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The application of Gompertz curve in the curve fitting of urbanization

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ABSTRACT

The urbanization of some developed countries, such as America, Britain, Japan and Germany, indicates that the urbanization process shows significant S-type characteristics, and Gompertz curve is a good choice to simulate them. Combining with some of Gompertz's characters such as monotonicity, periodicity, and having an inflection point, this article tries to apply Gompertz curve fitting to the urbanization process of Shandong province. The results show that the urbanization process of Shandong province can be divided into four stages by 1975, 2003, and 2030, and 2003 is a turning point. After 2003, the urbanization level is still rising, but growth rate began to decline. 85.58% is the saturation value of urbanization, and after reaching this value, urbanization rate will be difficult to grow. Future urbanization in Shandong province must conform to this rule, and it is more important to pay attention to the quality of urbanization rather than the speed of growth.

Key words: Urbanization; Curve fitting; Gompertz Curve; periodicity

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INTRODUCTION

Over the more than 30 years since the reform and opening up, China has experienced the fastest and largest urbanization in the world history. In 1978, only 172,450,000 Chinese lived in urban areas, which was less than 18% of the total population. In 2012, the urbanization rate increased to 52.6%. The increase in urban population is equivalent to the total of the current populations of US, Japan and Germany, all of which are major world economies. Shandong Province is one of the most populous provinces of China. According to the sixth national population census, permanent population of Shandong Province is 95,793,000, only next to that of Guangdong Province. Meanwhile, Shandong Province has a large economic size and is a major agricultural province. Therefore, urbanization process and the trend of Shandong Province is a typical example of China's urbanization.

It is universally recognized by academia that urbanization has several stages: primary stage, middle stage and late stage. During the primary stage, the urbanization rate is low, development is slow and agriculture dominates the economy. The middle stage is a stage when the urbanization accelerates. Improvement in agricultural labor productivity results in large numbers of surplus rural workers; meanwhile, acceleration of urbanization increases city's ability to absorb population, therefore, many people in rural areas are attracted to cities. In the late stage of urbanization, much depends on the development of the city itself. Basically all of surplus rural labor forces have already moved to the cities and the population growth rate of urban areas slows down. But in terms of specific rules of urbanization, the academia has not reached any consensus on whether there are reasonable models to describe and whether there are curves suitable to simulate the development of urbanization.

CURRENT CURVE FITTINGS OF URBANIZATION

Mathematical methods has been used to demonstrate that there is a significant correlation between the level of urbanization and per capita GDP[1].Apart from GDP, economic structure and non-economic factors also have noticeable effect on urbanization[2].Industrialization, export and foreign aid will influence a country's urbanization path[3].The process of urbanization can be described by an S-type curve and divided into three stages [4]. There are

also three periods of China's urbanization in the pattern of world [5].In simulating China's urbanization, logarithmic curve and logistic–type S curve are mainly used.

Logarithmic curve simulation

Research based on sectional data of 137 countries in the world shows that there is a significant logarithmic curve relationship between urbanization level and per capita GDP[6]. Further research demonstrates this logarithmic relationship and divides the curve on a time basis [7].

Logarithmic curve is used for verification on data of China's urbanization during 1987-2002 [8]. The expression of logarithmic curve is:

$$URBANR = a_0 + a_1 \ln(GDPPC)$$

In the expression, URBANR represents Urbanization Rate, which reflects urbanization level; GDPPC represents GDP per capita, which reflects economic development level; a_0 and a_1, are coefficients.

Logistic-type S curve simulation

Logistic growth curve was first proposed by P.E. Verhulst, a biological mathematician, in 1838. The curve was originally used to describe growth of animals and plants, reproduction process and dose-response relationship. Later it was widely used in predictions, demography, information science, social interaction, spatial diffusion and other areas. At the beginning, it grows slowly, but increases rapidly for a period later and then its growth slows down. It is a typical S-curve. Although Northam envisaged S-curve of urbanization in as early as 1979, he did not specify the mathematical model and cut-off points. In the macro sense, urbanization is a Logistic process and can be described by S-curve [9].The simulation curve can be divided into four stages and given geospatial explanations. Some researchers worked out an equation expression about the time variable t of the Logistic growth model of urbanization S-curve. They chose historical data in urbanization of Japan and 22 other countries to conduct curve fitting [10]. The basic expression of Logistic S-curve is:

$$y = \alpha/[1+ \exp(\beta - \gamma * x)]$$

In the expression, a ,β, γ are parameters, a represents the saturation value of urbanization. Urbanization rate is often the dependent variable y; time or GDP per capita is the independent variable x.

