



Research Article

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## Study on multi type-insurance compound Poisson-geometric risk model

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### ABSTRACT

*Risk theory is current fine and mathematics territories research of hot topic, which bankruptcy on the of research has is strong of actual application background, and has its probability on the Shang of significance. First, a single compound poisson process insurance extended to the double dual compound Poisson-Geometric insurance risk model. Second, previous scholars studies mostly the risk model under constant interest force or stochastic interest rate ,but this article has carried on the promotion, establishes fuzzy interest with interference double type-insurance compound Poisson-Geometric risk model. Finally, under fuzzy interest, double type-insurance compound Poisson-Geometric risk model extend to multi type-insurance compound Poisson-Geometric risk model.*

**Key words:** fuzzy interest rate; compound Poisson–Geometric; interference; ruinprobability

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### INTRODUCTION

Risk theory is widely applied in the field of insurance and investment. In particular, for those who want to get big profits through a small amount of investment of investors, the risk theory in the aspect of risk quantitative analysis and prediction plays an important role [1]. Everywhere in our life is full of risks, but for some weak individuals, independent risk especially in some big risk will be very hard, so people chose to cast protects. This also will promote the development of the insurance industry, established close and there are more and more insurance companies, but not the insurance company will not risk. They are not only collected the applicant to pay the premiums, but also for policy-holder is possible to meet the relevant risks. Therefore, the real financial situation and its compensation of insurance company ability is closely related [2]. Therefore, whether the insurer himself or policy-holder are very concerned about the solvency of insurance company. Risk theory is the core content of the ruin probability is used to evaluate an important indicator of the insurance company solvency. From the early Swedish actuary Lundberg to later a lot of scholars have done quite a lot of research on bankruptcy theory and made a lot of gratifying results. At the same time, to promote the development of the theory of risk and make the development of the theory of stochastic process is also fully [3]. And at the same time, Cramer will bankrupt theory based on stochastic process theory the basic theorem of work for the classical theory of bankruptcy. Today, we have used in mathematical model to describe and study by the insurance company will face the risk of bankruptcy theory, as a very important subject to study and research.

It was originally to set up in the classical risk model and too idealistic. It uses the same random process, homogeneous Poisson process to describe the risk events and compensation and make the two events are equal, but the reality is not what we want is what [4]. There will always be a certain gap, the insurance company in order to prevent the bankruptcy of gain high profits at the same time. In making compensation way, with deductibles the classical risk model cannot be very good reaction this situation. In order to research this kind of insurance affairs, many scholars introduced the compound Poisson-Geometric count process to describe the situation, such as Interference under the condition of compound Poisson-Geometric process more risk model under the bankruptcy probability [5]. With Poisson-Geometric Brown movement risk model of bankruptcy. A claim number for compound Poisson process-Geometric model and the risk of bankruptcy is, and so on, they not only get the ruin probability formula of the model also gives a corresponding update equations. On the other hand, due to the characteristics of

the insurance business of insurance company, it led to the development of the insurance product pricing is different also both are closely linked. In previous insurance business, the foreign insurance industry a predetermined interest rate to be decided according to the bank interest rates. The main use of insurance funds way is bank deposits. The main risk faced by insurance companies is to as for its insurance underwriting risk of the project [6]. Therefore, for insurance companies, scientific prediction of the premium income, evaluate the possibility of the amount of payments and the probability of bankruptcy is very important. Therefore, in the insurance business, the problem is an important means of business to a reasonable set of insurance rate. The benefits and risks of the future make scientific prediction.

### RELEVANT BASIC KNOWLEDGE

In recent years, many scholars have studied the compound Poisson-risk of Geometric model and has obtained certain achievements. The paper also made a further research to its. This chapter first describes the compound Poisson-Geometric process introduced simply. Secondly, it introduces the important tool of risk theory martingale deals with in the paper need to use the Brownian motion, ruin probability and fuzzy theory and related knowledge.

The bankruptcy probability of classical risk model research led to the bankruptcy of theory study. The following is referred to as simply introduced L-c model of classical risk model of some relevant conclusions and important results.

$\{T_i, i \geq 1\}$  is parameters as random variables are independent and satisfy with the condition of exponential distribution. The T said the interval between two claims occur.

$$S_n = T_1 + T_2 + T_3 + \dots + T_n, \forall n \geq 1 \quad (1)$$

And there,  $S_n$  is the Nth occurrence time of the claim.

