



Research Article

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The realization of automatic adaptive channel equalization

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ABSTRACT

This paper introduces the automatic adaptive equalization based on SMI, LMS, NLMS and RLS. Matlab tool can be used to simulate the above algorithms in linear dispersion channels. Also the effects of the delayed time ,channel parameter and filtering parameter to the convergence rate and precision are analysed.

Key words: automatic adaptive filtering; SMI; LMS; NLMS; channel equalization

INTRODUCTION

Since b. Widrow et al., 1967, put forward the adaptive filter, due to its advantages of small amount of calculation[1], easy to implement, it is developing rapidly in recent years. At present it has been widely used in channel equalization, including communications system of radar and sonar beam forming, signal detection and many other aspects of the background noise. In the field of signal and information processing, the performance of various adaptive filtering algorithm is put forward and the convergence has become the key to the application of adaptive filtering[2]. This article mainly research the SMI in channel equalization problem such as LMS, NLMS, RLS algorithm. The learning curve is obtained by simulation, and the influence of various parameters on the convergence and steady-state accuracy is also analyzed[3-5].

2. The equalization principle of adaptive filter algorithm

Adaptive filter is established on the basis of learning supervision. The best adjust the weighting coefficient makes a regulation of minimum cost function. In most practical applications, the adaptive process is through the input training sequence, according to minimizing the mean square signal error or minimizing the average power[6]. For a discrete time system, it will be hoped that the output of the adaptive system is defined as the expected response $r(n)$. Considering the stationary random signals, the adaptive system under the condition of minimum mean square error output of optimal weight vector meet wiener hoff equation:

$$\mathbf{h}_{opt} = R^{-1} \cdot \mathbf{p}^* \quad (1)$$

R stands for the autocorrelation of input signal x . \mathbf{p} means the expected response and $r(n)$ is cross-correlation between the input signal x , which conforms to the fact that the solution of wiener hope equation minimize objective function: $J(n) = E\{e(n)e^*(n)\} = E\{|e(n)|^2\}$

The most commonly used method in equalization currently is to insert a transversal filter w between the transmission signal $d(n)$ and the reception filter \mathbf{h} . It is composed of a stripe tap delay line, and each tap delay signal is weighted to a combined circuit output after the summary. Due to the randomness of wireless

communication channel, which requires the equalizer required to real-time tracking time-varying wireless channel, can adjust the tap coefficient automatically according to the channel response. For linear adaptive filter, the method of cumulative coefficient basic can be divided into two kinds, one is based on the gradient algorithm, such as LMS, NLMS in this paper; the other is based on the least squares algorithm, such as RLS in this article. Based on iterative gradient algorithm by searching the error performance of the surface to achieve optimum performance measurement, the least squares algorithm by making a right to seek the optimal cost function minimum value. The article introduces four kinds of adaptive filtering algorithm respectively.

2.1 Sampling matrix inversion algorithm (SMI)

SMI algorithm directly sample from received signal $z(n)$, and calculate the correlation matrix by sampling value. For stationary stochastic signal and the ergodic resistance, the average time averaging set can be gotten by the sampling value. So the input signal autocorrelation matrix and input and expected output of cross-correlation matrix are estimated as follows:

$$\hat{R}_z = \frac{1}{K} \sum_{n=1}^K z(n) \cdot z^T(n) \quad (2)$$

$$\hat{P} = \frac{1}{K} \sum_{k=1}^K z(n) \cdot r(n) \quad (3)$$

K is the observation dimension, which takes the equals length of the equalizer in the simulation.

$$\text{The estimation of weighted vector is: } \mathbf{w} = \hat{R}^{-1} \hat{\mathbf{p}} \quad (4)$$

The estimation of weighted vector is used to estimate the output signal :

$$\hat{d}(n) = \hat{\mathbf{w}}^H(n) z(n) \quad (5)$$

2.2 The least mean square algorithm (LMS)

The LMS algorithm is linear adaptive filter approach. Its most distinguishing feature is Simplicity. the criterion of LMS algorithm is minimum mean square error, which means the expectation value of the difference $e(n)$ between the ideal signal $d(n)$ and the output of filter the square $y(n)$ reach to minimum. And according to this criterion to modify weight coefficient, so it called the minimum mean square error.

