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# Study on one new grey similarity correlation degree model and its applications

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**ABSTRACT**

*Grey correlation analysis is not only the important part of grey system theory, but also the cornerstone of grey forecasting, grey policymaking, grey clustering and grey estimating. In this paper, a new computing model of grey similarity correlation degree is constructed based on analyzing grey correlation axioms and the existing computing models of grey correlation degree, then some of its properties is analyzed and proved. The example shows that the models algorithm is simple and has good applicability.*

**Key words:** Grey correlation analysis; grey system theory; similarity; grey correlation degree.

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## INTRODUCTION

The method of grey similarity correlation degree, which is an important method of grey system theory for system analysis, is based on qualitative analysis and quantitative analysis. Grey correlation degree represents a measure of correlation degree among the internal factors of grey system. After proposing grey system theory by the professor Julong Deng, many scholars have engaged in the research of grey correlation degree since 1982. Some new computing models of grey similarity correlation degree have been constructed such as Deng's correlation degree (proposed by Julong Deng [1]), the model of grey absolute correlation degree (constructed by Zhenguo Mei [2]), the model of relative correlation degree (obtained by Sifeng Liu [3]), the model of B-mode relational degree and the model of C-mode relational degree (presented by Qingyin Wang, et al [4, 5]), the model of T's correlation degree (proposed by Wuxiang Tang [6]), the model of grey entropy relation grade (obtained by Qishan Zhang [7]) , grey Euclidean relation grade (built by Yanlin Zhao [8]), the model on degree of grey slope incidence (proposed by Yaoguo Dang [9]), and so on (see [10, 11, 12, 13, 14, 15]). Looking from the existing research literature material, we can find that a large number of literatures study the close degree among the factors, i.e., the model of close correlation degree. But the study on the model of similarity correlation degree remains relatively rare. In this paper, a new computing method of grey similarity correlation degree is constructed based on the basic idea of grey correlation analysis which is proposed by the professor Julong Deng and the model deficiencies of existing grey correlation degree. Its basic properties are discussed meantime from the comprehensive microscopic view, and the proofs of the properties are given. Example analysis shows that the proposed new model of grey similarity correlation degree takes on simple calculations, low algorithm complexity and good applicability.

## 1. THE MODEL OF DENG'S GREY CORRELATION DEGREE

**Definition 1:** Assume that the system feature data sequence is $X_0 = (x_0(1), x_0(2), \cdots, x_0(n))$. And the correlation factor data sequence is assumed to be

$X_1 = (x_1(1), x_1(2), \cdots, x_1(n))$,

$X_2 = (x_2(1), x_2(2), \cdots, x_2(n))$,

…

$X_m = (x_m(1), x_m(2), \cdots, x_m(n))$,
where $x_i(j) > 0$, $i = 0,1,2,\cdots,m,\ j = 1,2,\cdots,n.$ Suppose that $\gamma(x_0(j), x_i(j))$, $j = 1,2,\cdots,n,$ are the given real numbers. If a real number

$$\gamma(X_0, X_i) = \frac{1}{n}\sum_{j=1}^{n}\gamma(x_0(j), x_i(j))$$

satisfies the following four properties:

**(i) Normality:** $0 < \gamma(X_0, X_i) \le 1$, and $X_0 = X_i \Leftrightarrow \gamma(X_0, X_i) = 1$.

**(ii)Integrity:** $\gamma(X_p, X_q) \ne \gamma(X_q, X_p)$ when $p \ne q$, where
$X_p, X_q \in X = \{x_k \mid k = 0,1,2,\cdots,m, m \ge 2\}$.

**(iii)Even symmetry:** $\gamma(X_p, X_q) = \gamma(X_q, X_p) \Leftrightarrow X = \{X_p, X_q\}$.

**(iv) Proximity:** the smaller $|x_0(j) - x_i(j)|$, the bigger $\gamma(x_0(j), x_i(j))$.

Then $\gamma(X_0, X_i)$ is called grey correlation degree of $X_0$ and $X_i$, $\gamma(x_0(j), x_i(j))$ is named correlation coefficient of $X_0$ and $X_i$ at the *j*-th point, the four properties (i), (ii), (iii) and (iv) is called the four axioms of grey correlation (see [1]).

**Definition 2:** Assume that $X_i = (x_i(1), x_i(2), \cdots, x_i(n))$ which is defined by the above Definition 1, where $i = 0,1,2,\cdots,m.$ Let

$$\Delta_{0i}(j) = |x_0(j) - x_i(j)|,$$

$$\Delta_{\min} = \min_i \min_j \Delta_{0i}(j)$$

and

$$\Delta_{\max} = \max_i \max_j \Delta_{0i}(j).$$

Then $\Delta_{0i}(j)$ is called absolute deviation, $\Delta_{\min}$ and $\Delta_{\max}$ are called bipolar minimum deviation and bipolar maximum deviation respectively (see [1]).