At the moment t claim number is defined as the insurance company what had happened

$$N(t) = \max_{n \geq 0} \{n \mid 0 < S_n \leq t\} \quad (2)$$

It is known that  $\{N(t), t \geq 1\}$  is parameter  $\lambda$  for Poisson process. It is shown that in Fig. 1 is this situation a sample of a typical of the orbit.

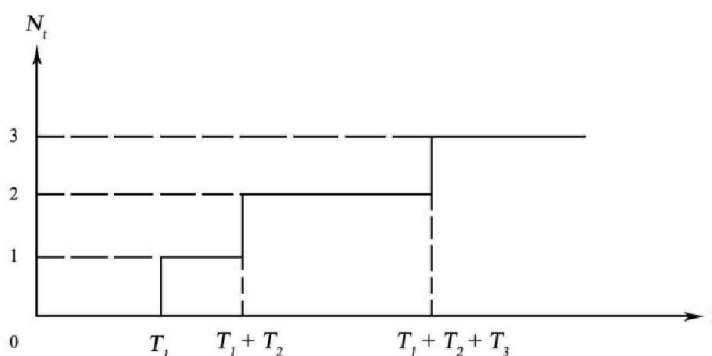


Fig. 1: Sample track

$\{X_i, i \geq 1\}$  and  $\{T_i, i \geq 1\}$  are independent of each other.  $\{X_i, i \geq 1\}$  is independent and identically distributed random variables. Among them,  $X_i$  is the Ith claim amount. The averages are  $P_1$ . Make until time t the total amount of the insurance company will have to claim to  $C(t)$ , then

$$C(t) = \sum_{i=1}^{N(t)} X_i \quad (3)$$

$\{C(t), t \geq 0\}$  is a parameter for  $\lambda$  compound Poisson process. We call it the total amount of the claim process.

To the insurance company in surplus to time t is  $U(t)$ , expressed as

$$U(t) = u + ct + C(t) \quad (4)$$

Among them,  $u \geq 0$  is the initial reserve,  $C(t)$  is the total amount of the claim, said process  $\{U(t) | t \geq 0\}$  surplus process for insurance. C is insurance company insurance rate per unit time, about a sample of the surplus process track as

shown in Fig.2.

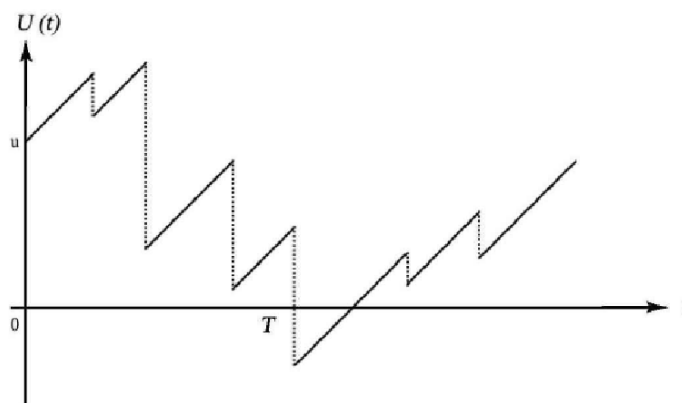


Fig. 2: Surplus process samples

Further, we put the insurance surplus negative moment to remember the first time

$$T = \inf\{t \mid U(t) < 0\} \quad (5)$$

$T$  for the insurance companies are facing bankruptcy moment (when  $T$  tends to  $\infty$ , explain the insurance company's business is not good Bankruptcy will happen).

By the parameter for the Poisson process  $\{N(t), t \geq 0\}$ , then

$$E[N(t)] = \lambda t \quad (6)$$

In order to ensure the insurance company will not occur bankruptcy, we assume that

$$c > \lambda p_1 \quad (7)$$

$$\rho = \frac{c}{\lambda p_1} - 1 \quad (8)$$

The  $\rho > 0$ , referred as relatively safe load.

when  $C > \lambda p_1$ , there are the same time the insurance company have to pay the damages of expected value is less than its received premium amount, its relatively safe load is positive. So, we got the ultimate ruin probability of insurance company:

$$\psi(u) = \Pr\{T < \infty\} \quad (9)$$

To understand the insurance company operating in a specific period of time, we make finite time  $(0, t)$  within its insurance. The ruin probability of insurance company is

$$\psi(u, t) = \Pr\{T < t\} \quad (10)$$

When the initial surplus is zero, the insurance company for the ultimate ruin probability

$$\psi(0) = \frac{1}{1 + \rho} \quad (11)$$

Thus, when the initial surplus is zero, a relatively safe load the final bankruptcy shall decide the size of the insurance company rate. So the relatively safe load control, is crucial for the insurance company.