Deficiencies in simulation by Logarithmic curve and Logistic S-curve

In current researches, the uses of both logarithmic curve and Logistic S-curve have achieved good simulation results (having passed significance test and high goodness of fit). But studied closely, these two models have some deficiencies in simulation. Logarithmic curve is suitable for simulation of primary and middle stages of urbanization, but if the urbanization level is high or in the late stages of urbanization, simulation results of logarithmic curve are far from ideal. Logistic curve is a typical S-curve and is basically in line with the characteristics of urbanization process. But it has a centrosymmetric point (β/γ,a/2), which is also the inflection point (the turning point from which urbanization slows down). The mathematical formula of Logistic curve requires that the urbanization process is completely symmetrical around the center of the inflection point, which cannot be agreed by economic theories.

MATHEMATICAL CHARACTERISTICS OF GOMPERTZ S-CURVE OF URBANIZATION

Gompertz curve was proposed in 1825 by B. Gompertz, a British statistician and mathematician. Gompertz curve is similar to Logistic curve. As a common S-curve, it is often used to describe growth of some plants and economic rules. The significant difference between Gompertz curve and Logistic curve lies in that Gompertz has inflection point but no centrosymmetric point. The growth in the main growth area of Gompertz curve is noticeably more than that of Logistic curve. Therefore, Gompertz curve is more suitable for simulating urbanization process. Mathematical expression of Gompertz curve is:

y = a*exp(-exp(-b*(x-c)))

The first, second and third derivatives of the function are as follows:

$$y' = abe^{-e^{-bx}((bx-bc)e^{bx}+e^{bc})}$$

$$y'' = -(ab^2e^{bx+bc} - ab^2e^{2bc})e^{-e^{-bx}(2bxe^{bx}+e^{bc})}$$

$$y''' = (ab^3e^{2bx+bc} - 3ab^3e^{bx+2bc} + ab^3e^{3bc})e^{-e^{-bx}(3bxe^{bx}+e^{bc})}$$

Let $y''=0$, we can get the inflection point $x_m = c$

Let $y'''=0$, we can get the maximum curvature point of Gompertz curve:

$$x_1 = \frac{1}{b}[\ln(\frac{3-\sqrt{5}}{2}) + bc], \quad x_2 = \frac{1}{b}[\ln(\frac{3+\sqrt{5}}{2}) + bc]$$

From these three feature points and the function expression, we can decide that Gompertz curve has the following basic features:

Table 1. Basic features of Gompertz curve

Feature	Meaning
Montonicity	y monotonically increasing, and 0 <y< a
Periodicity	x<x1， primary stage x1 <x<xm， accelerated growth stage xm<x <x2， decelerating growth stage x>xm， late stage
Inflection point	（c，a/e）is the inflection point
Order of feature points	x1 <xm< x2
Maximum rate	ab/e

FITTING VERIFICATION OF URBANIZATION PROCESS OF SEVERAL DEVELOPED COUNTRIES

Historical data of urbanization processes of the US, Britain, Japan and Germany are used in the verification (The data are from UN website, WDI Statistics Database and Japan Statistical Yearbook). UR represents urbanization rate, t represents time. The results are shown in Table 2.