The relationship between three basic form of LMS algorithm is as the following:

$$1. \text{ The output: } y(n) = \hat{\mathbf{w}}^H(n) z(n) \quad (6)$$

$$2. \text{ Estimate errors: } e(n) = r(n) - y(n) \quad (7)$$

$$3. \text{ The adaptive weight vector: } \hat{\mathbf{w}}(n+1) = \mathbf{w}(n) + \mu z(n) e^*(n) \quad (8)$$

μ stands for the step parameter.

2.3 The normalized least mean square error (NLMS)

The disorder of LMS algorithm is directly proportional to the tap input vector $z(n)$. When the $z(n)$ become large, the LMS filter come across gradient noise amplification problems. In order to overcome this difficulty, the standard LMS step generation such as an intuitive, the normalized LMS filter (NMLS) can solve this problem:

$$\begin{aligned} \hat{\mathbf{w}}(n+1) &= \hat{\mathbf{w}}(n) + \frac{\mu}{\|z(n)\|^2} z(n) e^*(n) \\ &= \hat{\mathbf{w}}(n) + \mu_N z(n) e^*(n) \end{aligned} \quad (9)$$

So the NLMS algorithm can be treated as the variable steps parameter LMS algorithm.

2.4 The recursive least squares algorithm (RLS)

The LS algorithm take place the average time with the average set. It do not need the statistical features of the known input, which means it is the not best , nor is it a wiener solution. Due to the time average is associated with observation length, by introducing the forgetting factor beta, the filter can be used for nonstationary environment.the The cost function is defined as follows:

$$J(n) = \sum_{i=1}^n \beta(n,i) |e(i)|^2 = \sum_{i=1}^n \lambda^{n-i} |e(i)|^2 . \quad (10)$$

The weight vector $w(n)$ is the use of all the observation data between $i = 1 \sim n$ to estimate. β is the forgetting factor. What makes the smallest weight vector is not the solution of WF equation, but normal equation:

$$\Phi(n)\hat{w}(n) = p(n) \quad (11)$$

$$\text{Then } \Phi(n) = \sum_{i=1}^n \beta^{n-i} \mathbf{z}(i)\mathbf{z}^H(i), \quad p(n) = \sum_{i=1}^n \beta^{n-i} \mathbf{z}(i)d^*(i)$$

Eventually the recursion method of RLS was derived formula is as follows:

$$\mathbf{K}(n) = \frac{\beta^{-1}P(n-1)\mathbf{z}(n)}{1 + \beta^{-1}\mathbf{z}^H(n)P(n-1)\mathbf{z}(n)} \quad (12)$$

$$\hat{\mathbf{w}}(n) = \hat{\mathbf{w}}(n-1) + \mathbf{K}(n)[r^*(n) - \mathbf{z}^H(n)\hat{\mathbf{w}}(n-1)] \quad (13)$$

$$P(n) = \beta^{-1}P(n-1) - \beta^{-1}\mathbf{K}(n)\mathbf{z}^H(n)P(n-1) \quad (14)$$

3 Experiments

Taking advantage of the sending sequence for weight training, which means as the expected response $r(n)$ and the input $z(n)$ known, through the weights of different algorithms we can get estimates. According to the learning curve of different algorithms , we study the transient and steady state performance of the algorithm.

The sending sequence add noise by simulating the channel response , then the output is gotten filtering by inputting the adaptive channel equalizer , as shown in Fig.1. \mathbf{h} is the channel impulse response: $\mathbf{h} = [h(1) \quad h(2) \quad h(3)]^T$. $h(i)$ can be described by the up-cosine function:

$$h(i) = \begin{cases} \frac{1}{2} \cos[\frac{2}{a}(i-2)], & i = 1, 2, 3 \\ 0, & \text{其它} \end{cases}$$

The parameters a control the channel distortion produces, and it also determines the eigenvalue spread $\chi(R) = \lambda_{\max} / \lambda_{\min}$, $R = E\{\mathbf{z}\mathbf{z}^H\}$, λ is the eigenvalue of matrix R .

