



The p-sets based on the matrix and its application

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ABSTRACT

The P-sets $(A^{\bar{F}}, A^F)$ is introduced to study the dynamic characteristics of the set better. The packet sets $(A^{\bar{F}}, A^F)$ is a collection of sets which is composed of the internal packet set $X^{\bar{F}}$ and the outer packet set X^F ; in this paper, we not only introduce the attribute value matrices in the finite case, but also illustrate the relationship between the attribute value matrices and the packet sets. Finally we give the algorithm and examples using the attribute value matrices for the P-sets.

Key words: P-sets, attribute value matrices, algorithm, application

INTRODUCTION

For example, If $x_1 - x_5$ is 5 apples, with "red, sweet taste". Which can be described using the mathematical language as follows, the apple set $X = \{x_1, x_2, x_3, x_4, x_5\}$, its corresponding attributes set $\alpha = \{\alpha_1, \alpha_2\}$, where α_1 represents red, α_2 represents the sweet taste. ① If we require finding the apple "made in Yantai" in X , which is equivalent to complementing the partial attribute properties to α , α become α^F , that is $\alpha \subseteq \alpha^F$. Accordingly X becomes $X^{\bar{F}} = \{x_1, x_2, x_4\}$, and also $X^{\bar{F}} \subseteq X$. ② If the "red" apple is not needed, the "sweet taste" apple is only needed, which is equivalent to deleting the partial attribute properties to α , α becomes $\alpha^{\bar{F}}$, that is $\alpha^{\bar{F}} \subseteq \alpha$. Accordingly X becomes $X^F = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$, and $X \subseteq X^F$. From the above we can obtain the fact that ordinary set X changes dynamically with the attribute set's dynamical change (widening or narrowing), as the set X becomes a set pair $(X^{\bar{F}}, X^F)$. It is an interesting phenomenon, thus literature proposed the concept, structure and application of the packet sets [1-3].

This paper explain the packet sets in a view of the attribute matrix, then put forward an effective algorithm to construct the packet sets, so as to get ready for people to understand the packet sets further.

THE PACKET SETS

Suppose X be a general finite nonempty set in U , in which U is a finite element universe, V is a finite attribute universe composed of the attribute of the elements in U . In 2008, literature proposes the fact as follows [4, 5]:

Let $X = \{x_1, x_2, \dots, x_m\} \subseteq U$ be a general set, $\alpha = \{\alpha_1, \alpha_2, \dots, \alpha_k\} \subseteq V$ is the corresponding attribute set of X , $X^{\bar{F}}$ is called the internal packet set generated by X and is shorted for the internal packet set. Furthermore:

$$X^{\bar{F}} = X - X^- \quad (1)$$

X^- is called the \bar{F} - element deleted set of X , meanwhile:

$$X^- = \{x \mid x \in X, \bar{f}(x) = u \bar{\in} X, \bar{f} \in \bar{F}\} \quad (2)$$

If the attribute set α^F of $X^{\bar{F}}$ satisfied

$$\alpha^F = \alpha \cup \{\alpha' \mid \beta \in V, f(\beta) = \alpha' \in \alpha, \beta \bar{\in} \alpha, f \in F\} \quad (3)$$

In which $X^{\bar{F}}$ is nonempty.

Let $X = \{x_1, x_2, \dots, x_m\} \subseteq U$ be a general set, $\alpha = \{\alpha_1, \alpha_2, \dots, \alpha_k\} \subseteq V$ is the corresponding attribute set of X , X^F is called the outer packet set generated by X and is shorted for the outer packet set. Furthermore

$$X^F = X \cup X^+ \quad (4)$$

X^+ is called the F – element supplementary set of X , meanwhile

$$X^+ = \{u \mid u \in U, u \bar{\in} X, f(u) = x' \in X, f \in F\} \quad (5)$$

If the attribute set $\alpha^{\bar{F}}$ of X^F satisfy

$$\alpha^{\bar{F}} = \alpha - \{\beta_i \mid \beta_i \in V, \bar{f}(\alpha_i) = \beta_i \bar{\in} \alpha, \bar{f} \in \bar{F}\} \quad (6)$$

In which $\alpha^{\bar{F}}$ is nonempty.