Table 2.Gompertz simulation of urbanization of US, Britain, Japan and Germany

Gompertz function , UR=a*exp(-exp(-b*(t-c)))						
2-1 US, 1850-2012						
Prob> F=0.0000		R^2= 0.9996	Adj R^2 =0.9996			
UR	Coef.	Std. Err.	t	P>t	[95% Conf.	Interval]
a	94.29776	1.801879	52.33	0.000	90.69468	97.90084
b	.0148707	.0007008	21.22	0.000	.0134694	.016272
c	1882.735	1.562096	1205.26	0.000	1879.611	1885.859
2-2 Britain, 1800-2010						
Prob> F=0.0000		R^2= 0.9989	Adj R^2 =0.9988			
UR	Coef.	Std. Err.	t	P>t	[95% Conf.	Interval]
a	97.6181	2.780652	35.11	0.000	92.04323	103.193
b	.011783	.0013072	9.01	0.000	.0091622	.0144038
c	1804.313	3.501223	515.34	0.000	1797.293	1811.332
2-3 Japan, 1900-2012						
Prob> F=0.0000		R^2= 0.9968	Adj R^2 =0.9967			
UR	Coef.	Std. Err.	t	P>t	[95% Conf.	Interval]
a	95.10041	2.651212	35.87	0.000	89.80071	100.4001
b	.031496	.0023314	13.51	0.000	.0268356	.0361564
c	1934.664	1.421882	1360.64	0.000	1931.822	1937.507
2-4 Germany, 1871-2012						
Prob> F=0.0000		R^2= 0.9998	Adj R^2 =0.9997			
UR	Coef.	Std. Err.	t	P>t	[95% Conf.	Interval]
a	75.09659	.4233721	177.38	0.000	74.2488	75.94437
b	.0284513	.0015304	18.59	0.000	.0253868	.0315158
c	1862.445	1.278531	1456.71	0.000	1859.884	1865.005

We can see that Gompertz curve can simulate urbanization process of UK, Britain, Japan and Germany very well. Goodness of curve fitting are all above 99%, and all of the parameters are significant. 1883, 1804, 1935 and 1862 are turning points of urbanization processes of the US, Britain, Japan and Germany. As the birthplace of the industrial revolution, Britain is the first developed country that began its urbanization. Judging from the time span of main development stages (the second and third stages), Britain's urbanization was relatively slow. Britain's main development stages lasted 160 years, and that time span of US, Japan and Germany lasted 130 years, 62 years and 68 years, respectively. Simulation curves of the four countries are as follows.

