



The application research of ARMA forecasting model in prediction of medals and ranking for 2016 Olympic Games

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ABSTRACT

The medal number and ranking of previous Olympic Game is the focus of people's attention. By studying the time series method, this paper applies it in predicting the number of medals. It respectively predicts the medal numbers of China, the U.S. and Russia, arrives at the medal number and ranking of China in the 2016 Olympic, obtains that China will respectively win 40 gold medals, 25 silver medals and 27 bronze medals with a total of 92 medals at the 2016 Olympic Games by using the weighted moving average method, uses autoregressive^(AR) model, moving average^(AM) model and autoregressive average ARMA model to respectively predict the medal condition of American and Russian in the 2016 Olympics, uses SAS software to test the stationary of the data, tests the autocorrelation coefficient model, and ultimately determines the predicted and estimated value. It is predicted that U.S. will get 44 gold medals, 36 silver medals and 23 bronze medals, a total of 103 medals; Russia will get 32 gold medals, 36 silver medals and 23 bronze medals, a total of 91 medals; thus China will be the second.

Key words: Medals amount, time series analysis, autoregressive model, ARMA prediction model, Olympic Games

INTRODUCTION

The first modern Olympic Games in 1896 were organized in Athens by the Greek. Olympic takes solidarity, peace and friendship as the purpose for a long time [1-3]. The Olympic Games is the competition opportunity to show the comprehensive strength of the nation, it represents not only the sport development of a nation, but also represents the condition in all aspects of a country's humanistic quality, political system, economic development and social harmony [4-6]. The performance of a country at the Olympics directly depends on the number of Olympic medals. Olympic medal not only represents the sports competition ability of individual, but also bears people's love for sports, and anticipation of the masses [7-9]. The number of medals directly reflects whether a country's sports, economy, policy is advanced or not. China's performance at the Olympics is more prominent, which is a miniature of the rapid development after China's reform and opening up [10-12]. Especially in the 2008 Beijing Olympics, China ranked first in the world with the advantage of 51 gold medals; which effectively promotes the development of sports undertakings in China at the same time fulfills China's Olympic dream, but also shows the Chinese powerful comprehensive national strength and the humanistic quality to the world [13].

The number of Olympic medals in the Olympic Games is the focus of people's attention around the world. People often incorporate a lot of feelings in medal. China's outstanding performance in the Olympic Games promotes the awareness unity and patriotism of the people. The medal number prediction can make China develop better sports-related policies, which is conducive to the development of sports and economic cause [14]. Therefore, the forecast for the number of medals has important significance for the economic sports development. Most of the papers are now predicting the number of gold medals at home and abroad, and papers to predict the total number of medals and Chinese ranking on the whole are very little. The most powerful opponents for China to have good ranking in the Olympics are the United States and Russia. This article respectively predicts the medals number of China, the U.S. and Russia in 2016 as a whole thereby deduces China's ranking. From the data observation, the data

of the three countries have their own characteristics. For the different characteristics of nine data sets, this article uses different methods of time series to predict, which is more targeted and more accurate.

THE APPLICATION AND RESULTS ANALYSIS OF ARMA FORECASTING MODEL IN THE OLYMPICS PERFORMANCE PREDICTION

Weighted moving average method

Suppose the time sequence is $y_1, y_2, \dots, y_t, \dots$; the weighted moving average formula:

$$M_{tw} = \frac{w_1 y_t + w_2 y_{t-1} + \dots + w_N y_{t-N+1}}{w_1 + w_2 + \dots + w_N}, \quad t \geq N \quad (1)$$

In the formula M_{tw} is a t period weighted moving average value; w_i is the weight of y_{t-i+1} . The prediction formula:

$$\hat{y}_{t+1} = M_{tw} \quad (2)$$

That is to take the t period weighted moving average value as the predictive value of the $t+1$ period.