The set pair generated by the internal packet set $X^{\bar{F}}$ and the outer packet set X^F is called the packet set generated by general set X and short for the P-sets. While the general set X is called the ground set of the set pair $(X^{\bar{F}}, X^F)$. Due to the P – set's dynamical property, its genera; representation form is as follows:

$$\{(X_i^{\bar{F}}, X_j^F) \mid i \in I, j \in J\} \quad (7)$$

Where i, j is the index set, the formula (7) is the representation form about the set pair of the P – set. From the formulas (1), (2), (3), (7), we can obtain: $X_n^{\bar{F}} \subseteq X_{n-1}^{\bar{F}} \subseteq \dots \subseteq X_2^{\bar{F}} \subseteq X_1^{\bar{F}} \subseteq X$

From the formulas (4), (5), (6), (7), we can obtain: $X \subseteq X_1^F \subseteq X_2^F \subseteq \dots \subseteq X_{n-1}^F \subseteq X_n^F$

Especially, $(X^{\bar{F}}, X^F)_{F=\bar{F}=\phi} = X \quad \{(X_i^{\bar{F}}, X_j^F) \mid i \in I, j \in J\}_{F=\bar{F}=\phi} = X$

Where \bar{F} and F are called the respectively element emigration function set and the element immigration function set, $f \in F, \bar{f} \in \bar{F}$ are called respectively the element emigration function and the element immigration function.

THE RELATION BETWEEN THE ATTRIBUTE MATRIX AND THE P – SETS

Because U is a finite element universe, V is a finite attribute universe composed of the attribute of the elements in U . Let the cardinal number of U and V respectively be n and m . Then $\forall \alpha_i \in V, x_j \in U, i = 1, 2, \dots, m, j = 1, 2, \dots, n$, suppose $\alpha_j(x_j)$ be \bar{r}_{ij} , which represents how many is the attribute value which x_j have the attribute α_i . For example, if someone's height is 1.78m, the height of attribute value is $\bar{r} = 1.78m$, while if the other person's height is 1.82M, its height attribute values are $\bar{r} = 1.82m$. It shows that different elements having the same attribute value is not consistent. Therefore under the assumption that in the explicit finite element universes or the finite attribute universes, all the elements of all property values are known (For the simplicity of discussion, when some element doesn't have some attribute, the attribute value is assumed to be zero.). Thus $\bar{R} = (\bar{r}_{ij})_{m \times n}$ is expressed as the attribute matrix of the finite attribute universe V . Specifically,

$$\begin{array}{cccc}
 x_1 & x_2 & \dots & x_n \\
 \\
 \alpha_1 & \left(\begin{array}{cccc} \bar{r}_{11} & \bar{r}_{12} & \dots & \bar{r}_{1n} \\ \bar{r}_{21} & \bar{r}_{22} & \dots & \bar{r}_{2n} \\ \dots & \dots & \dots & \dots \\ \bar{r}_{m1} & \bar{r}_{m2} & \dots & \bar{r}_{mn} \end{array} \right) \\
 \alpha_2 & & & \\
 \dots & & & \\
 \alpha_m & & &
 \end{array}$$

Due of the difference of the different attributes, \bar{R}^U is standardized to facilitate the analysis and comparison. Generally, we suppose $r'_{ij} = \frac{\bar{r}_{ij} + \max(\bar{r}_{ij}) - 2 * \min(\bar{r}_{ij})}{\max(\bar{r}_{ij}) - \min(\bar{r}_{ij})}$, where \bar{r}_{ij} the raw data is, $\min(\bar{r}_{ij})$ is the minimum in i line,

$\max(\bar{r}_{ij})$ is the maximum in i line, while the part of which value is zero doesn't participate the standardizing calculation. Thus the normalized matrix $R^U = (r_{ij})_{m \times n}$ is obtained, in which $r_{ij} = 0$ expresses that the element J doesn't have the attribute i , or $r_{ij} \in [1,2]$ expresses that the element j have the attribute i .