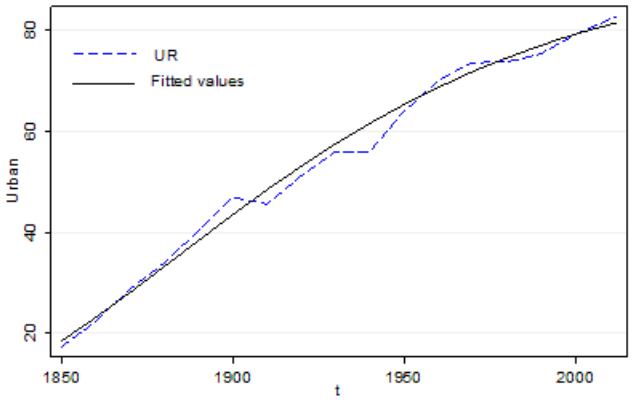

Fig.1. Urbanization curve fitting of US,1850-2012

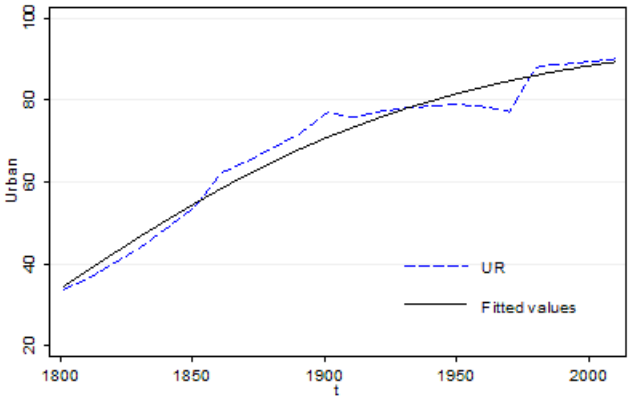

Fig.2. Urbanization curve fitting of Britain,1800-2010

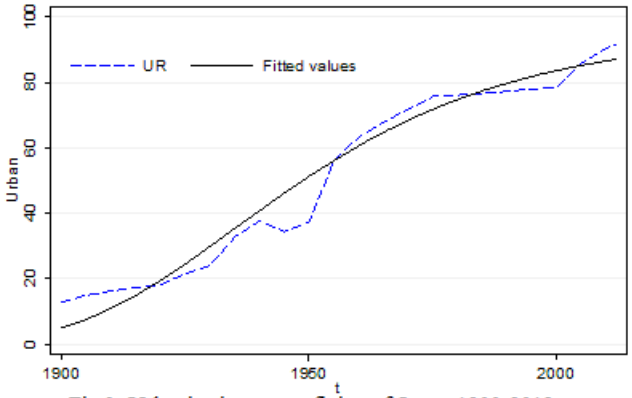

Fig.3. Urbanization curve fitting of Japan,1900-2012

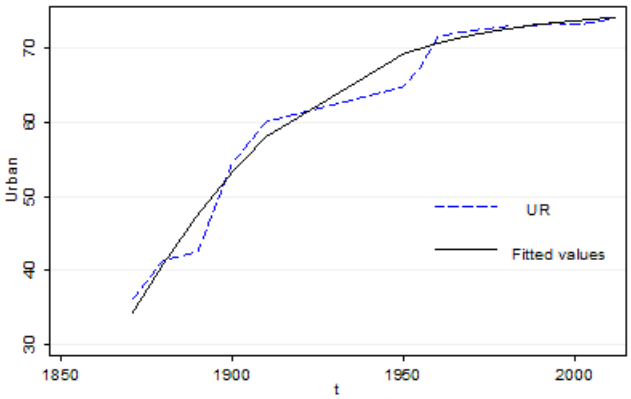

Fig.4. Urbanization curve fitting of Germany,1871-2012

GOMPERTZ CURVE FITTING OF URBANIZATION OF SHANDONG PROVINCE

During the period from the founding of PRC to 1978, when China adopted the reform and opening up policy, urbanization of Shandong Province is slow, low-end and fluctuational. During the ten years of the Cultural Revolution, the urbanization nearly stagnated. Since the reform and opening up, with the rapid development of the national economy and adjustment of urbanization policy, urbanization of Shandong Province was gradually put on the right track. In 2011, construction of Shandong Peninsula Blue Economic Zone was formally made a national strategy. To facilitate the construction of the Economic Zone, the government has published Urban System Planning of Shandong Peninsula Blue Economic Zone (2009-2020). The urban cluster will greatly facilitate the development of Shandong's marine economy. At present, Shandong Province is working on building A Cluster, A Circle, A Zone and A Belt.

Given the data continuity, this dissertation uses the proportion of non-agricultural population in the overall population as the indicator of urbanization level. The data is from Statistical Yearbook of Shandong Province of each year. Urbanization rates during 1978-2011 are shown in Table 3.

In the Gompertz curve of urbanization of Shandong Province, the fitting equation is:

y = 85.58*exp(-exp(-0.0348*(x-2002.76)))

Significance test of the fitting results is included in Table 3. The goodness of fit is 0.9983, the revised goodness of fit is 0.9981. All parameters and the equation have passed the significance test.

Table 3 Urbanization Rate of Shandong Province during 1978-2011
(Estimates based on proportion of non-agricultural population)

Year	Total popu.	Agri.Popu.	Non-agriPopu.	Proportion of non-agr. Popu.（%）	Year	Total Popu.	Agri.Popu.	Non-agriPopu.	Proportion of non-agr. Popu.（%）
1978	7160	6533	627	8.76	1995	8701	6531	2170	24.94
1979	7232	6570	661	9.14	1996	8747	6484	2263	25.87
1980	7296	6605	691	9.47	1997	8810	6500	2310	26.22
1981	7395	6659	736	9.95	1998	8872	6575	2296	25.88
1982	7494	6720	774	10.33	1999	8922	6600	2322	26.03
1983	7564	6753	811	10.72	2000	8975	6566	2409	26.84
1984	7637	6701	936	12.26	2001	9024	6507	2517	27.89
1985	7695	6676	1017	13.22	2002	9069	6435	2634	29.04
1986	7776	6797	979	12.59	2003	9108	6275	2833	31.10
1987	7889	6844	1045	13.25	2004	9163	6212	2951	32.21
1988	8009	6702	1307	16.32	2005	9212	6066	3147	34.16
1989	8181	6698	1483	18.13	2006	9282	6055	3228	34.78
1990	8424	6846	1578	18.73	2007	9346	5909	3436	36.76
1991	8534	6884	1650	19.33	2008	9392	5860	3532	37.61
1992	8580	6819	1761	20.52	2009	9449	5902	3548	37.55
1993	8620	6724	1896	22.00	2010	9536	5698	3839	40.26
1994	8653	6574	2079	24.03	2011	9591	5646	3945	41.13

