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Research Article

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The research of CV model by discrete kalman filtering

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ABSTRACT

Radar target trace estimation is an important application of Kalman filtering. This paper uses the CV model in discrete Kalman filtering to simulate the trace of moving target in 3-D space. We generate the real trace under orthogonal set, transform the observe trace under polar set to the orthogonal set, then apply it to Kalman filtering. We analyze the performance using the mean error and deviation of the observation, prescreen and filtered trace, the effects of original values and system parameters to Kalman filtering are also presented..

Key words: Kalman filtering, CV model, trace, performance analysis

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INTRODUCTION

Kalman filter (KF) is proposed by the American scholar Kalman and Bucy as a linear, unbiased and minimum mean square error recursive filtering algorithm. It is considered the estimator and the statistic characteristics of observed quantity, according to the principle of minimum error unbiasedness and estimation in real time from the measured noise pollution observation to estimate the system state variables. KF has two character[1-5]. One is real-time recursion method, and the second is the state variable is introduced for filtering theory. KF are not required to save the measurement data in the past, when new data are measured, based on the new data and the valuation of the previous moment, a set of recursion formula can work out the new valuation. It is suitable for stationary random process, and it is also fit to non-stationary random process. It has the strong adaptability to implement the state estimation and easy to use a computer to solve, which is suitable for real-time processing[6].

As the development of aerospace technology, radar can detect the target in the background noise and interference path. It not only can measure the target position now, and can predict the future position. Due to the target movement, this is a matter of time-varying state estimation. The kalman filter can be used to get the best estimate. In this paper, by using the CV model, the establishment of uniform motion in a straight line and uniform circular motion model, estimation of target flight position, and the filtering results for this paper[7-10].

EXPERIMENTAL SECTION

The discrete kalman filter should follow conditions below:
(1) Information model

$$\mathbf{x}_{k+1} = \Phi_k \mathbf{x}_k + G_k \mathbf{u}_k \tag{1}$$

n dimensional vector $\mathbf{x}_k$ stands for information, $\mathbf{u}_k$ is for r dimensional noise vector. For Φ_k, which means the system matrix or transfer matrix as $n \times n$ matrix, and G_k is $n \times r$ matrix.

(2)measurement model

$$\mathbf{z}_k = H_k \cdot \mathbf{x}_k + \mathbf{w}_k \tag{2}$$

Among them $\mathbf{z}_k$ is the observation vector with m dimension, and $\mathbf{w}_k$ is observation noise with m dimension.

(1) apriori information

$$E\{\mathbf{u}_k\} = \overline{\mathbf{u}}_k, \quad Cov\{\mathbf{u}_k, \mathbf{u}_j\} = Q_k \cdot \delta_{kj} \tag{3}$$

$$E\{\mathbf{w}_k\} = \overline{\mathbf{w}}_k, \quad Cov\{\mathbf{u}_k, \mathbf{u}_j\} = R_k \cdot \delta_{kj} \tag{4}$$

$$Cov\{\mathbf{u}_k, \mathbf{w}_k\} = 0 \tag{5}$$

Eq.（3）, Eq.（4）, Eq.（5）separately show the influence noise and observation noise of message model . They are gaussian white noise, and orthogonal to each other.

Change the coordinate（x，y，z）of rectangular coordinate system to the coordinates（R，α，β）as polar coordinate system as :

$$\begin{aligned} R &= \sqrt{x^2 + y^2 + z^2} \\ \alpha &= \arctan(y/x) \\ \beta &= \arcsin(z/\sqrt{x^2 + y^2 + z^2}) \end{aligned} \tag{6}$$

Then Change the coordinate （R，α，β）of polar coordinate system to the coordinates （x，y，z） as rectangular coordinates system.

$$\begin{aligned} x &= R\cos\beta\cos\alpha \\ y &= R\cos\beta\sin\alpha \\ z &= R\sin\beta \end{aligned} \tag{7}$$