Supposing $X = \{x_1, x_2, \dots, x_k\} \subseteq U$ is the basic set, $\alpha = \{\alpha_1, \alpha_2, \dots, \alpha_h\} \subseteq V$ is the corresponding attribute set, where $k \leq n, h \leq m$. Then the relation between the attribute matrix and the P -sets is as follows.

(1) Based on the definition of the packet sets, the internal packet set $X^{\bar{F}}$ compared to the basic set X removed part of the element $\{x | x \in X, \bar{f}(x) = u \in X, \bar{f} \in \bar{F}\}$, at the same time, the attribute set α^F of $X^{\bar{F}}$ compared to the attribute set α added part of the attribute $\{\alpha' | \beta \in V, f(\beta) = \alpha' \in \alpha, \beta \in \alpha, f \in F\}$, this is reflected in the standardization of the attribute matrix R^U can be explained as follows. At first, suppose R^X be the attribute matrix of the basic set X , which is composed of some values of R^U and zero, the basic set X decreased to $X^{\bar{F}}$ and the attribute set α increased to α^F . To illustrate easily, only one attribute α_{h+1} is added, and $\alpha_{h+1}(x_k) = 0$, $\alpha_{h+1}(x_j) \neq 0, j = 1, \dots, k-1$ is obtained, which equals to that the element of the k column of R^X is changed into 0 and the element of the $h+1$ line is revised into nonzero. Therefore R^X is changed into $R^{X^{\bar{F}}}$ which is composed of some values of R^U and zero.

(2) Based on the definition of the packet sets, the outer packet set X^F compared to the basic set X added part of the element $\{u | u \in U, u \in X, f(u) = x' \in X, f \in F\}$, at the same time, the attribute set $\alpha^{\bar{F}}$ of X^F compared to the attribute set α removed part of the attribute $\{\beta_i | \beta_i \in V, \bar{f}(\alpha_i) = \beta_i \in \alpha, \bar{f} \in \bar{F}\}$, this is reflected in the standardization of the attribute matrix R^U can be explained as follows. At first, suppose R^X be the attribute matrix of the basic set X , which is composed of some values of R^U and zero, the basic set X increased to $X^{\bar{F}}$ and the attribute set α decreased to $\alpha^{\bar{F}}$. To illustrate easily, only one attribute α_h is deleted, and $\alpha_h(x_{k+1}) = 0, \alpha_h(x_i) \neq 0, i = 1, \dots, k$ is obtained, which equals to that the element of the $k+1$ column of R^X is revised and the element of the h line is revised into zero. Therefore R^X is changed into R^{X^F} which is composed of some values of R^U and zero. The following expression is which of the attribute matrix of the basic set X in the attribute matrix of the universe U .

$$\begin{array}{ccccccc}
 & x_1 & x_2 & \dots & x_k & \dots & x_n \\
 \alpha_1 & \left(\begin{array}{cccccc}
 r_{11} & r_{12} & \dots & r_{1k} & \dots & r_{1n} \\
 r_{21} & r_{22} & \dots & r_{2k} & \dots & r_{2n} \\
 \dots & & & & & \\
 \alpha_h & r_{h1} & r_{h2} & \dots & r_{hk} & \dots & r_{hn} \\
 \dots & & & & & & \\
 \alpha_m & r_{m1} & r_{m2} & \dots & r_{mk} & \dots & r_{mn}
 \end{array} \right)
 \end{array}$$

THE GENERATION ALGORITHM AND THE APPLICATION OF THE PACKET SETS

Firstly, we put forward an algorithm based on the attribute matrix. Let $R^X \leq R^U$ represent $r_{ij}^X \in R^X \leq r_{ij}^U \in R^U$ and $\forall r_i \in R^U$ represent that r_i is the i line in R^U . The concrete algorithm is as follows.