$d(n)$ is the emission data symbols (message), such as probability distribution ± 1 (as the random numbers with the average to be 0 and the variance to be 1). $v(n)$ is the white noise with the mean to be 0 and the variance to be $\sigma_v^2 = 0.001$. $d(n)$ and $v(n)$ keep statistical independence.

Message after delay time as a reference signal (i.e., for training sequence), the error signal. The length of the equalizer $M = 11$.

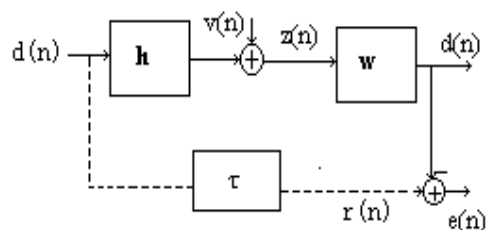


Fig.1 The principle of channel equalization

4. Results and analysis

Four different algorithms have been used for channel equalization in matlab simulation. For every parameter, 100 Monte carlo computer simulation experiments have been done. Through the average of relationship curve between the instantaneous mean square error (mse) $e^2(n)$ and n , we get the average learning curve set of adaptive filter.

4.1 SMI curve

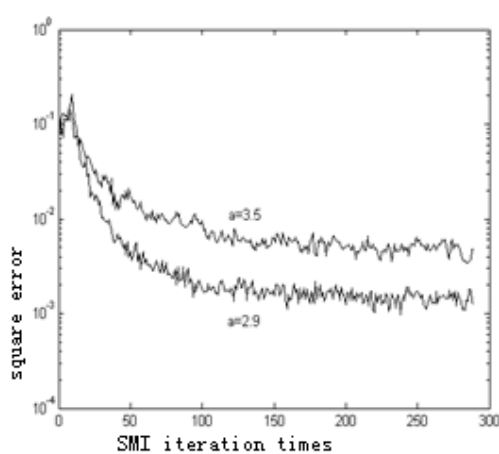
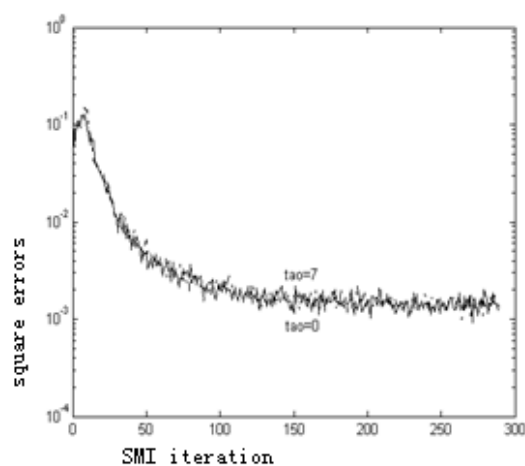
Fig.2 learning curve with $a = 2.9$ and 3.5 Fig.3 learning curve with $\tau = 7$ and 0

Fig.2 is when the channel distortion parameters are 2.9 and 3.5. It can be seen that the greater the eigenvalues spread, the slower convergence rate of SMI adaptive filtering algorithm is, and the greater the steady-state value of mean square error will be.

Fig.3 is learning curve with fixed channel distortion parameters when the delay time is 7 and 0 respectively. It can be seen that the selection of delay time τ has almost no effect to SMI convergence speed and steady-state value of the adaptive filter.

4.2 LMS curve

Fig.4 shows the LMS learning curve with fixed step length parameters, fixed delay time, and changed channel distortion parameters (corresponding eigenvalue spread rate). From the figure, we can see that the change of expanding the range of the eigenvalues reduces the convergence rate of the adaptive equalizer, and also improves the mean square error of steady-state value.

From Fig.5, we can see that the convergence rate of the LMS adaptive filter equalizer to a large extent depends on the step size parameter μ . When μ is large (such as $\mu = 0.075$), the equalizer reaches to the steady state after about 120 times of iteration. When μ is small ($\mu = 0.0075$), convergence speed slows down, it still doesn't reach to the steady state even after one thousand iterations. It can also be seen from the figure that the average mean square error of steady-state value increases with the enhance of μ . This fully shows that the step size of LMS algorithm parameters need a compromise between the convergence speed and steady-state accuracy.

