Available online www.jocpr.com

Journal of Chemical and Pharmaceutical Research, 2013, 5(9):333-340

Research Article

ISSN : 0975-7384
CODEN(USA) : JCPRC5

The design of a kind of multi-wavelet signal filtering algorithm and its application to precision optical tracking servo system

Yang Pu, Ni Jiangfan and Li Xiao

Automation Department Nanjing University of Aeronautics & Astronautics, China

---

ABSTRACT

This paper is aiming at non-stationary characteristics of gyroscope noise in precision optical tracking servo system, using the advantages of orthogonal wavelet transform, to present a multi-wavelets fast filtering method which is based on an energy weighted threshold, it can simplify multi-wavelets preprocessing method, and automatically change filtering threshold of different wavelet coefficients on the same scale through the design of a self adjusting energy weighted threshold, optimize multi-wavelets signal filtering algorithm. The practical application to optical servo system shows, this filtering method can take obvious inhibitory effect on gyroscope noise, the filtering effect is superior to Kalman filter which is based upon model and single wavelet filtering method with fixed threshold..

Keywords: multi-wavelets, optical tracking, gyroscope noise, signal processing.

INTRODUCTION

As common speed feedback measuring sensor of optical tracking servo system, the gyroscope is directly related to the precision of target tracking in system and the control performance of the whole system, however, the precision of output signal is affected by many factors, self zero drift and outside environment interference , etc[1]. Therefore, in practical application, designing a reasonable and effective method for error compensation and filtering processing of the gyroscope output signal is the fundamental means to guarantee the stability and tracking accuracy of high precision optical tracking system. Multi-wavelets is a new development of wavelet theory, because it can simultaneously satisfy the symmetry, short support, second orders of vanishing moments and orthogonality, not only compensate for the lack of orthogonality and symmetry which wavelet can not have at the same time, but also remain the good localization of wavelet transform in time and frequent domain, so it has an advantage over wavelet in the field of signal processing, and it is more suitable for the application to precision optical tracking servo system[2].In this paper, we introduce the Multi-wavelet theory to the gyro signal filtering algorithm in optical tracking system, design a self-tuning energy weighted threshold multi-wavelets fast filtering method, and get better result in the application of actual gyro feedback servo system.

Orthogonal multi-wavelets and its fast pyramid algorithm.

Definition1[3] Let $\Phi(t)=[\varphi_1(t),\varphi_2(t),\cdots,\varphi_s(t)]^T{}_{s\in N}$ to be a set of orthogonal multi-scaling function in the multiresolution analysis space $\{V_j\}_{j\in Z}$, $\Psi(t)=[\psi_1(t),\psi_2(t),\cdots,\psi_s(t)]^T{}_{s\in N}$ is the multi-wavelets function corresponding with the scaling function, and satisfy the orthogonal matrix of an orthogonal subspace $\{W_j\}_{j\in Z}$,which formed from its panning and zooming $\Psi_{j,k}(t)=\left\{\begin{matrix}2^{j/2}\psi_1(2^jt-k),2^{j/2}\psi_2(2^jt-k)\\ ,\cdots,2^{j/2}\psi_s(2^jt-k)\end{matrix}\right\}_{j,k\in Z}$,

namely, $V_{j+1}=V_j\oplus W_j$, then $\Phi(t)$ and $\Psi(t)$ satisfy two-scale matrix equation as follows:

$$\begin{cases}\Phi(t)=\sum_{k\in z}\sqrt{2}H_k\Phi(2t-k)\\ \psi(t)=\sum_{k\in z}\sqrt{2}G_k\Phi(2t-k)\end{cases} \tag{1}$$

Where H_k, G_k is a $s\times s$ matrix. On frequency, the frequency response of the matrices corresponding with H_k, G_k .

$$\begin{cases}H(\omega)=\sum_{k\in z}H_k e^{-ik\omega}\\ G(\omega)=\sum_{k\in z}G_k e^{-ik\omega}\end{cases} \tag{2}$$