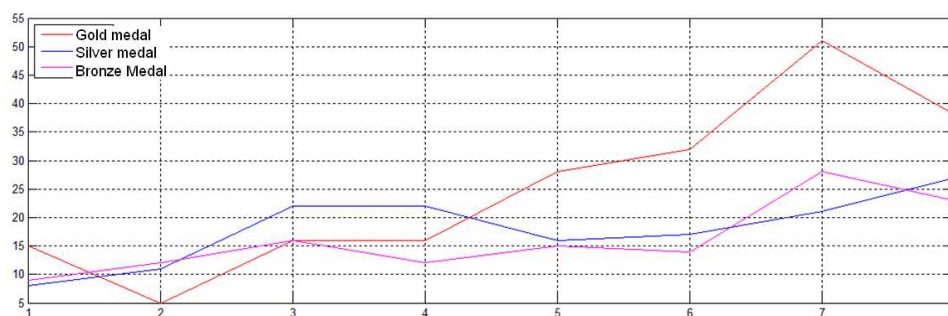


Figure 1: Chinese medal case of all previous

Figure 1 shows that the 2008 Olympic Game was organized in China, the number of medals obtained is obvious abnormal, so round the Chinese medal data in 2008.

Table 1: The medal ranking of China's Olympic Games and the weighted moving average predicted value table

Years	1984	1988	1992	1996	2000	2004	2012
t	1	2	3	4	5	6	7
Gold	15	5	16	16	28	32	38
The weighted moving average predicted value of three years				16.1667	27.1667	31	32
Relative error (%)				0.01042	0.0298	0.03125	0.1579
Silver	8	11	22	22	16	17	27
				16	20.1667	19	17.5
Relative error (%)				0.2727	0.2204	0.1176	0.3519
Copper	9	12	16	12	15	14	23
				8.5	10	12.5	20
Relative error (%)				0.2917	0.3333	0.1071	0.3913
The total number of medals	33	30	57	54	64	69	96
				44	61	69.5	91
Relative error (%)				0.1252	0.0341	0.0026	0.0179

Take $w_1=3$, $w_2=2$, $w_3=1$, the prediction formula is:
$$\hat{y}_{t+1} = \frac{2y_t + 2y_{t-1} + y_{t-2}}{3+2+1}$$

According to the above equation, after the weighted average, the results of the calculated prediction value are shown in Table 1. The predictive value for the number of China winning the 2016 Olympic gold medal is (number):

$$\hat{y}_{2016} = \frac{3 \times 38 + 2 \times 32 + 28}{6} = 34.3333$$

As the overall trend tends to rise, the predicted values lag. It is necessary to correct the predictive value, the method is: first calculate the relative error of the predicted value and the actual value of each session, such as the 1996

$$\text{session: } \frac{16-16.1667}{16} = 1.042\%$$

The relative error is shown in Table 1, and then calculates the total average error 0.932749122.

$$\left(1 - \frac{\sum \hat{y}_t}{\sum y_t}\right) \times 100\% = \left(1 - \frac{106.3334}{114}\right) \times 100\% = 6.725\%$$

From the above the average of the total predicted value is lower than the actual value^{33.04%}, thus the predicted value

$$\text{in 2016 can be corrected as: } \frac{34.333}{1-6.725\%} = 39.8084$$

Similarly draw the number of silver and bronze, and fill it in Table 1. In the 2016 session it is predicted that China will win 40 gold medals, 25 silver medals, 27 bronze medals, totally 92 medals.

Stochastic time series models:

- (1) The auto-regression ^(AR) model;
- (2) The moving average ^(AM) model;
- (3) The auto-regressive moving average ^(ARMA) model;

Introduce backward operator B and difference operator ∇ : $B^k X_n = X_{n-k}$, $B^k C = C$, $k = 0, 1, \dots$; C is a constant. And denote it as:

$$\phi_p(B) = (1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p) \quad (3)$$

First order difference:

$$\nabla X_n = X_n - X_{n-1} \quad (4)$$

Sequence $AR(p)$: suppose $\{X_t, t = 0, \pm 1, \pm 2, \dots\}$ is a zero mean stationary series, which meets the following:

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \dots + \phi_p X_{t-p} + \varepsilon_t \quad (5)$$

Parameter ε_t is the stationary white noise with zero mean and σ_ε^2 variance. Then X_t is the auto-regression sequence of order P , which is denoted as $AR(p)$ sequence, whereas:

$$\phi = (\phi_1, \phi_2, \dots, \phi_p)^T \quad (6)$$

It is more convenient to describe the formula after the introduction of backward shift operator. Operator B is defined as follows:

$$BX_t \equiv X_{t-1}, B^k X_t \equiv X_{t-k} \quad (7)$$

Computing sub-polynomial:

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p \quad (8)$$

The formula (8) can be rewritten as:

$$\phi(B)X_t = \varepsilon_t \quad (9)$$

Polynomial equation $\phi(\lambda) = 0$ is called the characteristic equation of $AR(p)$ model. The test of the model: its p roots $\lambda_1, \lambda_2, \dots, \lambda_p$ are called eigenvalues of the model. Eigenvalue may be real, and also could be plural. According to the conditions, if the eigenvalues λ_i are outside the unit circle, namely:

$$|\lambda_i| > 1, i = 1, 2, \dots, p \quad (10)$$

Model $AR(p)$ is called stable or stationary. The above formula is called the stable condition.

