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The further discussion of convergence of exchangeable random variables sequence

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ABSTRACT

In many practical problems, samples are not independent, so the concept of dependent random variables in probability and statistics is mentioned. Exchangeable random variables is a major type of dependent random variable. As the fundamental structure theorem of infinite exchangeable random variables sequences, the De finetti's theorem states that infinite exchangeable random variables sequences is independent and identically distributed with the condition of the tail σ-algebra. So some results about independent identically distributed random variables is similar to exchangeable random variables. By using reverse martingale approach, some scholars have given some results. In this paper we do some researches about the similarity and difference of identically distributed random variables and exchangeable random variables sequences, mainly discuss the limit theory of exchangeable random variables.

Key words: identically distributed random variables; dependent random variables; exchangeable random variables

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INTRODUCTION

If the joint distribution of $X_1, X_2, \cdots, X_n$ is permutation invariant, for each permutation π of $1, 2, \cdots, n$, the joint distribution of $X_1, X_2, \cdots, X_n$ is the same of the joint distribution of $X_{\pi(1)}, X_{\pi(2)}, \cdots, X_{\pi(n)}$, so the finite random variable sequence $X_1, X_2, \cdots, X_n$ is exchangeable。Obviously, the independent and identically distributed random variable sequence is the simplest exchangeable random variable sequence.The concept of exchangeability was first proposed by De Finett[1] in 1930, people using the De Finetti theorem has made some results (see [2]-[5]). In this paper we extend the results of Katz and Baum theorem in the condition of independent and identically distributed random variable sequence to the results of Katz and Baum theorem in the condition of exchangeable random variables(see [6]-[8]).. We obtain the Katz and Baum theorem for specific forms of expression in the case of exchangeable random variables.

PRELIMINARY CONCEPTS

Definition [4]: when on the $[0, \infty)$ the positive function $l(x)(x \to \infty)$ is slowly varying function, as for all $c > 0$, $\lim_{x\to\infty} \frac{l(cx)}{l(x)} = 1$. About slowly varying function with the following properties: if it is slowly varying function when $l(x)(x \to \infty)$, so

$$(1)\quad \lim_{x\to\infty} \frac{l(tx)}{l(x)} = 1,\ \forall t > 0,\ \lim_{x\to\infty} \frac{l(x+u)}{l(x)} = 1,\ \forall u \ge 0;$$

(2) $\lim_{k\to\infty}\sup_{2^k\le x<2^{k+l}}\frac{l(x)}{l(2^k)}=\lim_{k\to\infty}\inf_{2^k\le x<2^{k+l}}\frac{l(x)}{l(2^k)}=1$；

(3) $\lim_{x\to\infty}x^{\delta}l(x)=\infty,\ \forall\delta>0,\ \lim_{x\to\infty}x^{-\delta}l(x)=0$。

Lemma 1 Suppose $\{X_n;n\ge1\}$ are exchangeable random variables, and satisfy

$Cov(f_1(X_1),f_2(X_2))\le 0$

$\forall m\ge2,\ A_1,A_2,\cdots A_m$ is $\{1,2,\cdots,n\}$ twenty-two disjoint non-empty set，if $f_i,i=1,2,\cdots,m$ each argument is a function of both the non-drop (non-liters，so

(1) if $f_i\ge0,i=1,2,\cdots,m$， we obtain

$$E\left(\prod_{i=1}^n f_i(X_j,j\in A_i)\right)\le\prod_{i=1}^n Ef_i(X_j,j\in A_i)$$

(2) $\forall\ x_i\in R,i=1,2,\cdots,m,$

$$P(X_1<x_1,\cdots X_m<x_m)\le\prod_{i=1}^n P(X_i<x_i)$$

Lemma 2 Suppose $\{X_n;n\ge1\}$ are exchangeable random variables,

$Cov(f_1(X_1),f_2(X_2))\le 0$

$EX_n=0,|X_n|\le b_n\quad a.s.\ (n=1,2,\cdots),\ t>0$ and $t\max_{1\le i\le n}b_i\le1$，so $\forall u>0$， we obtain

$$P\left(\left|\sum_{i=1}^n X_i\right|\ge u\right)\le2\exp\left\{-tu+t^2\sum_{i=1}^n EX_i^2\right\}$$