For arbitrary signal $f(t)\in V_J$, can be expressed as linear combination of $\{\Phi_i(t-k),1\le i\le s,k\in z\}$:

$$f(t)=\sum_{i=1}^{s}\sum_{k\in z}c_{i,J,k}\varphi_i(t-k) \tag{3}$$

Through further decomposition, we can get:

$$f(t)=\sum_{i=1}^{s}\sum_{k\in z}c_{i,J_0,k}2^{J_0/2}\varphi_i(2^{J_0}t-k) +\sum_{i=1}^{s}\sum_{J_0\le j\le J}\sum_{k\in z}d_{i,j,k}2^{j/2}\psi_i(2^j t-k) \tag{4}$$

From the multiresolution analysis theory of multi-wavelets, we can get multi-wavelets pyramid fast decomposition algorithm for any signal:

$$\begin{cases}C_{j-1,k}=\sum_{n\in Z}\bar{H}_{n-2k}C_{j,n}\\ D_{j-1,k}=\sum_{n\in Z}\bar{G}_{n-2k}D_{j,n}\end{cases},\ j,k\in Z \tag{5}$$

The corresponding signal Reconstruction formula:

$$C_{j,n}=\sum_{k\in z}H_{n-2k}C_{j-1,k}+\sum_{k\in z}G_{n-2k}D_{j-1,k} \tag{6}$$

Among them, $C_{j,k}=[c_{1,j,k},c_{2,j,k},\cdots,c_{s,j,k}]^T$, $D_{j,k}=[d_{1,j,k},d_{2,j,k},\cdots,d_{s,j,k}]^T$,represents the signal as (coarse signal and detail signals) in resolution of 2^j respectively.

Design of multi-wavelets pretreatment method

Signal pre-post processing is an important link in multi-wavelets analysis. Pretreatment is related to multi-wavelet properties, can not be selected arbitrarily, inappropriate pretreatment will not get satisfied decomposition results. In this paper, we use GHM multi-wavelets, design following simple pretreatment method, set the wavelet multiplicity s=2, then the scaling function $\Phi(t)=[\varphi_1(t),\varphi_2(t)]^T$, wavelet function $\Psi(t)=[\psi_1(t),\psi_2(t)]^T$, and the support interval of $\varphi_1(t)$ is [0,1], the support interval of $\varphi_2(t)$ is [0,2], besides, $\varphi_1(t)$ and $\varphi_2(t)$ are symmetrical, have second vanishing moments, so we can calculate the coefficients of Eq.1:

$$h_1=\begin{bmatrix}\frac{3\sqrt{2}}{10} & \frac{4}{5}\\ \frac{-1}{20} & \frac{-3\sqrt{2}}{20}\end{bmatrix},\ h_2=\begin{bmatrix}\frac{3\sqrt{2}}{10} & 0\\ \frac{9}{20} & \frac{1}{\sqrt{2}}\end{bmatrix},$$

$$h_3=\begin{bmatrix}0 & 0\\ \frac{9}{20} & \frac{-3\sqrt{2}}{20}\end{bmatrix},\ h_4=\begin{bmatrix}0 & 0\\ \frac{-1}{20} & 0\end{bmatrix}$$

$$g_1=\begin{bmatrix}\frac{-1}{20} & \frac{-3\sqrt{2}}{20}\\ \frac{-\sqrt{2}}{20} & \frac{-3}{10}\end{bmatrix},\ g_2=\begin{bmatrix}\frac{-9}{20} & \frac{-1}{\sqrt{2}}\\ \frac{-9\sqrt{2}}{20} & 0\end{bmatrix},$$

$$g_3=\begin{bmatrix}\frac{9}{20} & \frac{-3\sqrt{2}}{20}\\ \frac{9\sqrt{2}}{20} & \frac{-3}{10}\end{bmatrix},\ g_4=\begin{bmatrix}\frac{-1}{20} & 0\\ \frac{-\sqrt{2}}{20} & 0\end{bmatrix}$$