Sequence $MA(q)$: suppose $\{X_t, t = 0, \pm 1, \pm 2, \dots\}$ is the zero mean stationary series, which meets the following model:

$$X_t = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \dots - \theta_q \varepsilon_{t-q} \quad (11)$$

Parameter ε_t is the stationary white noise with zero mean and σ_ε^2 variance, which is denoted as sequence $MA(p)$, whereas:

$$\theta = (\theta_1, \theta_2, \dots, \theta_q)^T \quad (12)$$

It is called the moving average parameter vector, whose components $\theta_j, j = 1, 2, \dots, q$ is called average sliding coefficient. For the linear backward shift operator B , we have:

$$B\varepsilon_t = \varepsilon_{t-1}, B^k \varepsilon_t = \varepsilon_{t-k} \quad (13)$$

Re-introduce the operator polynomials:

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q \quad (14)$$

The formula (14) can be written as:

$$X_t = \theta(B)\varepsilon_t \quad (15)$$

Polynomial equation $\theta(\lambda) = 0$ is called the characteristic equation of $MA(q)$ models; its q roots are called the eigenvalue of $MA(q)$.

The model test: If the eigenvalue of $MA(q)$ are outside of the unit circle, then the $MA(q)$ model is reversible.

Sequence $ARMA(p, q)$: the definition of ε_n , $\phi_p(B)$ and $\theta_q(B)$ is the same with that of above, the wide stationary sequence X_n meets: $EX_n = 0$, $X_t - \phi_1 X_{t-1} - \dots - \phi_p X_{t-p} = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \dots - \theta_q \varepsilon_{t-q}$

Then X_n is called autoregressive moving average model of order (p, q) , which is referred to as $ARMA(p, q)$ model.

$ARMA(p, q)$ can be written in the form of operators:

$$\phi_p(B)X_n = \theta_q(B)\varepsilon_n \quad (16)$$

For $ARMA(p, q)$ model, we always assume $\phi_p(B)$ and $\theta_q(B)$ (as a polynomial of the variable B) have no common factor, and respectively satisfy stationary conditions and reversible condition.

Predict the gold medal number of U.S. in 2016 Olympic Game, as shown in Figure 2.

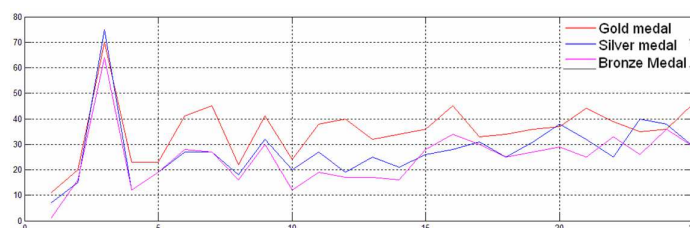


Figure 2: The U.S.'s medal condition of all previous

Use SAS software for data processing, and test stationary of the data:

Lag	Covariance	Correlation	Autocorrelations																	Std Error				
			-1	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6		7	8	9	1
0	6.478225	1.00000																						0
1	4.942171	0.76289																						0.277350
2	2.423277	0.37407		.																				0.407997
3	0.353969	0.05464		.	*																			0.433576

The calculation results of autocorrelation coefficients are given, it can be seen that with the increasing of delay deployment, the autocorrelation coefficients shows a declining trend. Autocorrelation coefficient of the sequence quickly reduces to 0; it is known that the sequence is the stationary time series. As can be seen from the table the autocorrelation P is 1.

Conduct correlation analysis and partial correlation analysis of the data to determine the model order:

Partial Autocorrelations																							
Lag	Correlation	-1	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	1	
1	0.76289																						
2	-0.49745									.													
3	0.02166									.													

The calculation results of partial correlation coefficient are given, wherein the first column is the delayed deployment, the second column is the partial correlation coefficient, the third column uses an asterisk to express the partial correlation coefficients. It can be seen that after one step delay, the partial correlation coefficients are all between twice of the standard errors. And the value of correlation coefficient is much smaller. So we can say that the model ends after the one step delay.