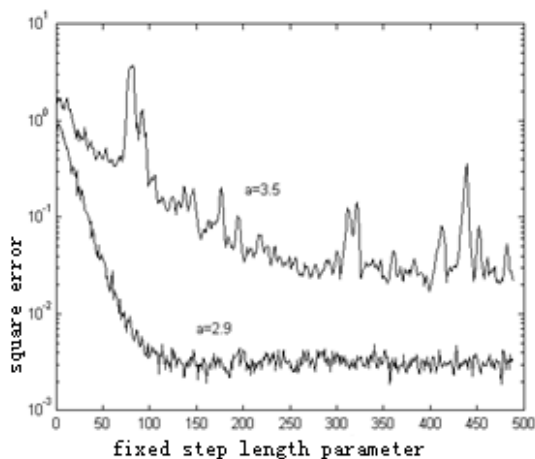


Fig. 4 changed distortion parameters LMS learning curve

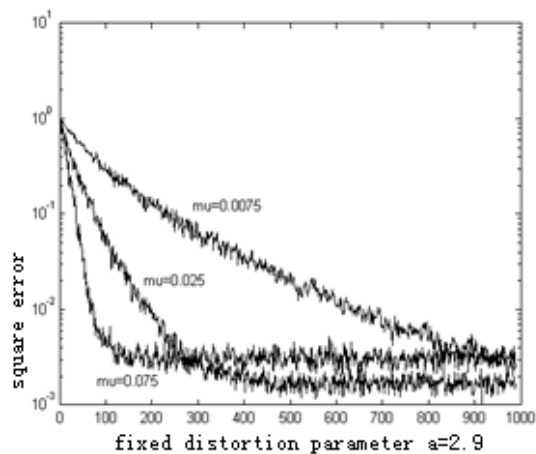


Fig.5 changed step parameters LMS learning curve

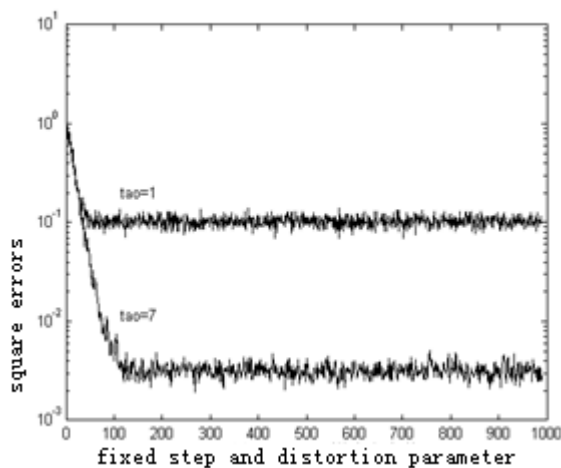


Fig. 6 changed delay time LMS learning curve

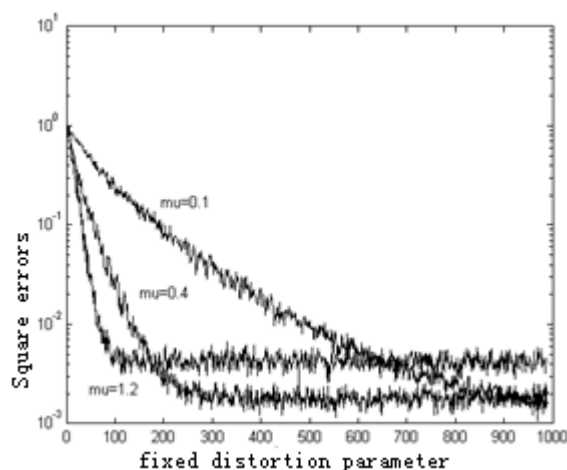


Fig.7 changed step parameters NLMS learning curve

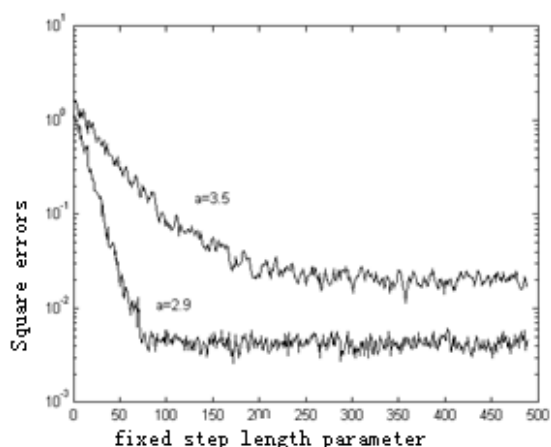


Fig.8 changed distortion parameters NLMS learning curve

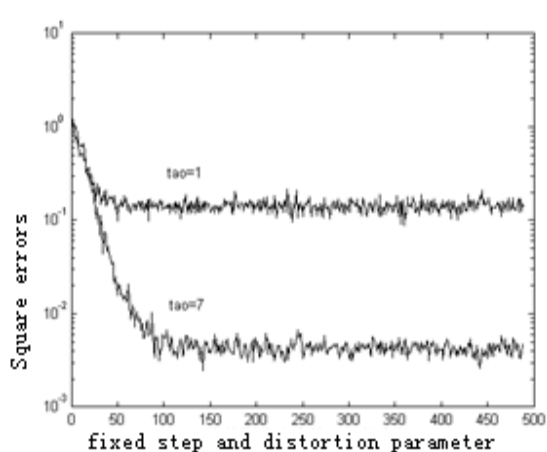


Fig.9 changed delay time LMS learning curve

From Fig.6 we can see that the selection of delay time τ has no obvious effect to LMS adaptive filter convergence rate. But with the appropriate delay time (such as $\tau = 7$), it can reduce the steady-state value of mean square error to very small (as 0.002). If the delay time is not appropriated chosen (such as $\tau = 1$), the steady state value is significantly large(as 0.1).

4.3 NLMS curve

Fig. 7 shows that the bigger the step size parameter is, the slower convergence rate will be, and the smaller the steady-state error value is. This is the same with LMS situation. NLMS has slightly faster convergence speed than the LMS.

Fig.8 shows that the greater channel distortion parameters(or the eigenvalues spread degrees) is, the slower convergence rate the NLMS adaptive filter will have, and the bigger the steady state error value is. This conclusion is the same with LMS algorithm.

Fig.9 shows that the choice of the delay time has a great influence on the average steady state mean square error (mse) value. By choose appropriate delay time (such as $\tau = 7$), the steady state values will be small, otherwise the steady-state value will be great (such as $\tau = 1$).

