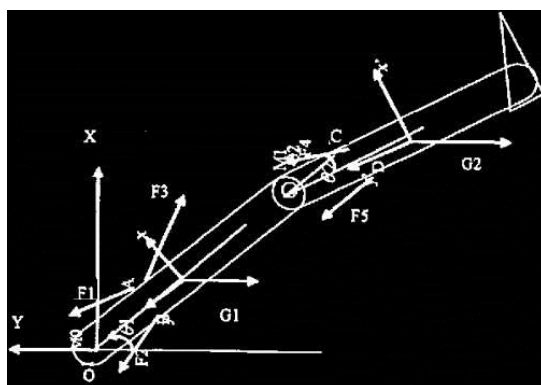


Figure 4: The simplified diagram of the knee joint model

In Figure 4 take the hip joint as the coordinate origin, take the front and upward of people as the axis to establish global coordinate system. The movement of the moving system (x, y) relative to the fixed coordinate system can be represented by the angle θ_1 between the femur centerline and the y axis. The movement of the moving system (x', y') relative to the moving system (x, y) can be represented by the angle θ_2 between the center line of the tibia and the center line of the femur. Thus the coordinate transformation relation between the three coordinate systems can be obtained as shown in the formula (8), (9) and (10):

$$\begin{Bmatrix} I \\ J \end{Bmatrix} = [T_{01}] \begin{Bmatrix} i \\ j \end{Bmatrix} = \begin{bmatrix} \cos \theta_1 & \sin \theta_1 \\ -\sin \theta_1 & \cos \theta_1 \end{bmatrix} \begin{Bmatrix} i \\ j \end{Bmatrix} \tag{8}$$

$$\begin{Bmatrix} i \\ j \end{Bmatrix} = [T_{12}] \begin{Bmatrix} i' \\ j' \end{Bmatrix} = \begin{bmatrix} \cos(-\theta_2) & \sin(-\theta_2) \\ -\sin(-\theta_2) & \cos(-\theta_2) \end{bmatrix} \begin{Bmatrix} i' \\ j' \end{Bmatrix} \tag{9}$$

$$\begin{Bmatrix} I \\ J \end{Bmatrix} = [T_{01}] [T_{12}] \begin{Bmatrix} i' \\ j' \end{Bmatrix} = [T_{02}] \begin{Bmatrix} i' \\ j' \end{Bmatrix} = \begin{bmatrix} \cos(\theta_1 - \theta_2) & \sin(\theta_1 - \theta_2) \\ -\sin(\theta_1 - \theta_2) & \cos(\theta_1 - \theta_2) \end{bmatrix} \begin{Bmatrix} i' \\ j' \end{Bmatrix} \tag{10}$$

In the above three formulas the subscript 0,1,2 respectively represent pelvis, femur and tibia, thus the dynamic governing equation of the lower limbs is shown in formula (11), (12), (13) and (14):

$$\sum_{j=4}^5 F_j (\lambda_j)_x + N_x = m_2 \ddot{x} \tag{11}$$

$$\sum_{j=4}^5 F_j (\lambda_j)_y + N_y = m_2 \ddot{y} \tag{12}$$

$$-M_1 + [T_{02}] \{ \rho'_B \} \times (N) + \sum_{j=4}^5 [T_{02}] \{ \rho'_j \} \times (F_j \lambda_j) = J_2 (\ddot{\theta}_1 - \ddot{\theta}_2) \tag{13}$$

$$-M_0 + [T_{01}] \{ \rho_B \} \times (N) + \sum_{j=1}^3 [T_{01}] \{ \rho_j \} \times (F_j \lambda_j) + l'_1 m_1 g \sin(\theta_1) - M_p = J_1 \ddot{\theta}_1 \tag{14}$$

In the above four formulas(11) - (14), λ_j means the unit vector along the longitudinal direction of the j th muscle;; J_1 means the rotational inertia of the mass center of the femur relative to the hip joint; J_2 means the rotational inertia of the calf and bone's mass central of foot; F_j represents the j th muscle tension of the knee joint; ρ_B means the position vector of the knee hinge point B in the (x, y) coordinate system; ρ_j denotes the position vector of j th muscle's attachment point in the (x, y) coordinate system; ρ'_j denotes the position vector of j th muscle's attachment point in the (x', y') coordinate system; ρ'_B means the position vector of the knee hinge point B in the (x', y') coordinate system; l'_1 means the length from the mass center of femur to the hip joint; M_0 means the soft tissue effects at the structure of hip joint; M_1 means the moment caused by the soft tissue effect of human knee joint.

4. Data empirical analysis

4.1 Parameter Value situation

All parameter values of the governing equations are as shown in Table 3.

Table 3: All parameter values of the governing equations

Parameter	Parameter Value	Parameter	Parameter Value
λ_1	$(\sqrt{2}, \sqrt{2})$	ρ_2	$(-0.062, 0.13) m$
λ_2	$(\sqrt{2}, -\sqrt{2})$	ρ'_3	$(0.081, 0.142) m$
λ_3	$(0, 1)$	ρ'_4	$(0.06, 0.033) m$
λ_4	$(0, -1)$	ρ'_5	$(-0.061, 0.12) m$
λ_5	$(0, 1)$	ρ'_B	$(0, -0.247) m$
ρ_1	$(0, 0.15) m$	ρ_B	$(0, 172) m$