From the first two observation value Z_1 and Z_2, we can get the estimation of initiation value $\hat{X}_0$:

$$\hat{X}_0 = \begin{bmatrix} \hat{x}(0) & v_x(0) & \hat{y}(0) & v_y(0) & \hat{z}(0) & v_z(0) \end{bmatrix}', \quad Z_1 = \begin{bmatrix} x_o(1) \\ y_o(1) \\ z_o(1) \end{bmatrix}, \quad Z_2 = \begin{bmatrix} x_o(2) \\ y_o(2) \\ z_o(2) \end{bmatrix}$$

Then $\hat{x}(0) = x_o(2)$, $\hat{y}(0) = y_o(2)$, $\hat{z}(0) = z_o(2)$

$$\hat{v}_x(0) = \frac{\hat{x}_0(2) - \hat{x}_0(1)}{T}, \quad \hat{v}_y(0) = \frac{\hat{y}_0(2) - \hat{y}_0(1)}{T}, \quad \hat{v}_z(0) = \frac{\hat{z}_0(2) - \hat{z}_0(1)}{T} \tag{8}$$

As know the information model and the measurement model, the observation can be put into the following equation to do KF.

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{\Phi}_{k-1}\hat{\mathbf{x}}_k + G_{k-1}\overline{\mathbf{u}}_{k-1} \tag{6}$$

$$P_{k|k-1} = \mathbf{\Phi}_{k-1} \cdot P_{k-1} \cdot \mathbf{\Phi}_{k-1}^T + G_{k-1} \cdot Q_{k-1} \cdot G_{k-1}^T \tag{7}$$

$$P_k = (P_{k|k-1}^{-1} + H_k^T \cdot R_k^{-1} \cdot H_k)^{-1} \tag{8}$$

$$K_k = P_{k|k-1} \cdot H_k^T (H_k \cdot P_{k|k-1} \cdot H_k^T + R_k)^{-1} \tag{9}$$

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{k|k-1} + K_k \cdot \left[\mathbf{z}_k - (H_k \hat{\mathbf{x}}_{k|k-1} + \overline{w}_k)\right] \tag{10}$$

This article use the average error and the mean variance on filtering performance analysis:

average error： $m_R = \frac{1}{N}\sum_{i=1}^{N}\Delta R_i$ (13)

mean variance： $\sigma_R{}^2 = \frac{1}{N}\sum_{i=1}^{N}(\Delta R_i - m_R)^2$ (14)

2.1 Producing target track and message model

(1) Uniform motion in a straight line flight tracking

$$x_T(k) = x_0 + v_x kT$$
$$y_T(k) = y_0 + v_y kT$$
$$z_T(k) = z_0 + v_z kT$$

(2) Uniform circular motion tracking

$$x_T = x_0 + v\cos\{wk(T - T_0)\}$$
$$y_T = y_0 + v\sin\{wk(T - T_0)\}$$
$$z = z_0$$

This is the real track of target motion, the actual processing of the unknown, need to estimate. From the above two kinds of motion track can generate message model: $\mathbf{x}_{k+1} = \Phi_k \mathbf{x}_k + G_k \mathbf{u}_k$

By the CV model，we can get the parameters for 6 dimension CV model：

$$X_k = \begin{bmatrix} x_T(k) \\ v_x \\ y_T(k) \\ v_y \\ z_T(k) \\ v_z \end{bmatrix}, \Phi = \begin{bmatrix} 1 & T & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & T & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, G = \begin{bmatrix} T^2/2 & 0 & 0 \\ T & 0 & 0 \\ 0 & T^2/2 & 0 \\ 0 & T & 0 \\ 0 & 0 & T^2/2 \\ 0 & 0 & T \end{bmatrix} \quad Q = \sigma_u{}^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

, $\sigma_u{}^2$ is the variance of launch noise.