4.4 RLS curve

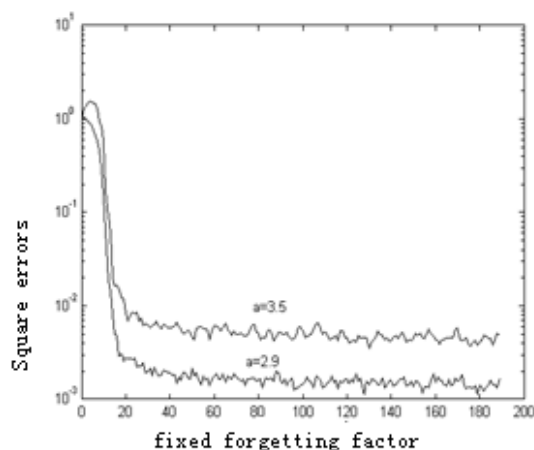


Fig.10 changed distortion parameters RLS learning curve

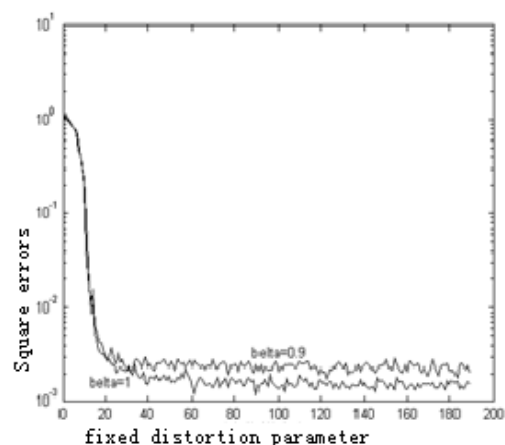


Fig.11 changed forgetting factor RLS learning curve

By Fig.10, we can see that the greater the channel distortion parameters is, the faster the RLS adaptive filter converge, and the greater the average mean square error of steady-state value will be. Fig.11 shows that the greater the forgetting factor β is, the smaller the steady-state value will be, but the convergence speed changes little. Fig.12 shows that choosing the appropriate value of the steady state value for the delay time τ should be very small, otherwise the error will be great.

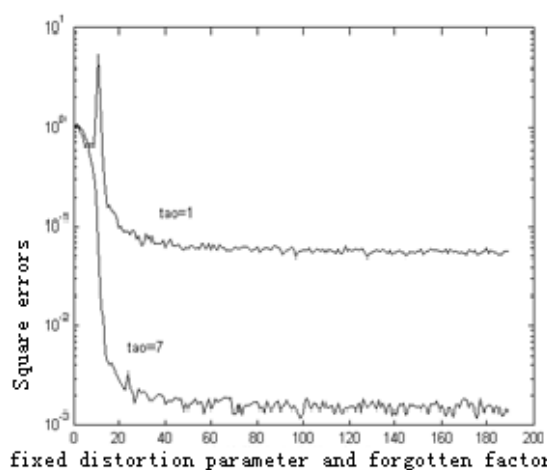


Fig.12 changed delay time RLS learning curve

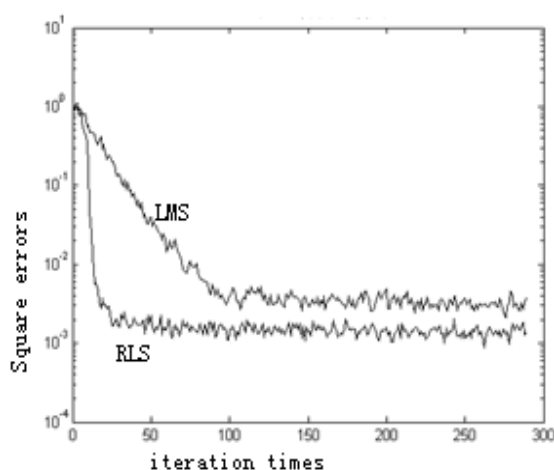


Fig.13 LMS and RLS learning curve

From Fig.10, Fig.11 and Fig.12 we can see that the RLS algorithm will be convergence after about 20 iterations which is about 2 times than the number of transversal filter tap.

Fig.13 shows the learning curve of LMS algorithm and RLS algorithm. As we can see that compared with the

convergence of LMS algorithm, the RLS algorithm convergence of relatively insensitive on the eigenvalues spread of transformation. The RLS algorithm has the much faster convergence rate than LMS algorithm. The RLS algorithm sets has much smaller mean square error value of the steady state than LMS algorithm. It shows that the disorder of RLS algorithm is 0 in theory.

CONCLUSION

Through the simulation of SMI, LMS, NLMS and RLS algorithm, we can get the following conclusion: SMI algorithm has common convergence speed. Subjected to the influence of eigenvalues spread, when the inverse of covariance matrix is morbid matrix, the matrix inversion will be irreversible. So it is rarely used in the practical application. LMS algorithm is very simple, but in order to obtain good steady-state performance, the needed step is small, so the convergence speed is slow, and the operation time is longer. Increasing the amount of calculation is a great disadvantage for real-time processing requirements. In addition, the learning curve of LMS is sensitive to eigenvalues spread. NLMS is the improvement on the LMS algorithm, by using the changed step length, the convergence speed of LMS algorithm is increased, but the steady state error value is not too big to change. RLS algorithm converge quickly, and the error is very small. It is better than the other three clock algorithm. But It usually needed large for the RLS algorithm, and the program structure is complicated.

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