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Research Article

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Tennis 'Hawk-Eye' technical research

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ABSTRACT

'Hawk-Eye' system started to apply into tennis competition as early as 2003, now almost all large-scale tennis competitions use 'Hawk-Eye' system to define tennis drop point so as to ensure competition accuracy and fairness. By kinematics knowledge, mathematical differential, adopt several shapes combination, and with the help of MATLAB software, it researches and discusses 'Hawk-Eye' system working principle and accuracy. In research process, it discusses by dividing tennis into taking spin force into consideration and ignoring spin force such two cases. Firstly, it converts tennis three -dimensional space coordinate into motion plane's two dimensional coordinate, and then utilizes kinematics knowledge, it gets tennis kinematical equation, uses mathematical differential and Bernoulli equation to simplify it and then can get tennis drop point. To judge the system accuracy, select one competition tennis positions records, input them into model, and get that tennis drop point is nearly the same as tennis actual drop point in competition, it is clear the system accuracy is high.

Key words: 'Hawk-Eye' system, Biomechanics, Mathematical differential, Bernoulli equation, Kinematics

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INTRODUCTION

'Hawk-Eye' system is also called timely response system, which assists to define referee penalty accuracy. 'Hawk-Eye' system principle is relative simple, but is very accurate. It is composing of 8~10 pieces of high speed cameras, four computers and large screen, it defines whether tennis is outside or not by different orientation ball positions records. 'Hawk-Eye' system has become an indispensible tracking system in some large-scale tennis courts. In tennis competitions, tennis moves at high speed, after landing it is prone to generate issues disputes about whether tennis is in line or out of line as well as other similar ones. And by 'Hawk-Eye' system it can relative correct record and judge ball landing point positions, and eliminate similar problems occurrence. 'Hawk-Eye' system working principle is not complicated. It firstly cuts competition field space into measurement unit that counts by millimeter with the help of computer calculation. Then, utilize high speed cameras to capture tennis motion process different positions, and generate images. Finally make use of timely imaging, use large screen to indicate tennis motion trajectory and drop point. The whole process only takes less than 10 seconds. 'Hawk-Eye' system has been approved by British royal television institution as early as 2001. In 2003, due to its widely used in tennis, it has achieved national American television highest Emmy award 'Outstanding contribution award in science and technology'. 'Hawk-Eye' system has been widely used in tennis competition field, and it has overturned 30%~40% disputed calls. The system applied degrees are different in different competitions. Such as in baseball and hemisphere, the system hasn't been widely used. While in tennis, many top athletes show that they like 'Hawk-Eye' system, because it let penalty to be more accurate.

'Hawk-Eye' system provides professional standard for ball and athlete follow shooting. Its existence provides more accurate judgment for ball is in bounds or out of bounds. Official has ever carried out 250 times testing on 'Hawk-Eye' system; the accuracy rate is 100%. But tennis competition sponsors haven't certainly expressed while only put forward accuracy is 'inside 3 millimeters' .The paper makes research on 'Hawk-Eye' system ball drop point defining principle. And define the system accuracy.

2 'Hawk-Eye' working principle researches

2.1 'Hawk-Eye' system working principle

Tennis makes projectile motion in the air. Regard the whole tennis court as tennis motions located stereo space.

Firstly set tennis speed as v_0, radius as R, tennis mass as m, tennis located point as (x_0, y_0, z_0), tennis each direction component speed as v_x, v_y, v_z, speed direction as (a, b, c).

In order to simple compute, regulate tennis located point as coordinate system origin, regard speed direction horizontal projection direction as x axis direction, z axis as y axis direction, then tennis speed direction is $(\sqrt{p^2+q^2}, 1)$.

According to kinematics, tennis suffered air resistance is:

$$f = kv^2 \tag{1}$$

Tennis flying instant, except for air resistance impacts, it also suffers gravity impacts that vertical to ground, as Figure 1 show:

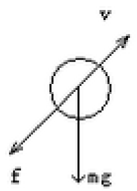

Figure1: Tennis force schematic diagram

Then now x axis direction:

$$m\frac{\partial^2 x}{\partial t^2} = -k(\frac{\partial x}{\partial t})^2 \tag{2}$$

Y axis direction:
When tennis makes upward motions, its motion direction is opposite to speed and gravity direction, so:

$$m\frac{\partial^2 x}{\partial t^2} = k(\frac{\partial x}{\partial t})^2 - mg \qquad (\frac{\partial y}{\partial t} \geq 0) \tag{3}$$