2.2 produce the observation model

In this article the observation vector is real track after the observation noise under the polar coordinate. So first by using the real Eq.(11) to produce polar track [$R_T(k)$, $\alpha_T(k)$, $\beta_T(k)$]' , then get the polar radar target under observation vector [$R_o(k)$, $\alpha_o(k)$, $\beta_o(k)$]'by the following equations. The variances of observation noise is σ_R , σ_α , σ_β .

$$R_o(k) = R_T(k) + w_R(k) = R_T(k) + \sigma_R \cdot v_R(k$$
$$\alpha_o(k) = \alpha_T(k) + w_\alpha(k) = \alpha_T(k) + \sigma_\alpha \cdot v_\alpha(k)$$
$$\beta_o(k) = \beta_T(k) + w_\beta(k) = \beta_T(k) + \sigma_\beta \cdot v_\beta(k)$$

Get the observation vectors in the cartesian coordinates by Eq.(12): [$x_o(k)$, $y_o(k)$, $z_o(k)$]'

The observation model can be built as $\mathbf{z}_k = H_k \cdot \mathbf{x}_k + \mathbf{w}_k$

And $Z_k = \begin{bmatrix} x_o(k) \\ y_o(k) \\ z_o(k) \end{bmatrix}$, $H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

Because of the noise variance is in spherical coordinates, so need to turn to the right Angle coordinate system:

$$\sigma_x^2 = \sigma_R^2 \cos^2\beta \cos^2\alpha + R^2\cos^2\beta \sin^2\alpha\sigma_\alpha^2 + R^2\sin^2\beta\cos^2\alpha\sigma_\beta^2$$

$$\sigma_y^2 = \sigma_R^2 \cos^2\beta \sin^2\alpha + R^2\cos^2\beta \cos^2\alpha\sigma_\alpha^2 + R^2\sin^2\beta\sin^2\alpha\sigma_\beta^2$$

$$\sigma_z^2 = \sigma_R^2 \sin^2\beta + R^2\cos^2\beta\sigma_\beta^2$$

$$\sigma_{xy} = (0.5\sin 2\alpha)(\sigma_R^2\cos^2\beta - R^2\cos^2\beta\sigma_\alpha^2 + R^2\sin^2\beta\sigma_\beta^2)$$

$$\sigma_{yz} = (0.5\sin 2\beta)(\sigma_R^2 - R^2\sigma_\beta^2)\sin\alpha$$

$$\sigma_{xz} = (0.5\sin 2\beta)(\sigma_R^2 - R^2\sigma_\beta^2)\cos\alpha$$

This can be the observation noise covariance matrix in rectangular coordinate system: $R = \begin{bmatrix} \sigma_x{}^2 & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_y{}^2 & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_z{}^2 \end{bmatrix}$.

After determining the initial value P_0 of the filtering error covariance matrix and the initial value $\hat{x}_0$ of the filtering , the observation can be turned into kalman filter equation, and the target state estimation can be done.

RESULTS AND DISCUSSION

Target motion simulation in this paper is by the coordinates of target in the discrete sampling time operation. The movement of motor noise is through MATLAB which generates in the superposition of gauss distribution of multidimensional random Numbers on the trajectory.

The simulation of kalman filtering is done by six dimensional CV model. The observation noise and the bold noise are assumed to unrelated white noise, so the values of variance is considered to be the filtering effect parameter to do results analysis. The initial state and initial variance matrix as a recursive analysis of the initial value is as a parameter too. When finally get the estimated conversion of polar coordinates, we just take the distance between target and radar as dimension to do the analysis. The results of transient characteristics focus on its convergence speed, steady state feature through the forecast , the estimated error and variance calculated respectively in the comparison. In the process of using the monte carlo experiment method to obtain the mean, the use of MATLAB software in gaussian distribution function of the random number of superposing noise simulate N times monte carlo experiments.

Uniform motion in a straight line:
The initial position of target（x, y, z） in three-dimensional space is（1000，1000，2000）.
The speed in the three direction （v_x，v_y，v_z） is （100，100，0）.