Table 3.Fitting results of Shandong Province during 1978-2011

UR	Coef.	Std. Err.	t	P>t	[95% Conf. Interval]	
a	85.57984	14.61566	5.86	0.000	55.77101	115.3887
b	.0347939	.0048365	7.19	0.000	.0249298	.044658
c	2002.759	5.009464	399.80	0.000	1992.542	2012.976

The fitting figure is seen in Figure 5.

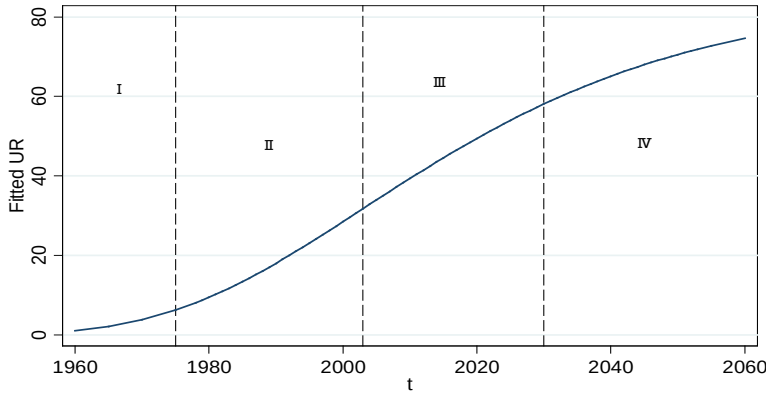

Fig.5: Curve fitting of Shandong's urbanization

CONCLUSION

Urbanization processes of developed countries such as America, Britain, Japan and Germany have shown that Gompertz curve can describe the "S" feature of the urbanization process very well. By using curve fitting, we can reach some conclusions about Shandong's urbanization.

Stages of urbanization of Shandong Province
1975, 2003 and 2030 are cut-off points of Shandong's urbanization. Therefore, the process can be divided into four stages:

Stage 1: The period before 1975 is the primary stage of Shandong's urbanization. In this stage, the urbanization hardly started. The urbanization level is low and agricultural population is the majority.

Stage 2: The period of 1975-2003 is a period when Shandong's urbanization accelerated. The urbanization rate grew increasingly rapidly.

Stage 3: The period 2004-2030 is a period when Shandong's urbanization decelerated. The growth of urbanization rate started to decline after 2004.

Stage 4: After 2030, the urbanization of Shandong Province enters its late stage. Compared to its earlier stages, urbanization is mature in this stage and the growth rate is small.

1975-2030 is the main development stage, in this time span, urbanization rate increases from 6.2% to 58.9%,at around 2050,the urbanization rate will reach 70%; at around 2080, the urbanization rate will reach 80%.According to these estimates, there are nearly 20 years left to the end of the main development stage.

Turning point of urbanization of Shandong Province

In mathematics, an inflection point is a point where convexity changes. In economics, a turning point is a point where trend in economic development starts to change. The curve fitting results show that 2003 is the turning point of urbanization of Shandong Province: In 2003, the urbanization rate is 31.48%; after 2003, the urbanization still proceeds but begins to slow down.

Saturation value of urbanization of Shandong Province

The simulation results show that 85.58% is the saturation value of Shandong's urbanization. After reaching this value, the urbanization rate will hardly grow. From this we can see that urbanization has its own laws. Urbanization is driven by industrialization; it also needs support of agriculture. Simply seeking to expand the urban area, even setting a target urbanization rate as the objective of development and blindly pursuing development speed is very unfavorable for the economy. Therefore, urbanization of Shandong Province must comply with the law, and we need to focus on the improvement of the quality, not the speed of the growth, of urbanization.

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