**Theorem 1:** For an arbitrary real number $\lambda \in (0,1)$, let

$$\gamma(x_0(j), x_i(j)) = \frac{\Delta_{\min} + \lambda\Delta_{\max}}{\Delta_{0i}(j) + \lambda\Delta_{\max}}$$

(1)

and

$$\gamma = \gamma(X_0, X_i) = \frac{1}{n}\sum_{j=1}^{n}\gamma(x_0(j), x_i(j)).$$

(2)

Then the real number $\gamma = \gamma(X_0, X_i)$ satisfies the four axioms of grey correlation (see [1]).

## 2. THE SHORTCOMINGS OF DENG'S GREY CORRELATION DEGREE MODEL

The impact of the distance between two points on correlation degree is considered emphatically when Deng's correlation degree is calculated. Only by large quantity of data calculating and analyzing can we find that Deng's correlation degree exists some shortcomings.

**(1)** Non-uniqueness of the value $\gamma$ of grey correlation degree.

There are numerous factors affecting the value $\gamma$, such as the reference sequence $X_0$, the transform methods of comparative sequence $X_i$, the distinguishing coefficient $\lambda$, and so on. Especially, the magnitude on grey correlation degree is various when $\lambda$ is substituted various values or when the transform method is different. So the value $\gamma$ of grey correlation degree is not uniqueness.

**(2)** The variation of grey relational order depending on the value $\lambda$.

For example, let

$X_0 = (2, 2, 2, 2, 2, 2)$,

$X_1 = (2, 1, 3, 1, 3, 2)$,

$X_2 = (\frac{8}{5}, \frac{12}{5}, \frac{8}{5}, \frac{12}{5}, \frac{8}{5}, \frac{12}{5})$,

i.e., $x_0(j) = 2$ and $x_2(j) = x_0(j) + (-1)^j \frac{2}{5}$, $j = 1, 2, \cdots, 6$. After computing by adopting the mean method, we obtain an expression as below

$$\gamma_1 - \gamma_2 = \frac{1 - 2\lambda}{3(1+\lambda)(1+\frac{5}{2}\lambda)}.$$

Therefore, according to the above equality we have the following conclusions:

(a): $\gamma_1 < \gamma_2$ when $\lambda > 0.5$.

(b): $\gamma_1 > \gamma_2$ when $\lambda < 0.5$.

(c): $\gamma_1 = \gamma_2$ when $\lambda = 0.5$.

**(3)** Assume that $\lambda = 0.5$. Then the value $\gamma$ of grey correlation degree is greater than $\frac{1}{3}$.

**Proof:** Suppose that

$$a = \Delta_{\min} = \min_i \min_j \Delta_{0i}(j)$$

and

$$b = \Delta_{\max} = \max_i \max_j \Delta_{0i}(j).$$

Substituting $a$, $b$, and $\lambda = 0.5$ into Eq. (1), we get

$$\gamma(x_0(j), x_i(j)) = \frac{a + 0.5b}{\Delta_{0i}(j) + 0.5b} = \frac{a}{\Delta_{0i}(j) + 0.5b} + \frac{0.5b}{\Delta_{0i}(j) + 0.5b}.$$

Because $a > 0$ and $b > 0$, it is obvious to see that $\gamma(x_0(j), x_i(j))$ has a minimum when $b = \Delta_{0i}(j)$, that is, for arbitrary $i$ and $j$, one can get

$$\gamma(x_0(j), x_i(j)) = \frac{a}{b + 0.5b} + \frac{0.5b}{b + 0.5b} > \frac{0.5b}{b + 0.5b} = \frac{1}{3}.$$

(3)

According to the above inequality (3) and Eq. (2), one has

$$\gamma = \frac{1}{n} \sum_{j=1}^{n} \gamma(x_0(j), x_i(j)) > \frac{1}{3}.$$

Hence the minimum of $\gamma$ is greater than $\frac{1}{3}$. The proof is completed.

Obviously, it is not reasonable that grey correlation degree of $X_0$ and $X_i$ is close to medium value which is not less than 0.333.