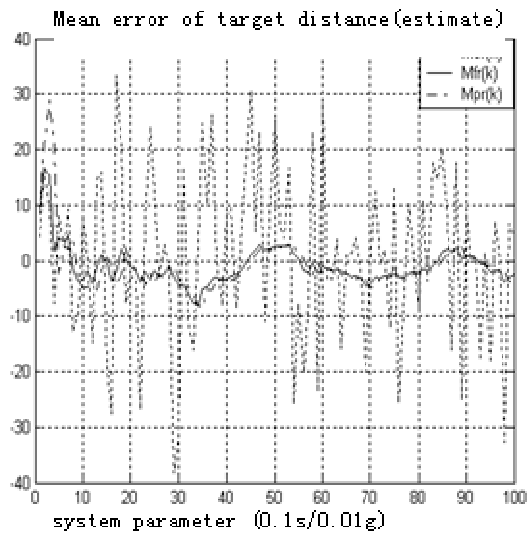
The radar do observation to target every time T , and then estimate the next moment until continuous observation of 100 or 200 points. Using the monte carlo experiments, repeat the observation of 50 times.

（1） The influence of initial value to convergence speed

At the moment of filtering begin, it should contain the initiate value $\hat{X}_0$ and P_0 to ensure the process. The initiate value can be assumed or estimated by Eq(8).

Fig.1 and Fig.2 show the mean distance target errors of assuming initiation and estimated initiation.

From the comparison of the two figures, we can see that when selecting initial value according to the two previous observation, beginning in the filtering estimation error is very small, the transient error convergence speed quickly; When the arbitrary selection of initial value in the early stages have larger average error, error of transient slower convergence speed. This consistent with theoretical analysis, so in the process of the simulation should be selected to estimate the initial value, then the value make filtering convergence as soon as possible.



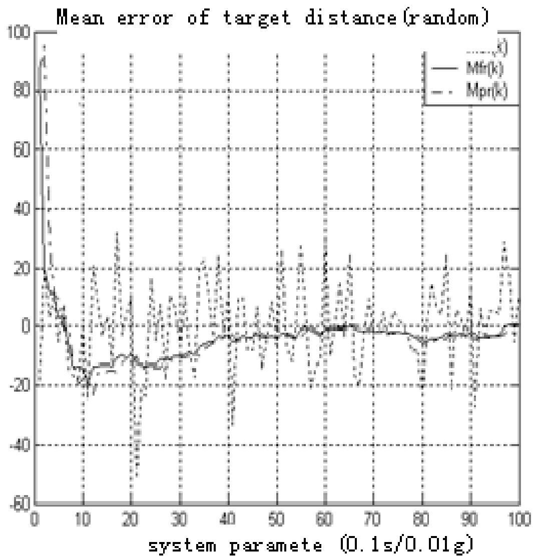
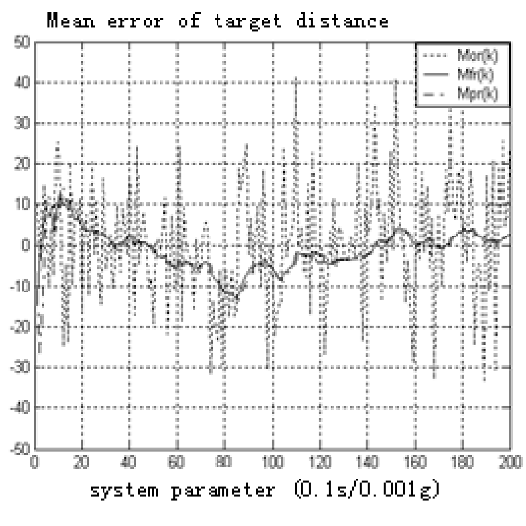


Fig.1 distance errors of estimated initiation **Fig.2 distance errors of assuming initiation**