Let signal $f(t)\in L^2(R)$, using non-repeated strict sampling, and let the number of sampling points $n=2m$, sampling signal $y(k)$ is:

$$y(k)=f\left(\frac{k}{2}\right),\quad k=1,2,\cdots,n \tag{7}$$

We can get:

$$y(2k)=f(k),\ y(2k+1)=f\left(k+\frac{1}{2}\right) \tag{8}$$

According to Eq.3 we can further get the pretreatment formula of GHM multi-wavelets:

$$\begin{cases} c_{1,0,k}=\dfrac{y(2k+1)}{\varphi_1(1/2)}-\dfrac{y(2k+2)\varphi_2(1/2)}{\varphi_1(1/2)\varphi_2(1)} \\ -\dfrac{y(2k)\varphi_2(3/2)}{\varphi_1(1/2)\varphi_2(1)} \\ c_{2,0,k}=\dfrac{y(2k+2)}{\varphi_2(1)} \end{cases} \tag{9}$$

The corresponding treatment formula:

$$\begin{cases} y(2k)=c_{1,0,k-1}\varphi_2(1) \\ y(2k+1)=c_{1,0,k}\varphi_1(1/2)\quad +c_{2,0,k}\varphi_2 \\ (1/2)+c_{2,0,k-1}\varphi_2(3/2) \end{cases} \tag{10}$$

Design of multi-wavelets energy weighted threshold

Three methods are usually used in wavelet filtering: modulus maxima method, method based on spatial correlation , and threshold method[5].Because the threshold filtering algorithm is relatively simple, has small amount of calculation, so it is particularly suitable for practical application of gyro signal filtering. Considering the common soft-threshold and hard-threshold function, in this paper, we use an improved modulus square threshold function[6]:

$$\hat{d}_{j,k} = \begin{cases} \operatorname{sign}\left(d_{j,k}\right) \cdot \\ \sqrt{\left(d_{j,k}\right)^2 - \alpha\lambda^2}, & \left|d_{j,k}\right| \ge \lambda \\ 0, & \left|d_{j,k}\right| < \lambda \end{cases} \tag{11}$$

In the formula, the adjustment factor $\alpha = \left(\lambda / \left|d_{j,k}\right|\right)^j$.

Because the estimated coefficient which is got form Eq.11 is closer to non-noise coefficients than the simply soft and hard threshold method, so it is better than common modulus square threshold processing method.

In threshold filtering method, the selection of threshold is very important. For different thresholds, the SNR (signal-to-noise ratio) of the filtered signal has obvious differences. So far, there is not a "universal" threshold which is suitable for any situation. Considering requirements of the precision optical tracking system for algorithm real-time ,in this paper, we design a variable threshold based on 3σ rule[7],it has better filtering result and simple calculation process.

By the wavelet transform theory, the wavelet transform coefficients of white Gaussian noise are independent and still keep Gauss distribution. Through probability and statistics, if $X \sim N\left(\mu, \sigma^2\right)$,then

$$P\left\{\mu - 3\sigma \le X \le \mu + 3\sigma\right\} = 0.9974 \tag{12}$$

Through statistical characteristics of the system gyro output signal, the noise signal is consistent with the distribution characteristics $X \sim N\left(0, \sigma^2\right)$, Therefore, if the threshold $\lambda = 3\sigma$,then

$$P\left\{-3\sigma \le X \le 3\sigma\right\} = 0.9974 \tag{13}$$

Namely, the probability that X fall in the interval $\left[-3\sigma, 3\sigma\right]$ is 0.9974, energy coefficient of white noise is almost all filtered.

In the practical application, 3σ threshold can not effectively reflect different propagation characteristics of signal and noise wavelet coefficients at each decomposition level, In order to make the change of threshold and wavelet coefficients of the noise consistent in propagation characteristics at each scale, we design the following variable threshold:

$$\lambda_j = \frac{3\sigma_j}{\ln\left(j+1\right)} \tag{14}$$

Where σ_j is the variance of j th level noise wavelet coefficients.