When individuals claim amount counting process is exponential distribution and the distribution parameters of  $\beta$ , for eventually ruin probability

$$\psi(u) = \frac{\lambda}{c} p_1 \exp\left(\frac{\lambda}{c} u - \frac{u}{p_1}\right) \quad (12)$$

So we can draw the conclusion: the ruin probability of insurance company is subject to the individual claims

obedience distribution parameters. For the most common form of exponential distribution, the individual claim amount to the parameters of the distribution and expected claim into reverse. It also decided to the size of the possibility of bankruptcy. The parameters of the distribution is smaller but more likely bankruptcy, as a result and insurance [7]. In order to ensure not occur the bankruptcy of the company when the insured, we have very high request for underwriting.

In addition, the amount of initial reserve funds by insurance companies. It will also affect the bankruptcy probability values. The general situation is to its ruin probability. The initial surplus value is inversely proportional, therefore, it must have enough initial surplus to insurance. The insurance company management tend to be stable.

Losses of variables for insurance companies

$$L = \max_{t \geq 0} \{C(t) - ct\} \tag{13}$$

$$\psi(u) = \Pr\{L > u\} \tag{14}$$

In addition, the huge loss of another is expressed as the variable L

$$L = \sum_{n=1}^N L_n \tag{15}$$

The deficit  $L_1$  said the initial surplus  $u$  is greater than the surplus process for the first time gap, between adjacent two deficits phase delta as  $L_2, L_3, \dots, N$  for the surplus process  $\{R(t), t \geq 0\}$  and deficit number times. When the initial surplus  $u=0$ . As shown in Fig.3 L samples of the orbit. Similarly, when the initial surplus  $u = 0$ .

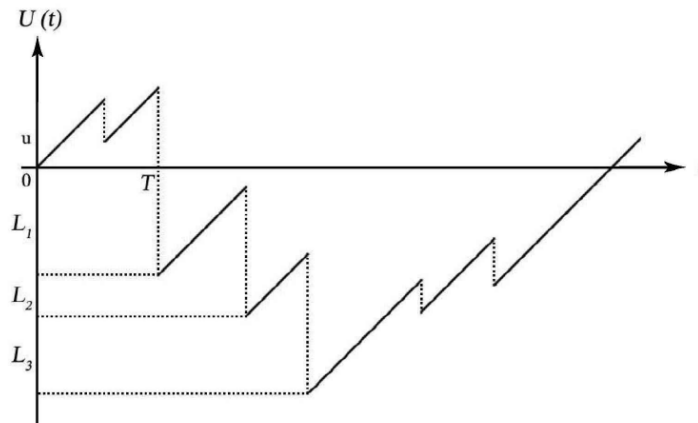


Fig. 3: The orbit of the sample

Similarly, when the initial surplus  $u=0, L_1$  rail as shown in Fig.4..

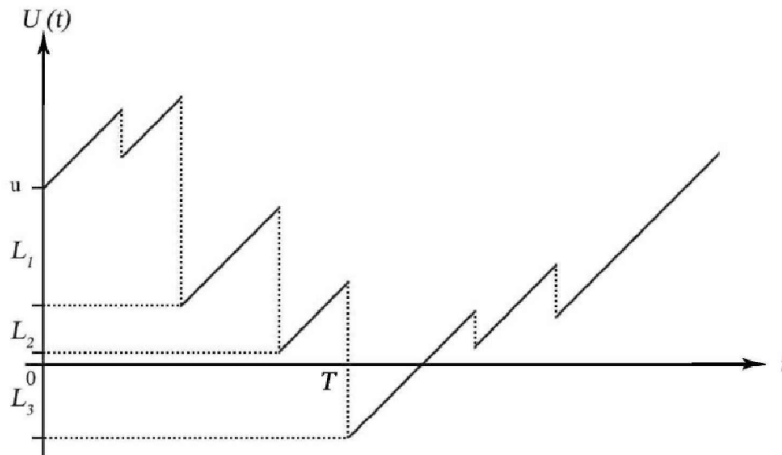


Fig. 4: The orbit of the sample

Classical risk model in risk theory continuously in the development of increasingly mature and gradually improve, first proposed the theory of a Swedish actuary Filip Lundberg. After Filip Lundberg and Harald Cramer studied the initial surplus is zero, and its corresponding ruin probability is obtained. On the time value of ruin, surplus of bankruptcy occurs and moments before and after some scholars to the bankrupt surplus. And it done a lot of feasibility study, also got a lot of valuable achievements. And on the two kinds of situations and the distribution function is given the corresponding analytical. Dickson made a further study on the distribution, bankruptcy deficit for the ruin probability distribution function and its corresponding before the bankruptcy and instantaneous surplus of the distribution function and so on.