Broadly speaking, the law of development of any objective things is all the function of time whose graph reflects the image of development of things under all possible development factors. The different graphs of functions reflect largely differences of thing's development processes [4]. In the reference [1], the professor Julong Deng has made insightful statements that the developing trends are more closer if the geometrical shapes are closer, which causes more bigger correlation degree. This property is called similarity of thing's development. In this paper, a new computing model of grey similarity correlation degree is constructed based on analyzing grey correlation axioms and the existing computing models of Deng's grey correlation degree, which can avoid some shortcomings of general grey correlation degree models.

## 3. THE COMPUTATIONAL METHOD OF THE NEW GREY CORRELATION DEGREE MODEL

**Definition 3:** Assume that $X_0 = (x_0(1), x_0(2), \cdots, x_0(n))$ and $X_i = (x_i(1), x_i(2), \cdots, x_i(n))$, where $x_i(j) > 0$, $i = 0,1,2,\cdots,m,\ j = 1,2,\cdots,n.$ Let

$$Y_{0,i} = (y_{0,i}(1), y_{0,i}(2), \cdots, y_{0,i}(n)),$$

where $y_{0,i}(j) = \dfrac{x_i(j)}{x_0(j)}$, $j = 1,2,\cdots,n.$ And suppose that

$$\gamma_i^* = \frac{1}{n}\sum_{j=1}^{n} \gamma^*(x_0(j), x_i(j)), \tag{4}$$

where

$$\gamma^*(x_0(j), x_i(j)) = \frac{1}{1 + \left| y_{0,i}(j) - \overline{Y}_{0,i} \right|}, \tag{5}$$

$$\overline{Y}_{0,i} = \frac{1}{n}\sum_{j=1}^{n} y_{0,i}(j),\quad i = 1,2,\cdots,m. \tag{6}$$

Then $\gamma_i^*$ is called the grey similarity correlation degree of $X_0$ and $X_i$.

It is obvious to see that $\gamma_i^*$ gives the computational method of the new grey correlation degree model according to Definition 3.

## 4. THE PROPERTIES OF THE NEW GREY SIMILARITY CORRELATION DEGREE MODEL

**Property 1 (Normality):** $0 < \gamma^*(X_0, X_i) \le 1$, and $X_0 = X_i \Leftrightarrow \gamma^*(X_0, X_i) = 1$.

**Proof:** It is easy to see that $0 < \gamma^*(X_0, X_i) < 1$ according to Definition 3. When $X_0 = X_i$, it is clear that $y_{0,i}(j) = 1$ and $\gamma^*(x_0(j), x_i(j)) = \dfrac{1}{1 + \left| y_{0,i}(j) - \overline{Y}_{0,i} \right|} = 1$.

Therefore, $\gamma_i^* = \dfrac{1}{n}\sum_{j=1}^{n} \gamma^*(x_0(j), x_i(j)) = 1$ satisfies the property of normality. Property 1 is proved.

We can find easily that $\gamma_i^*$ reflects the correlation between two variables. A bigger value for $\gamma_i^*$ indicates a higher correlation degree. $\gamma_i^* = 1$ suggests a perfect correlation of two variables. The values of the grey correlation degree presented by the literature [10] could be equal or less than zero, which contradicts philosophical thought of grey system that internal factors of system affect each other to generate that correlation degree is greater than zero. In this paper, the new computational method of grey similarity correlation degree is presented to avoid its value equivalent to or less than zero, which is consistent with philosophical thought of grey system.

**Property 2 (Isotonicity):** $\gamma_i^*$ is isotonic for scalar-multiplicative transformation.

**Proof:** Let $X_0' = (\alpha x_0(1), \alpha x_0(2), \cdots, \alpha x_0(n))$ and $X_i' = (\alpha x_i(1), \alpha x_i(2), \cdots, \alpha x_i(n))$, where $\alpha \ne 0$ is a constant. Then we get

$$y_{0,i}'(j) = \frac{\alpha x_i(j)}{\alpha x_0(j)} = \frac{x_i(j)}{x_0(j)} = y_{0,i}(j).$$

It is easy to see that correlation coefficient of $X_0'$ and $X_i'$ at the $j$ -th point remains unchanged.

Therefore, this similarity correlation degree model has isotonicity for scalar-multiplicative transformation. The proof of Property 2 is completed.

**Property 3(Uniqueness):** $\gamma_i^*$ is unique.

**Proof:** According to Eq. (4) and Eq. (5), we can see easily that the model does not contain other parameters. $\gamma_i^*$ is not a relative value, but an unique absolute value. The proof of Property 3 is ended.

**Property 4(Proximity):** $\gamma_i^*$ approaches one.

**Proof:** Obviously, the smaller the value of $\left|y_{0,j}(j)-\overline{Y}_{0,i}\right|$, the closer the correlation coefficient of point is to one, which causes that grey similarity correlation degree approaches one. The proof of Property 4 is completed.