Comparing with the single wavelet filter, the high-frequency wavelet coefficients in S dimensions will be solved in the each layer of the multi-wavelet filters. The thresholds of the multi-wavelet coefficients in S dimensions will be the same, which of the multi-wavelet coefficients are selected as the traditional methods. here, to the principle of the analysis of multi-wavelet, the traditional methods for deciding the thresholds is unreasonable. In this paper, the multi-wavelet's filters for gyro's noise signal performs well with the pre-processing method and the multi-wavelet coefficients in S dimensions are not the same. So it is necessary to do the further study for the thresholds of the multi-wavelet coefficients in each layer for the better denoising effects. In this paper, the energy function is devised to self-adjust the weight of multi-wavelet coefficients. The weight of the thresholds for the coefficients will be self-adjusted according to the multi-wavelet coefficients in each level of decomposing the actual Gyro signal.

Calculate the energy function of high frequency coefficients of the j-lever s-dimension wavelet:

$$E_{i,j}=\sum_{k\in z}(d_{i,j,k})^2\ ,\quad i=1,2,\cdots,s \tag{15}$$

Without loss of generality, we assume that $E_{1,j}$ is the maximum, then calculate the corresponding wavelet threshold according to Eq.14:

$$\lambda_{1,j}=\lambda_j=\frac{3\sigma_j}{\ln\left(j+1\right)} \tag{16}$$

The energy weights of wavelet coefficients are designed as:

$$\beta_{i,j}=\frac{E_{i,j}}{E_{1,j}}\ ,\quad i=2,3\cdots,s \tag{17}$$

It automatically adjusts the threshold corresponding to the wavelet coefficients in the light of Eq.16, Eq.17, and the actual threshold of the coefficients are:

$$\lambda_{i,j}=\beta_{i,j}\cdot\lambda_{1,j}\ ,\quad i=2,3\cdots,s \tag{18}$$

Through the above analysis, we can obtain the actual procedure of the energy-weighted multi-threshold wavelet filtering algorithm is:

Step1: Get the sample value $\omega(n)$ by sampling gyro output rate signal, and preprocess the signal according Eq.7~Eq.9.

Step2: Let the signal pyramid decomposed by multi-wavelet according formula (5), we can get the multi-wavelet coefficients on different decomposition scale space.

Step3: Calculate according to Eq.16-Eq.18, and then we obtain the denoising threshold of the high frequency coefficients of wavelet with different decomposition scales $\lambda_{i,j}$.

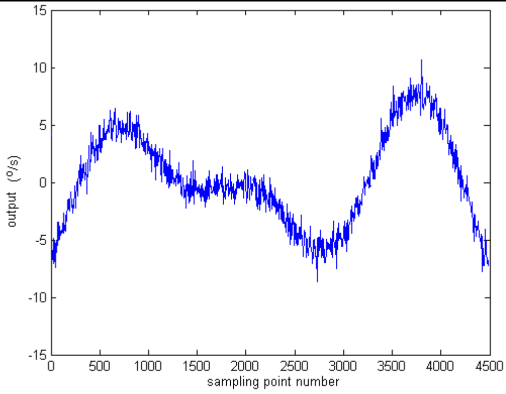
Step4: By substituting $d_{i,j,k}$ into Eq.11, we can obtain the new estimated wavelet coefficients $\hat{d}_{i,j,k}$.

Step5**:** Based on the estimated wavelet coefficients $\hat{d}_{i,j,k}$, reconstruct wavelet according to Eq.6, and using Eq.10 for data processing, then get the filtered gyro rate signal $\hat{\omega}(n)$.