4.2 The motion simulation data of knee joint

This research studies the knee movement situation during the starting process; the initial position of the study object is the fighting force posture, in which one leg is fixed on the ground, the other leg lifts taking the pelvis as fixed point. People's hip joint always makes actions of body bent at hips, and the knee joint first bends and then stretches. During the movement simulation, the muscles of the model are activated in stages, and the output function of the muscle force is as formula (15) below.

$$F = f_1(t)[kF_{\max} f_2(\theta_1, \theta_2) + F_0] \quad (15)$$

In Formula (15) $f_1(t)$ means the function of the muscle's activation time, $f_2(\theta_1, \theta_2)$ means the force expression functions of different muscles, and the values of the two functions during the simulation process are as formula (16) and (17) below.

$$\begin{cases} f_{11} = 1, f_{12} = 0, f_{13} = 0, f_{14} = 0, f_{15} = 0 & 0 < t \leq 0.2 \\ f_{11} = 1, f_{12} = 0, f_{13} = 1, f_{14} = 1, f_{15} = 1 & 0.2 < t \leq 0.4 \end{cases} \quad (16)$$

$$\begin{cases} f_{21} = \sin(\theta_1) \exp\left(\theta_1 - \frac{\pi}{2}\right) \\ f_{22} = \sin(\theta_1) \\ f_{23} = \exp\left(\theta_2 - \frac{\pi}{2}\right) \\ f_{24} = f_{23} + \exp\left(\theta_1 - \frac{\pi}{2}\right) \exp\left(\theta_2 - \frac{\pi}{2}\right) \\ f_{24} = \exp\left(\theta_1 - \frac{\pi}{2}\right) \exp\left(\theta_2 - \frac{\pi}{2}\right) \end{cases} \quad (17)$$

The remaining parameters corresponding to five muscles are shown in Table 4.

Table 4: The remaining parameters corresponding to five groups of muscles

	The first group of muscles	The second group of muscles	The third group of muscles	The fourth group of muscles	The fifth group of muscles
k	0.5	0.5	0.1	0.1	0.2
F_{\max}	4680	1050	4500	1500	500
F_0	200	100	20	20	20

The data of hip joint angle changing with time is shown in Table 5.

Table 5: The data of hip joint angle changing with time

Time(s)	0.0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
Angle(rad)	0.00	0.02	0.06	0.14	0.24	0.35	0.50	0.70	0.91

The change trend of the hip joint angle in Table 5 over time is shown in Figure 5.

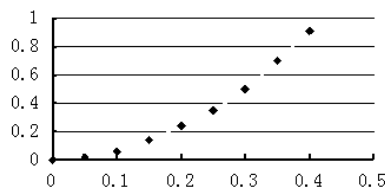


Figure 5: The change trends of the hip joint angle over time

The data of hip joint angular velocity changing with time is shown in Table 6.

Table 6: The data of hip joint angular velocity changing with time

Time(s)	0.0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
(rad/min)	0.00	0.65	1.25	1.75	2.12	2.65	3.50	4.25	4.75

The change trend of the hip joint angular velocity over time is shown in Figure 6.

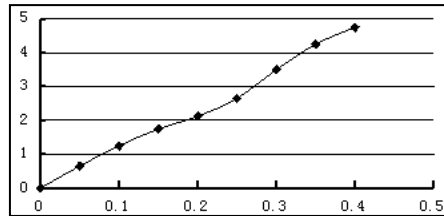


Figure 6: The change trends of the hip joint angular velocity over time

The data of knee joint angle changing with time is shown in Table 7.

Table 7: The data of knee joint angle changing with time

Time(s)	0.0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
Angle(rad)	0.00	0.05	0.18	0.37	0.60	0.76	0.75	0.57	0.20

The change trend of the knee joint angle over time is shown in Figure 7.

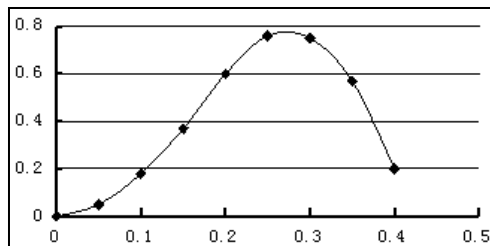


Figure 7: The change trend of the knee joint angle over time

The data of knee joint angular velocity changing with time is shown in Table 8.

Table 8: The data of knee joint angular velocity changing with time

Time(s)	0.0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
(rad/min)	0.00	1.75	3.42	4.13	4.18	1.61	-2.11	-6.51	-9.91

The change trend of the knee joint angular velocity over time is shown in Figure 8.

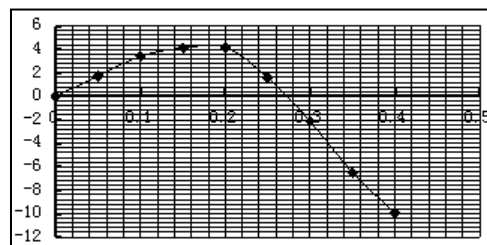


Figure 8: The change trend of the knee joint angular velocity over time

CONCLUSION

This study describes the dynamics model of computer multi-body system, discusses the physical model of the human body multi-system under normal circumstances through the model, and provides a theoretical basis for motion models of the knee joint. On the basis of analysis on the geometric model of the knee joint during exercise, it establishes the dynamic equations of knee joint motion, obtains the mechanics response function of knee joint muscle, discusses the change trends of the angle and the angular velocity over time by biomechanical simulation

data of knee joints and related systems in the starting process, and verifies the reliability of the model through the simulation results. This model established in this paper can simulate the movement situation of the knee over time, which is similar with the actual human movement and can be used as a mathematical model for simulation analysis.

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