Proof: because $Y_n \triangleq \sum_{k=0}^n\frac{(tX_i)^k}{k!}\to e^{tX_i}$， $|tX_i|\le1\ a.s.$ we obtain $|Y_n|\le e\ a.s.$.so

By Lebesgue Theorem control convergence，we obtain

$$E(e^{tX_i})=E\left(\lim_{n\to\infty}\sum_{k=0}^n\frac{(tX_i)^k}{k!}\right)=\lim_{n\to\infty}\sum_{k=0}^n\frac{E(tX_i)^k}{k!}$$

$$=\sum_{k=0}^n\frac{E(tX_i)^k}{k!}\le1+E(tX_i)^2\sum_{k=2}^\infty\frac{1}{k!}$$

$$\le1+t^2EX_i^2\le e^{t^2EX_i^2}$$

By Markov inequality and lemma1(1)，we obtain $\forall u>0,\ t>0$，有

$$P\left(\sum_{i=1}^n X_i>u\right)=P\left(e^{t\sum_{i=1}^n X_i}>e^{tu}\right)\le e^{-tu}Ee^{t\sum_{i=1}^n X_i}$$

$$\le e^{-tu}\prod_{i=1}^n Ee^{tX_i}\le e^{-tu}\prod_{i=1}^n e^{t^2EX_i^2}$$

$$=\exp\left(-tu+t^2\sum_{i=1}^n EX_i^2\right)$$

We use $-X_i$ instead of X_i

$$P\left(\sum_{i=1}^n(-X_i)>u\right)=P\left(\sum_{i=1}^n(X_i)<-u\right)\le\exp\left(-tu+t^2\sum_{i=1}^n EX_i^2\right)$$

$$P\left(\left|\sum_{i=1}^{n} X_i\right| \geq u\right) = P\left(\sum_{i=1}^{n} X_i > u\right) + P\left(\sum_{i=1}^{n} X_i < -u\right)$$
$$\leq 2\exp\left(-tu + t^2\sum_{i=1}^{n} EX_i^{\ 2}\right)$$

THE MAIN CONCLUSION

Theorem: Suppose $\{X_n; n \geq 1\}$ are exchangeable random variables, $0 < r < 2$ and when $r \geq 1$, $EX_1 = 0$, so

$$\lim_{n\to\infty} S_n / n^{1/r} = \lim_{n\to\infty} X_n / n^{1/r} = 0 \quad a.s,$$
$$\lim_{n\to\infty}\sup |S_n| / n^{1/r} = \lim_{n\to\infty}\sup X_n / n^{1/r} = \infty \quad a.s$$

proof: because $E|X_1|^r < \infty$ and $\sum_{n=1}^{\infty} P\left[|X_n|^r > n^{1/r}\right] < \infty$, so

$$\sum_{n=1}^{\infty} P\left[|X_n| > \varepsilon n^{1/r}\right] < \infty, \forall \varepsilon > 0$$

so $\lim_{n\to\infty} S_n / n^{1/r} = 0 \quad a.s$, wnen $\lim_{n\to\infty} X_n / n^{1/r} = 0 \quad a.s$

suppose $\sum_{n=1}^{\infty} P\left[X_n / n^{1/r} \to 0\right] > 0$, by Kolmogorov 0-1, for $\varepsilon > 0$, so

$$\sum_{n=1}^{\infty} P\left[|X_n|^r \varepsilon^r > n\right] = \sum_{n=1}^{\infty} P\left[|X_1|^r \varepsilon^r > n\right] = \infty$$

so $E|X_1|^r = \infty$, we obtain

$$\sum_{n=1}^{\infty} P\left[|X_n| > Mn^{1/r}\right] = \infty, \forall M > 0,$$

so $\lim_{n\to\infty}\sup |X_n| / n^{1/r} = \infty \quad a.s$, by Borel-Cantelli

$$\lim_{n\to\infty}\sup |X_n| / n^{1/r} = \lim_{n\to\infty}\sup |S_n - S_{n-1}| / n^{1/r}$$
$$\leq \lim_{n\to\infty}\sup |S_n| / n^{1/r} + \lim_{n\to\infty}\sup |S_{n-1}| / n^{1/r} = 2\lim_{n\to\infty}\sup |S_n| / n^{1/r}$$

so $\lim_{n\to\infty}\sup |S_n| / n^{1/r} = \infty \quad a.s.$

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