When tennis arrives at top point and makes downward motions, tennis suffered resistance direction is opposite to speed direction, so

$$m\frac{\partial^2 x}{\partial t^2} = -k(\frac{\partial x}{\partial t})^2 - mg \qquad (\frac{\partial y}{\partial t} \leq 0) \tag{4}$$

In x axis direction, it has $v_x = \frac{\partial x}{\partial t}$, input it into formula (2), it gets:

$$m\frac{\partial x}{\partial t} = -kv_x^2 \tag{5}$$

Set that when tennis is in the bound, its speed is v_{0x}, according to above formula, it can get $\frac{\partial v_0}{\partial t} = -\frac{kv_0^2}{m}$. Input the formula into formula (5), it can get:

$$v_x = \frac{mv_{0x}}{m + kv_{0x}t} \tag{6}$$

In tennis motion initial condition, it has $x = 0$, use method of separation of variables, it can get:

$$x = \frac{m}{k}\ln(\frac{kv_{0x}t}{m} + 1) \tag{7}$$

Similarly, in y axis, it has $v_y = \frac{\partial y}{\partial t}$, input it into formula (3)and get:

$$m\frac{\partial v_y}{\partial t} = -kv_y^2 - mg \tag{8}$$

Similar to formula (7), it can get:

$$v = \sqrt{\frac{mg}{k}}\tan[\arctan\left(\sqrt{\frac{mg}{k}}v_{0y}\right) - t\sqrt{\frac{kg}{m}}] \tag{9}$$

Assume that tennis releasing initial state to ground height is h, and then when t=0, $y_0 = h$.Make variables separation integral on above formula, it gets:

$$y = h - \frac{m}{k}\ln\left|\frac{\cos\left[\arctan\left(\frac{k}{mg}v_{0y}\right)\right]}{\cos\left[ars\tan\left(\frac{k}{mg}v_{0y}\right) - t\sqrt{\frac{gk}{m}}\right]}\right| \tag{10}$$

When tennis is in the top point, it has maximum y value $y_{\max}$. According to kinematics, it is clear that now $v_y = 0$. Tennis flying time in the air is t_1, then it has:

$$y_{\max} = h - \frac{m}{k}\ln\left\{\cos\left[\arctan\left(\sqrt{\frac{k}{mg}}v_{0y}\right)\right]\right\} \tag{11}$$

$$t = t_1 = \sqrt{\frac{m}{gk}}\arctan\left(\sqrt{\frac{k}{mg}}v_{0y}\right) \tag{12}$$

When $t = t_1$, tennis arrives at top point, formula(10)has maximum value. Therefore formula (10) t value range is $t \in [0, t_1]$.

Analyze y axis: input $v_y = \frac{\partial y}{\partial t}$ into formula(4), take $v_y = 0$, now $t = t_1$. It can get:

$$v_y = \frac{\sqrt{\frac{mg}{k}}\left(1 - e^{\sqrt{\frac{kg}{m}}(t-t_1)}\right)}{1 + e^{\sqrt{\frac{kg}{m}}(t-t_1)}} \tag{13}$$

In order to simplify computation, let $a = e^{\sqrt{\frac{kg}{m}}(t-t_1)}$. Input it into formula (13), it can get:

$$v_y = \frac{\sqrt{\frac{mg}{k}}(1-a)}{1+a} \tag{14}$$

And $v_y = \frac{\partial y}{\partial t}$, then it has:

$$\frac{\partial y}{\partial t} = \frac{\partial y}{\partial a}\frac{\partial a}{\partial t} = \frac{\partial y}{\partial a} \bullet a\sqrt{\frac{kg}{m}} = \frac{\sqrt{\frac{kg}{m}}(1-a)}{1+a} \tag{15}$$

Make integral on above formula, input $y_{\max}$ and $a = 1$, it gets:

$$y = y_{\max} + \frac{m}{k}\ln\left\{\frac{4e^{\sqrt{\frac{kg}{m}}(t-t_1)}}{\left[1+e^{\sqrt{\frac{kg}{m}}(t-t_1)}\right]^2}\right\} \tag{16}$$

Among them, $t \in [t_1, \infty]$. When tennis lands, y=0, input it into formula (16), set tennis total time from releasing to landing is T, it can get:

$$T = t_1 + \sqrt{\frac{m}{kg}}\ln\left(\frac{2-e^{-\frac{k}{m}y_{\max}} + 2\sqrt{1-e^{-\frac{k}{m}y_{\max}}}}{e^{-\frac{k}{m}y_{\max}}}\right) \tag{17}$$