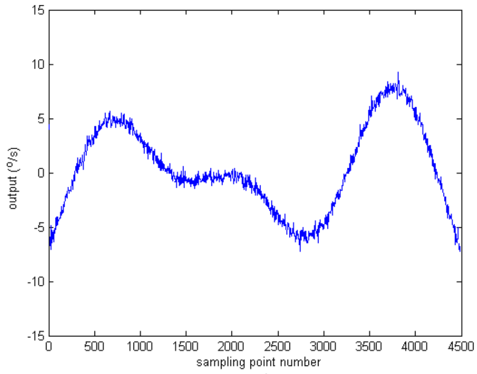
Experiment Results and Analyses

Experiments using a certain type of three-axis optical tracking servo stabilized platform, its azimuth axis, pitch axis and roll axis have adopted the type KT-11 SDOF (Single Degree Of Freedom) Liquid Floated Gyro, this paper takes azimuth axis as an example, in accordance with the requirements of the Monte Carlo experiment, the same experiments were carried out several tests on the remaining axes, the results are consistent with one in this paper. In order to verify the effects of the proposed multi-wavelet filtering algorithm, we process the same sample data of gyro output signal using four different filtering methods.

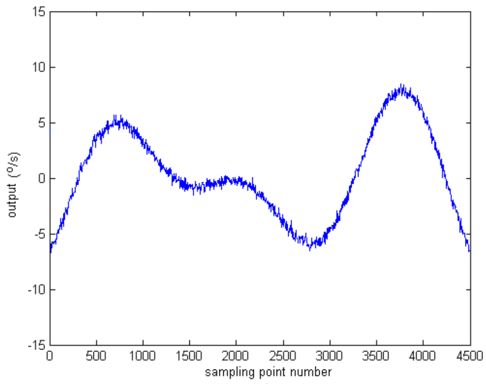
Experimental content: Random collect a group of Azimuth axis gyro feedback signals, the number of samples is 4500, using the above different filtering methods to deal with the samples. As shown in Figure 1. In which Fig.1(a) is the original gyro output signal; Fig.1(b) adopt moving filtering method which commonly used by general digital systems, and data length is taken as 5; Fig.1(b) use Kalman filtering method based on gyro random drift model, error model using the AR (2) model. Figure 1 (d) using the Daubechie-5 hard thresholding single-wavelet filtering method, the choice of threshold using Stein unbiased likelihood estimation rules; Figure 1 (e) using the self-tuning energy-weighted threshold multi-wavelet algorithm design in this paper, choose the GHM multi-wavelet. For the reason that wavelet filtering accuracy has a relationship with the decomposition level, therefore the decomposition level is taken as 5 in each method to compare the filtering effects apparently.



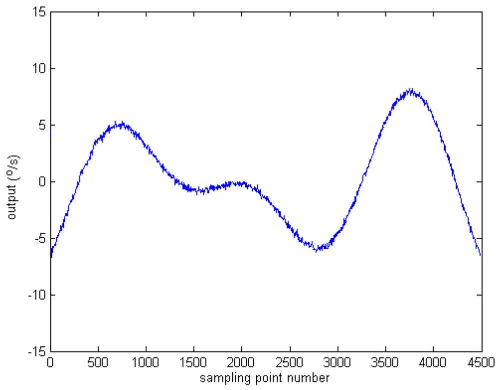
(*a*) Gyro output signal



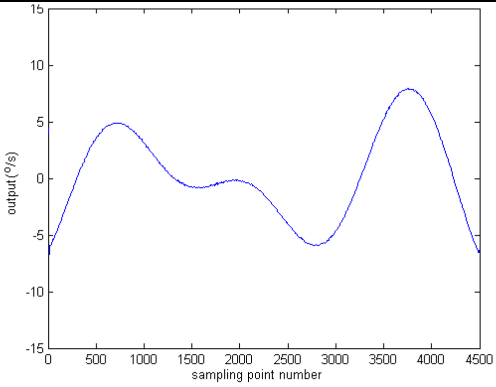
(*b*) Gyro moving filtered output signal