### GENERALIZATION OF THE CLASSICAL RISK MODEL

Researchers in recent years, risk theory is based on the model, add or change some influencing factors for further research to establish a risk model can response more practical. More meet the needs of the insurance company. Due to the classical risk model considered too little, it can't reasonable actual reaction, also can't reasonable description of the premium income of insurance company, etc. Therefore some scholars for the promotion of classical risk model in different directions.

In insurance financial markets, the income of insurance company to pay insurance cost. In addition to policy-holder and insurance business operators will spare money in the bank, purchases of Treasury securities or buy other forms of risk fund. In order to obtain bank interest, the part of the income is an important part of the insurance company, as a result. The interest is the effect on insurance company assets and liabilities should not be ignored. The important factors for the field of special attention, therefore, the uncertainty of claims and the market interest rate is also one of the risk. The risk model in the study if you don't consider the effect of interest rates is not conform to the actual. So in order to make the risk model in line with the actual, we introduced the renewal risk model with interest. In this kind of extension model mainly includes the constant interest rate and stochastic interest rate risk model under two kinds of situations. Poisson coordinates computation is seen as Fig. 5. We with constant interest rate risk model as the example to this kind of model are explained simply.

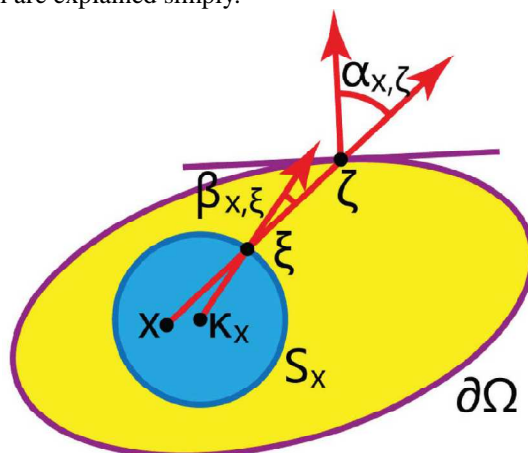


Fig. 5: Poisson coordinates computation

In insurance financial markets, the income of insurance company to pay insurance cost. In addition to policy-holder and insurance business operators will spare money in the bank. Purchases of Treasury securities or buy other forms of risk fund etc. In order to obtain bank interest, the part of the income is an important part of the insurance company as a result. The interest is the effect on insurance company assets and liabilities should not be ignored. The important factors for the field of special attention, therefore, the uncertainty of claims and the market interest rate is also one of the risk. The risk model in the study if you don't consider the effect of interest rates is not conform to the actual. So in order to make the risk model in line with the actual, we introduced the renewal risk model with interest in this kind of extension model mainly includes the constant interest rate and stochastic interest rate risk model under two kinds of situations. We with constant interest rate risk model as the example to this kind of model are explained simply.

Set up the model of interest rate as a constant  $\delta$ , its premium for  $dt$  time change the quantity to the  $cdt$ . The interest income of the insurance company is  $\delta U(t)dt$ . And at the same time compensation of happen is  $dC(t)$ , so its surplus in  $dt$  time change is:

$$\delta U(t) dt = cdt + \delta U(t) dt - dC(t) \quad (16)$$

For the problem, Brown suppose the risk management process to obey by Brownian motion. It finally has obtained certain achievement. 2D discrete Poisson coordinates is seen as Fig. 6. Jinm etc and CAI research such as the risk of interest rate for the random model of bankruptcy.