**5. NUMERICAL EXAMPLE**

In the national economy system, all industries such as agriculture, industry, commerce, transportation, and so on, are related and influenced with each other. Agriculture can offer a variety of raw materials and almost all the means of production, and provide a lot of means of livelihood which meet the manifold needs of the people. Therefore, analysing the magnitude on grey correlation degree between agriculture and other sectors of the national economy has an important reference significance to make different economic development policy for decision maker. Now, based on economic data of agriculture, industry, commerce and transportation for four consecutive years, we employ the grey similarity correlation degree model to make empirical analysis through an example. Shown in Table 1 is the original data.

**Table 1: The original data (unit: 10 million yuan)**

| | $j$ (initial year) | $j+1$ | $j+2$ | $j+3$ |
|---|---|---|---|---|
| Agriculture $X_0$ | 139.1 | 141.6 | 143.9 | 144.9 |
| Industry $X_1$ | 162.8 | 147.5 | 138.3 | 134.9 |
| Commerce $X_2$ | 23.7 | 22.7 | 17.4 | 15.2 |
| Transportation $X_3$ | 12.1 | 11.2 | 11.5 | 11.5 |

Let $X_0$ be the reference sequence. According to $Y_{0,i}=\left(y_{0,i}(1),\, y_{0,i}(2),\cdots,\, y_{0,i}(n)\right)$ where $y_{0,i}(j)=\dfrac{x_i(j)}{x_0(j)}$, we have

$Y_{0,1}=\left(1.17, 1.04,\, 0.96,\, 0.93\right)$,

$Y_{0,2}=\left(0.17, 0.16,\, 0.12,\, 0.105\right)$,

$Y_{0,3}=\left(0.087, 0.079,\, 0.08,\, 0.079\right)$.

From Eq. (6), we get

$\overline{Y_{0,1}}=1.026$, $\overline{Y_{0,2}}=0.139$ and $\overline{Y_{0,3}}=0.081$.

(7)

Substituting the above results (7) into the expression Eq. (5), one has

$\gamma^*(x_0(1),x_1(1))$=0.874 , $\gamma^*(x_0(2),x_1(2))$=0.986 ,

$\gamma^*(x_0(3),x_1(3))$=0.938 , $\gamma^*(x_0(4),x_1(4))$=0.912 ,

$\gamma^*(x_0(1),x_2(1))$=0.970 , $\gamma^*(x_0(2),x_2(2))$=0.980 ,

$\gamma^*(x_0(3),x_2(3))$=0.981 , $\gamma^*(x_0(4),x_2(4))$=0.967 ,

$\gamma^*(x_0(1), x_3(1))$=0.994, $\gamma^*(x_0(2), x_3(2))$=0.998 ,
$\gamma^*(x_0(3), x_1(3))$=0.999 , $\gamma^*(x_0(4), x_1(4))$=0.998 .

Using the grey similarity correlation degree defined by Eq. (4), one can easily verify that

$\gamma^*(X_0, X_1)$=0.9275 , $\gamma^*(X_0, X_2)$=0.9745 and $\gamma^*(X_0, X_3)$=0.9973 .
Therefore, we have
$\gamma^*(X_0, X_1) < \gamma^*(X_0, X_2) < \gamma^*(X_0, X_3)$,
which means that the influence degree of industry, commerce, transportation on agriculture is varied. That all values of $\gamma^*(X_0, X_1)$, $\gamma^*(X_0, X_2)$ and $\gamma^*(X_0, X_3)$ are greater than 0.9 show that the correlation degree between agriculture and industry or commerce or transportation is higher, which coincides with the reality. In economic systems, the agricultural economy is always dominant to some extent, but agriculture's development cannot be isolated from transportation's development. Commerce play a role of bridge and tie between the producer (or manufacturer) and the consumer. In the same way, commerce's development is closely correlated with transportation's development. Therefore, when formulating a series of industrial plans and policies, government departments at all levels should pay enough attention to reasonable policy decision according to the objective laws of development of things. Priority to the development of transport can promote the rapid development of transportation such that agriculture and even national economy will develop in the direction toward the depth.

## CONCLUSION

A new computing model of grey similarity correlation degree is constructed based on the idea of the theory of grey correlation presented by the professor Julong Deng. The new model has some important properties such as normality, uniqueness, proximity and rank preservation for multiplicative transformation. The detailed proofs of the properties are given by adopting qualitative and quantitative methods. At last, this paper demonstrates the application of the model in an example. The research result shows that the model’s algorithm is simple and has good applicability.

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