(*c*) Gyro Kalman filtered output



(*d*) Gyro single-wavelet filtered output



(*e*) Gyro multi-wavelet filtered output

Fig.1. Gyro dynamic sampling signal and the filtered output curve

As can be seen from Fig. 1, all the filtering methods can reduce the noise in the signal given by the gyro .AS is shown in Fig. 1 (b), moving filtering method doesn't work well, it just eliminates some regularity disturbance. Comparing Fig. 1 (c) to Fig. 1 (d), we can find: different from the single wavelet filtering, curves using Kalman filtering method is not as smooth as single wavelet smooth as a whole, especially in the velocity mutation section. Glitches in the curves is still obvious , indicating that, reducing the accuracy of the gyro error model and adding in the random noise greatly influence the accuracy of Kalman filtering. Compared with the other figures, Fig. 1 (e) shows that, by using the biorthogonal wavelet and multi-wavelet filtering methods mentioned in this paper, the dynamic gyro signal filtering still has a good effect——smooth curves, no major mutations.

Theoretically speaking, during the wavelet filtering process, the filtering effect will be getting better and better with the increase of decomposition level [8]. However, in practical applications, the amount of computation is also increasingly large and then resulting longer data processing time, produce output delay, and the calculation error of the actual results become bigger simultaneously. Therefore, we should choose the decomposition levels refer to the following criteria: On the premise of meeting tracking accuracy, minimize the decomposition levels. This will improve the real-time performance of the system. In this system, 5 decomposition levels can provide enough tracking accuracy. The experimental results are shown in the following table.

Table 1 the statistical results with different decomposition scales

Index / Dimension	multi-wavelets filtering based on energy weighted threshold	
	Mean	standard deviation
j=4	0.0137	0.0839
j=5	0.0128	0.0828
j=6	0.0112	0.0803
j=7	0.0108	0.0786
j=8	0.0084	0.0761

CONCLUSION

High-precision of the gyro output signal is the important safeguard for the normal operation of optical servo system. Considering the characteristics that the gyro output signal of system contains a large number of non-stationary random noise, combine with the basic principle of orthogonal multi-wavelet filtering, an energy-weighted threshold multi-wavelet filtering algorithm is proposed. For the different wavelet coefficients on the same scale, the filter threshold with self-tuning energy weights is designed which can effectively optimize the filtering algorithm of wavelet signal. The experimental results show that this algorithm can effectively suppress the gyro noise signal of the precision optical tracking servo system.

Acknowledgment
The author would like to thank the associate editor and the anonymous reviewers for their valuable comments and constructive suggestions. They were very helpful for this study. This work was supported by the Research Fund for the Doctoral Program of Higher Education of China (No.20103218120016), the Fundamental Research Funds for the Central Universities (No.NS2012087), the Natural Science Foundation of Jiangsu Province of China（No.BK2012384）, and the National Natural Science Foundation of China (No. 61203090).

REFERENCES

[1]. Bin liu, Guipin dai. *Chinese Journal Of sensors and Actuators*, **2005**, 18(3): 473-476
[2]. Charles K. Chuia, Jian-ao Lianb. *Applied Numerical Mathematics* **1996**, 20[3]: 273–298
[3]. Donoho D L. Johnstone I M. *Biometrika*, **1994**, 81: 524-455.
[4]. Pu Yang, Qi Li, *Measurement & Control Technology*, **2007**,26(4): 32-36
[5]. Qin Yi,Wang Jiaxu,Mao Yongfang. *Journal of Vibration, Measurement & Diagnosis*, **2011**, 31(5): 543-547
[6]. Yanhong Li. Application and construction of orthonormal symmetric multiwavelets[D]. Beijing, Beijing Jiaotong University, **2010**.
[7]. Yu Gan, Yufen Sui,. *Acta Geodaetica et Cartographica Sinica*, **2011**, 40(6): 343-346
[8]. Zhengxing Chenm Lingling zhang. *Journal Of Engineering Mathematics*, **2001**, 18(1): 99-106