Input T value into formula (7), set tennis drop point coordinate as $(x_t, y_t, 0)$, and known that tennis initial coordinate is (x_0, y_0, z_0), it can get:

$$\begin{cases} x_t = x_0 + x\dfrac{p_0}{\sqrt{p_0^2 + q_0^2}} \\ y_t = y_0 + x\dfrac{p_0}{\sqrt{p_0^2 + q_0^2}} \end{cases} \tag{18}$$

2.2 System simulation

In practical situation, tennis will also suffer air resistance and spin force impacts during flying process, when make practical simulation, it should take the two forces into calculation.

Regard tennis motion space as a three-dimensional stereo space, establish coordinate system, take origin as tennis initial position. Tennis suffers gravity, resistance and spin force impacts in motion process. When consider resistance and tennis gravity, tennis force schematic diagram is as Figure 2 show:

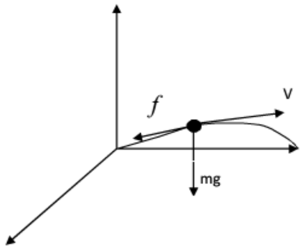

Figure 2: Tennis force in case only considers resistance and gravity

It can get tennis moves instant, resistance components in different directions as:

$$f_x = kx_x^2 \tag{19}$$

$$f_y = kx_y^2 \tag{20}$$

$$f_z = kx_z^2 \tag{21}$$

When consider spin force and tennis is flying, air forms into spin airflow in upwards and downwards of tennis. Assume tennis spins round tennis horizontal diameter in horizontal direction, and then spin airflow is consistent to tennis spin direction. When tennis makes upward motions, it suffers air resistance that in the opposite direction of air, top airflow generates bigger pressure on tennis than bottom airflow by comparing. When tennis makes downward motions, tennis top air speed will be bigger than tennis bottom .Big flow speed air generated pressure is bigger than small flow speed air. Therefore, on a whole, tennis suffers downward spin force, set it as F_p, tennis force figure is as Figure 3 show:

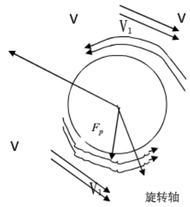

Figure 3: Tennis spin force schematic diagram

In figure, v_1 is spin force generated tennis speed variable quantity.

Under effect of spin force, tennis upper and lower two hemisphere surface suffered spin forces directions are different. Tennis first half suffered air flow speed is $v - v_1$, second half air flow speed is $v + v_1$.

By Bernoulli equation, it can get:

$$p + \rho gz + \frac{\rho v^2}{2} = c \tag{22}$$

According to above formula, when $v = 0$, it has $c = p_0 + \rho gz$

It can get tennis upper and lower surface suffered air pressures are respectively:

$$P_1 = p_0 - \frac{\rho(v - v_1)^2}{2} \tag{23}$$

$$P_2 = p_0 - \frac{\rho(v + v_1)^2}{2} \tag{24}$$

So tennis upper and lower surface suffered spin forces are respectively:

$$F_{P1} = \iint P_1 ds \tag{25}$$

$$F_{P2} = \iint P_2 ds \tag{26}$$

Therefore, tennis suffered total spin force is:

$$F_p = (P_1 - P_2)\iint ds = \pi\rho v_0 R^2 v_1 \tag{27}$$

Let $h = \pi\rho R^2 v_1$, then

$$F_p = hv_0 \tag{28}$$

So tennis force components in three directions are respectively:

$$\begin{cases} F_x = f_x = kv_x^2 \\ F_y = f_y = kv_y^2 \\ F_z = f_z + F_p + mg = kv_z^2 + hv_0 + mg \end{cases} \tag{29}$$

Firstly, analyze tennis component motion:

In x axis: By Newton's second law: $F = ma$, it gets that tennis accelerated speed a is:

$$a = \frac{F_x}{m} = \frac{dv}{dt} \tag{30}$$

By kinematics knowledge, it has $v_x = \frac{dx}{dt}$, so:

$$kv_x^2 = -m\frac{d^2x}{dt^2} \tag{31}$$

When $t = 0$, $x = x_0$, $v_x = v_{0x}$, it solves:

$$x_x = f_1(t) = \frac{m}{k}\log\frac{m}{m - kv_{0x}t} \tag{32}$$

Similarly, it can get in y axis:

$$y_y = f_2(t) = \frac{m}{k}\log\frac{m}{m - kv_{0y}t} \tag{33}$$

In z axis: When tennis is in the rising phase, tennis suffered gravity and air resistance directions are the same, by Newton's second law: $F = ma$, and $v_z = \frac{dz}{dt}$, then it has:

$$kv_z^2 + mg + F_p = -m\frac{d^2z}{dt^2} \tag{34}$$

When $t = 0$, $z = z_0$, $\frac{dz}{dt} = v_{0z}$, it can solve:

$$v_z = f_3(t) = \frac{v_{0z} - \tan(t\sqrt{F_p + kmg})\sqrt{(F_p + mg)/k}}{1 + \tan(t\sqrt{F_p + kmg})\sqrt{kv_{0z}^2/(F_p + mg)}} \quad (35)$$

When $v_z = 0$, it can calculate total time that tennis spent to raise to top point t_1, and get:

$$t_1 = \frac{m(\arctan(v_{0z}k/\sqrt{kF_p}) + \arctan(v_{0z}\sqrt{k/F_p}))}{\sqrt{kF_p}} \quad (36)$$

Let $F_z' = mg + hv_0$, by formula(36), it gets:

$$z_z = f_4(t) = \frac{m(\log(1 + v_{0z}2k/F_z') - \log(1 + \tan(\sqrt{kF_z}'(t - m \times \arctan(v_{0z}/\sqrt{kF_z}')/\sqrt{kF_z}')^2/m)}{2k} + z_0 \quad (37)$$

When tennis moves to top point and starts to fall, tennis suffered air resistance and gravity directions are opposite, so

$$-kv_z^2 + F_p + mg = m\frac{d_2 z}{dt^2} \quad (38)$$

Before tennis releasing, time $t = 0$, now tennis speed $v_z = \frac{dz}{dt} = 0$, according to above formula, it can get:

$$z_z' = f_5(t) = \frac{m\pi i + 2hk - m(\log(\frac{\tanh(t\sqrt{kF_p + kmg}/m) - 1}{\tanh(t\sqrt{kF_p + kmg}/m) + 1}))}{2k} \quad (39)$$

Set when tennis falls from top point to ground, spent time is t_2, then

$$t_2 = -\frac{m(a\tan(\frac{kv_{0z}}{\sqrt{kF_p}}) + a\tan(\frac{\sqrt{kFLp}v_{0z}}{F_p}))}{\sqrt{kF_p}} \quad (40)$$

Now $z_z' = 0$. Then tennis flight total used time is:

$$t_z = t_1 + t_2 \quad (41)$$

2.3 System implementation

During 'Hawk-Eye' system working, eight cameras will shoot tennis motions whole process positions. When system is working, it records every point position. It can select any point from these points to calculate. In the following, it makes simulation calculation on hawk-eye system drop point defining.

Assume that tennis rising phase sometime position is (x_1, y_1, z_1). Input the point into formula (37), it can get $z_1 = f_4(t)$, it can solve that tennis flies to top point used time t_{11} at this time. Respectively input t_{11} into formula (32) and formula (33), and simultaneous two equations, then it can solve h and k values.

For ball specific drop point defining, when tennis lands, $z = 0$. According to h and k values, it can solve tennis flight used total time t_z. Respectively input t_z、h and k into formula(32)and formula (33), then it can solve when tennis lands, its positions $x_x = f_1(t_z)$ and $y_y = f_2(t_z)$. It can define tennis drop point as $(x_x, y_y, 0)$.

With the help of MATLAB software, input points, and get drop point is:

$$\left(\frac{2m(Inf + \log(\frac{m}{v_{0x}}))}{c\rho s}, \frac{2m(Inf + \log(\frac{m}{v_{0x}}))}{c\rho s}, 0 \right) \qquad (42)$$

CONCLUSION

In real competitions, tennis except for suffering gravity and air resistance, it also suffers wind force that is not taken into consideration in the paper, little error may be generated .In order to define 'Hawk-Eye' system accuracy, we input one competition system recorded tennis position tennis one moment position point as (2, 16, 1) into model result. With the help of MATLAB software, it gets tennis drop point position is (1.075, 8.6002, 0), actual shot tennis drop point is(1.0785, 8.6015, 0). Compare and find that the two values differences is less than 3 millimeters, so 'Hawk-Eye' system accuracy is higher.

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