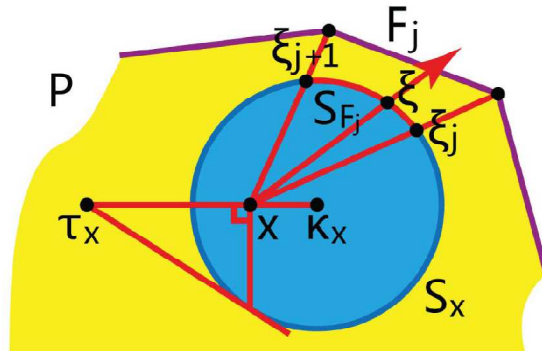


Fig. 6: 2D discrete Poisson coordinates

Different from the classical risk model. In practice, various types of business insurance company, so it is not the only source of revenue. There is a bank interest rates, investment income, and so on. Insurance companies only in the accurate understanding of its surplus situation. The company's managers decided to invest excess surplus in order to obtain profits. Due to the insurance company formulated the different way of claims, so in the case of a claim the claim is relatively is different and influenced by external factors. The investment cannot be determined to make the balance of the insurance company. We introduce the interference law, mainly refers to the deviation of the insurance company's management or business insurance company's financial situation. Set the surplus process is:

$$U(t) = u + ct - C(t) + W(t), \forall t \geq 0 \tag{17}$$

$W_t$  is for Brownian motion and noninterference between  $\{W(t), t \geq 0\}$  and  $\{C(t), t \geq 0\}$ . So, it can decompose the bankruptcy probability.

$$\psi_d(u) = \psi_d(u) + \psi_c(u) \tag{18}$$

Where  $x$  and  $y$  respectively expresses the bankruptcy was caused by the random disturbance and claim. Poisson coordinates and MVCs for 2D polygons and Poisson coordinates or MVCs and discrete harmonic coordinates are seen as Fig. 7.

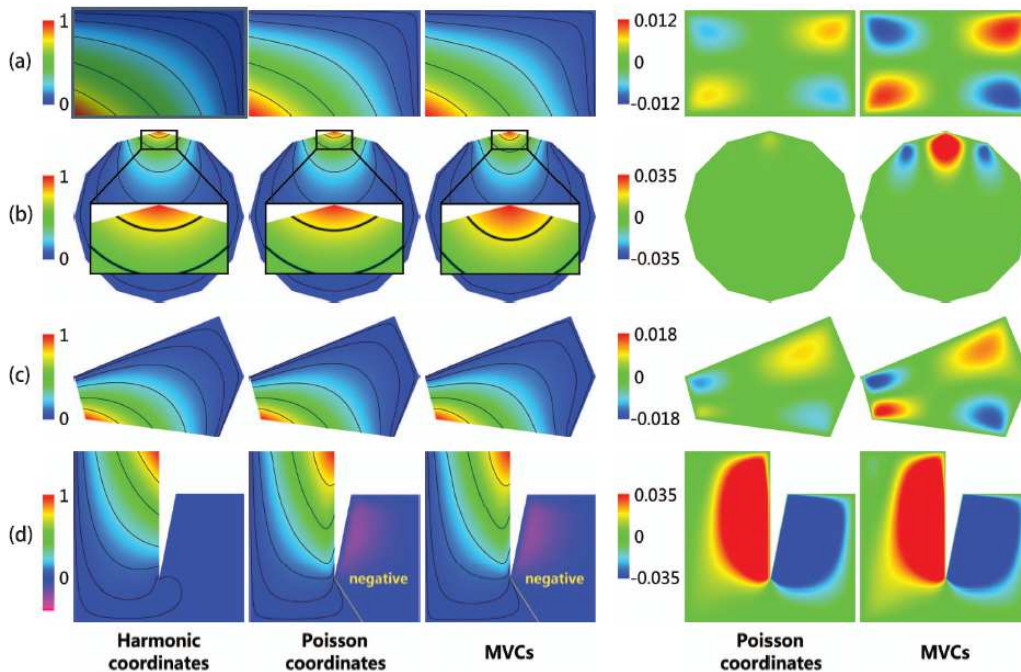


Fig. 7: Poisson coordinates and MVCs for 2D polygons and Poisson coordinates or MVCs and discrete harmonic coordinates

### CONCLUSION

This paper mainly studies the fuzzy rates under the compound Poisson risk - Geometric model. It will be part of the initial capital investment and consider its premium amount and claim amount and interest rate between membership function. It establish a bivariate double type-insurance risk model under fuzzy rates. Some conclusions about the ruin probability are obtained. It spread to the situation of more danger is planted. It also can get the similar conclusions. Previously under the stochastic interest rate of compound Poisson-Geometric risk models are generalized to fuzzy rates under the compound Poisson-Geometric risk model. Make it more practicalmore some maneuverability. The insurance company by understanding some features of the ruin probability under the model can better prevent and control the risk of insolvency.

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