(2) The influence of sampling interval T for prediction and filtering
Fig. (3) and Fig.(4) respectively show the average errors of target distance estimated, distance observation and distance forecasting when T = 0.1 s and 1 s ,

The comparison show that the smaller the T is , the smaller the error of initial time prediction will be, and the faster the transient convergent speed is. This is because that the smaller the T is, the more observation state (points) it has, and the greater correlation the estimation and the observation will have. Then the estimated results will become faster into the steady state. Gradually with the increase of T value, the process of convergence is not obvious, and the steady state accuracy is improved. This is because the filtering has been convergence, but the error variance matrix is still with the recursive operations gradually decreases, and eventually tend to be more theoretical minimum. In the simulation, when the T value is equal to the actual filtering time increased, the results of the steady-state performance is better.



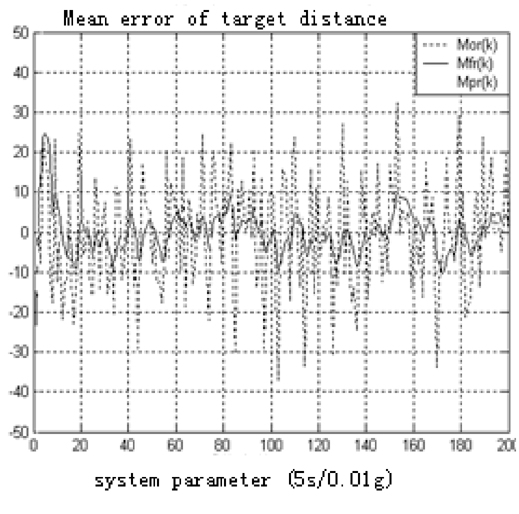


Fig.3 The mean error curve T=0.1s **Fig.4 The mean error curve T=5s**

(3) Noise variance influence on prediction and filtering

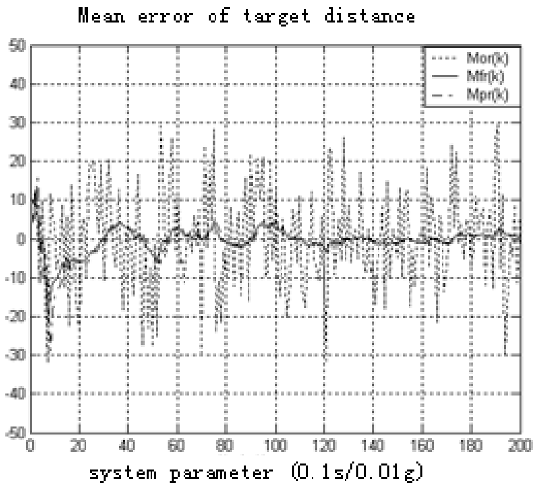


图 5 **The mean error curve** σ_u **=0.01g**

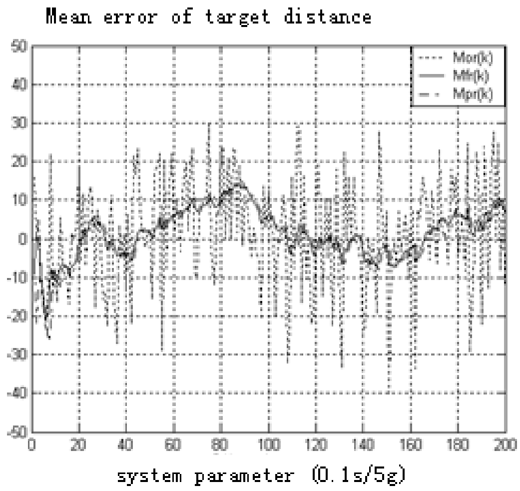


图 **6 The mean error curve** σ_u **=5g**

Working system noise increases the convergence time, down the steady-state accuracy, and makes the prediction error greater than the estimation error. This is because the CV model is based on the conduct of the uniform motion model . When the motor noise of the system increases, the forecast accuracy down, and the estimation depends more and more on the observation. The kalman filter gain adjustment go with the rule as convergence recursive time increasing with the convergence time increasing.