Through the above analysis it can be considered that the hypothetical model is applicable. According to the identified order, estimate the coefficients that the model establishes. It uses the estimate statement to establish a first-order autoregressive model, first-order moving regression model, as well as a first-order autoregressive moving average $ARMA(1,1)$ model.

Model test:

The fitting statistics gives the evaluation of the model results. In the given statistical parameters, it includes the estimated value of the model, the estimated value of the residual variance, the estimate value of standard deviation, the standard of information amount AIC and SBC, and the residual number. With the program running results, it establishes three models of different time series. We can study the models from the following two criteria: 1 the higher the likelihood function value is, the better; 2 the fewer the number of model position parameters is, the better.

The model that can make the function minimum is considered to be the best model. For criterion SBC, the smaller the better SBC is. By judging AIC and SBC, the third model is much better.

	AR(1)	MA(1)	ARMA(1, 1)
AIC	53.71279	51.77924	48.82854
SBC	54.84269	52.90913	50.52339

Significant test:

Conditional Least Squares Estimation

Parameter	Estimate	Standard Error	t Value	Approx	
				Pr > t	Lag
MU	69.33704	0.81936	84.62	<.0001	0
MA1, 1	-1.00000	0.28198	-3.55	0.0053	1
AR1, 1	0.65025	0.26055	2.50	0.0317	1

The value of P is less than 0.05, which meets the test requirements.

It is derived from SAS software that the development function of the gold medal that United States won is:

$$\hat{y} = 0.007219x^3 - 0.2985x^2 + 3.855x + 20.75$$

It is obtained that U.S. in the 2016 Olympic Game will win 44 gold medals, based on this model the number of silver medal is 36, the number of bronze medal is 23, a total of 103 medals.

Predict the number of gold medals for Russia at the 2016 Olympic Games, as shown in Figure 3.

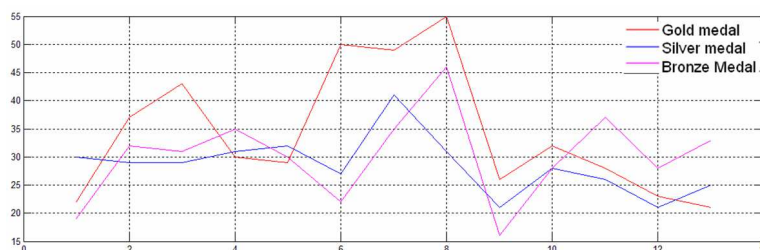


Figure 3: Russia's medal condition of all previous

The method is the same with the prediction of the number of gold medals the United States, it is calculated that p is 1 and q is 1. The model checking is the same with the above model:

	AR(1)	MA(1)	ARMA(1, 1)
AIC	63.88356	69.2892	64.03058
SBC	64.67935	70.08499	65.22426

By combining the value of AIC and SBC, it draws an optimal significant test of model one:

Conditional Least Squares Estimation

Parameter	Estimate	Standard Error	t Value	Approx	
				Pr > t	Lag
MU	85.35841	4.06892	20.98	<.0001	0
AR1, 1	1.00000	0.18187	5.50	0.0004	1

Value P is less than 0.05 and meets the test requirements.

It is derived from SAS software that the development function of the gold medal that the Russian won is:

$$\hat{y} = 0.01865x^3 - 0.9725x^2 + 9.688x + 15.8$$

It is obtained that Russia will win 32 gold medals, based on this model the number of silver medal is 36, the number of bronze medal is 23, a total of 91 medals.

CONCLUSION

Time series model is the common solution to solve the prediction problem that has a clear chronological order. In the previous Olympic Games, the strengths of China, the U.S. and Russia are nearly the same, which are all in the world top level. Aiming at the quantity of gold silver and bronze medals that the three countries got in the previous Olympic Games, according to the different characteristics of nine sets of data, this paper respectively selects the auto-regression (AR) model, moving average (AM) model, autoregressive average $ARMA$ model to conduct prediction, obtains the number of Olympic medals obtained in 2016, and derives the ranking. Thus each set of the data is more targeted, and the results are more accurate. Time series model can be widely used to chronological prediction problems, in addition to sports medal predictions; it also can be used for a variety of data forecasting problems, such as population prediction.

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