If $R^X \equiv R^U$ Then $X^{\bar{F}} = X$, $X^F = X$

Else if $R^X \leq R^U$ Then $\forall r_i \in R^U$

If $r_i \cap R^X = \emptyset$ Then $R^{X^{\bar{F}}} = R^X \cup r_i$

Else if $r_i \cap R^X \neq \emptyset$ Then $R^{X^F} = R^X - r_i$

End

End

The application as an example is as follows. In a company, there is an important link in the evaluation mechanism to evaluate the employee according to the various criteria, i.e. attribute. If there be 9 employees, denote the employee set $U = \{x_1, \dots, x_9\}$, the attribute set $\alpha = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$. Combining the above attribute set, the evaluation results on 9 employees were standardized as the matrix.

Due to the special case of the various departments, all the departments used the criteria as reference but not fully implementing. For instance, all of the employees in the sales department $X = \{x_1, x_3, x_4, x_6, x_7, x_9\}$, reached the interior standard formulated by the sales department using the company's criteria as reference, and the attribute set $\alpha = \{\alpha_3, \alpha_5\}$, the attribute matrix is as following.

$$R^X = \begin{pmatrix}
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 1.3 & 0 & 1.75 & 1.47 & 0 & 1.76 & 1.8 & 0 & 1.4 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 1.5 & 0 & 1.56 & 1.8 & 0 & 1.1 & 2 & 0 & 1
 \end{pmatrix}$$

That is to say that the staff did not fully meet all the evaluation criteria of the company, to select talents strictly according to the company's standard, the person in charge studied carefully each staff, then introduced the new attribution α_1, α_2 , this lead to that the number of employees who can meet the attribute requirement reduced to 3, the employee set were modified to $X^{\bar{F}} = \{x_4, x_6, x_9\}$, while the attribute set is added to $\alpha^F = \{\alpha_1, \alpha_2, \alpha_3, \alpha_5\}$. Thus the evaluation results of the 3 employees are given in the following attribute matrix.

$$R^{X^{\bar{F}}} = \begin{pmatrix}
 0 & 0 & 0 & 1.4 & 0 & 2 & 0 & 0 & 1.2 \\
 0 & 0 & 0 & 1.2 & 0 & 1.14 & 0 & 0 & 1.84 \\
 0 & 0 & 0 & 1.47 & 0 & 1.76 & 0 & 0 & 1.4 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 1.8 & 0 & 1.1 & 0 & 0 & 1
 \end{pmatrix}$$

The above matrix show that the employees who can meet the attribute decreased gradually, while the elite employees who can meet the company criteria appeared gradually.

On the contrary, because of the job's need, in order to attract more outstanding personnel, sales department adjusted the evaluation criteria, the attribute α_3 was cancelled, and the employee x_2 in other departments was attracted. Thus the number of the employees was added 1, the employees set was modified to $X^F = \{x_1, x_2, x_3, x_4, x_6, x_7, x_9\}$, the corresponding attribute $\alpha^F = \{\alpha_5\}$, and the evaluation results of the 7 employees are given in the following attribute matrix.

$$R^{X^F} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.5 & 1.6 & 1.56 & 1.8 & 0 & 1.1 & 2 & 0 & 1 \end{pmatrix}$$

The above matrix show that with the reduction of the attribute, the employees who can meet the attribute increased gradually, while more elite personnel were attracted to the sales department gradually.

SUMMARY

This paper explain the packet sets in a view of the attribute matrix based on the finite universe, put forward an effective algorithm to construct the packet sets, and the properties of the dynamic changes were embodied through the dynamic changes of the matrix, so the description of the packet set are more intuitive and deep with respect to other papers.

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