Because the system influence the estimation error caused by the increase of availability through better T improved (equivalent to lengthen estimated time). If it is really do uniform motion, the target constant parameters of CV model can be achieved through multiple recursive estimation variance minimum value, and the prediction error is still large.

(4) The comparison of actual filtering error and the theoretical value
The matrix P in filtering recursive algorithm is the filtering error covariance matrix . Through the independent Monte - Carlo experiments we can calculate the actual filter variance. After comparison the actual filtering error variance value with the theoretical value , we can see that the error variance value is greater than the theoretical value as the increase of estimated time, and the difference with theory errors is in reducing error.

(5)The influence of system parameters on the performance of maneuvering target tracking
As the target tracking time is 10 s, when T = 0.1 s and 1 s , the goal of actual distance and estimated distance is shown in Fig. (6) and Fig.(8). The forecasts value, estimates value and the true value of target azimuth is shown in Fig. (7) and Fig.(9) as T = 0.1 s and 1 s. From the comparision we can see that the smaller T is , the smaller of the value between prediction and filtering , filtering and true is. The differences between the predicted values have higher credibility, and also the estimates have higher accuracy. So it should increase the sampling frequency to filtering.

Fig. (6) and Fig (10) respectively show the actual distance and estimated distance when σ_u = 0.01 g and σ_u =5 g . Fig. (7) and Fig.(11) respectively show the goal of true azimuth and estimate azimuth when σ_u = 0.01 g and σ_u =5 g. It can be seen that, the greater the forecast is , the greater the estimate of the error will be. This is completely in line with the theory. σ_u is for the variance of noise, the greater σ_u is, the greater the influence of noise will have , and the predict and estimate will be less accurate.

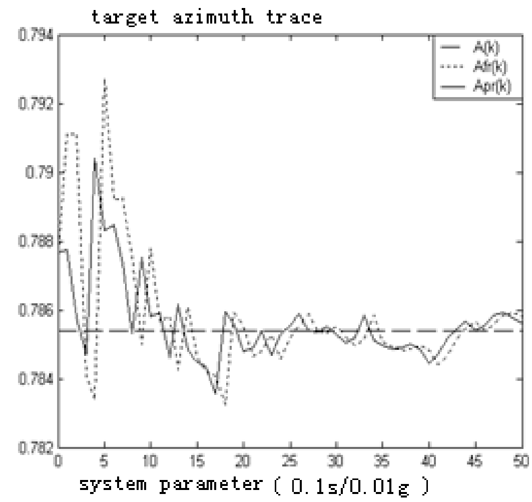


Fig.6 real and estimated distance T=0.1s

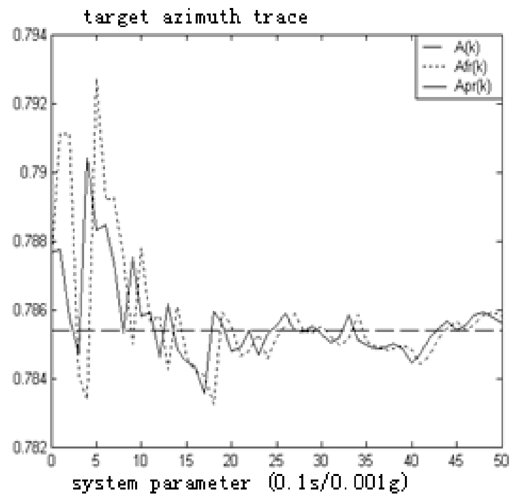


Fig.7 real and estimated direction azimuth T=0.1s

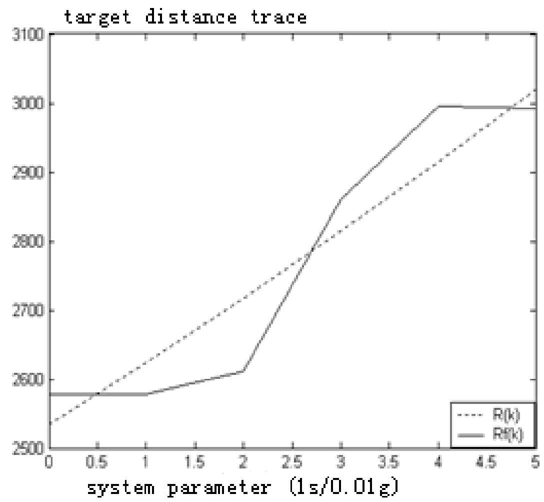


Fig.8 real and estimated distanceT＝1s

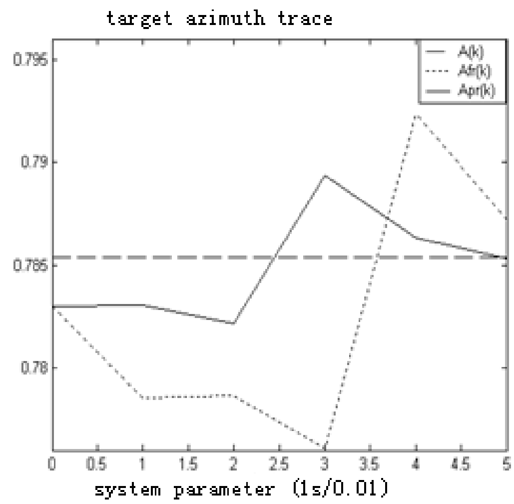


Fig.9 real and estimated direction azimuth T=1s

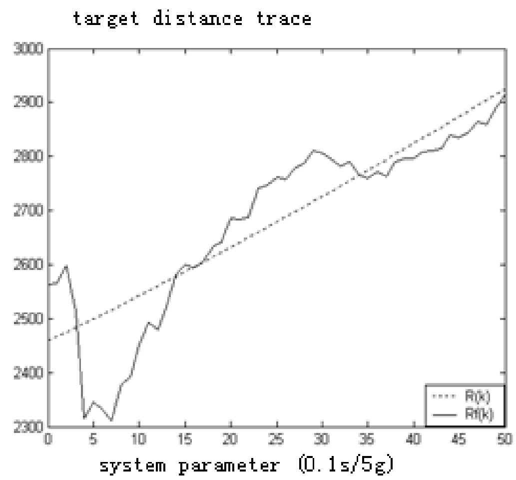


Fig.10 real and estimated distance σ_u =5g

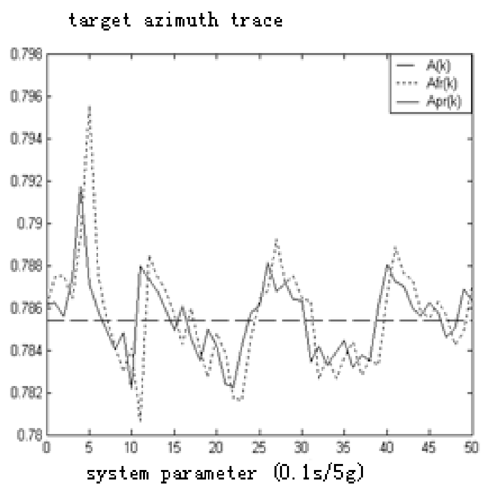


Fig.11 real and estimated direction azimuth σ_u =5g

CONCLUSION

This paper uses the CV model in discrete Kalman filtering to simulate the trace of moving target in 3-D space. We generate the real trace under orthogonal set, transform the observe trace under polar set to the orthogonal set, then apply it to Kalman filtering. We analyze the performance using the mean error and deviation of the observation, prescreen and filtered trace, the effects of original values and system parameters to Kalman filtering are also presented..

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