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**Research Article**

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# Slip flow and heat transfer through a rarefied nitrogen gas between two coaxial cylinders

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**ABSTRACT**

*The slip flow and heat transfer through a rarefied gas confined between concentric cylinders are investigated. The heat transfer and flow between a heated tungsten filament and a surrounded rarefied gas ($N_2$) has been studied numerically; gas temperatures range from 300 K in the envelope region to 3000 K at the wire. Temperature, density and velocity fields are calculated. This work has been carried out using the CFD (Computational Fluid Dynamics) computer code Fluent (Fluent® 6.3.26). The LPBS method proposed in Fluent for implementing slip conditions is used. The influence of the mesh type on the solution has been examined by using the CFD code Fluent and the analytical solution.*

**Keywords:** Rarefied nitrogen gas, slip flow regime, heat transfer, CFD, low pressure boundary slip.

## INTRODUCTION

The problem of flow and heat transfer through rarefied gases, confined between coaxial cylinders is very frequent in several technological applications, such as Pirani and diaphragm gauges for instrumentation, monitoring and control of vacuum processes or micro heat exchangers in microfluidics [1].

A gas inside the microsystems or the porous media is in its non-equilibrium state, due to the fact that the molecular mean free path is comparable to the characteristic dimension of the media. The same state of a gas, called rarefied, is found at high altitude or in the vacuum equipment working at low pressure. [2]

The slip flow regime is a slightly rarefied regime of grand importance for gas flow [3]. It corresponds to a Knudsen number between $10^{-3}$ and $10^{-1}$. The Knudsen layer plays a fundamental role in the slip flow Regime [4]. In the slip flow regime, the Navier-Stokes equations are still valid in the mass flow; the gas is not in local thermodynamic equilibrium near the wall in said layer Knudsen. In this region, different effects of rarefaction are exposed, including the presence of a non-negligible temperature jump at the walls [5] or solid interfaces.

Even though the Navier-Stokes equations are not valid in the Knudsen layer, due to a nonlinear stress/strain-rate behaviour in this small layer [6], their use with accurate boundary velocity slip and temperature jump conditions confirmed to be precise for calculating mass flow rates [7-8] out of the Knudsen layer.

Rarefied gas flow simulation is feasible with the commercial CFD code Fluent by the use of the "Low Pressure Boundary Slip" (LPBS). Pitakarnnop *et al* [4], used the LPBS method in the slip flow regime for modeling triangular and trapezoidal microchannels for predicting slip velocity.

In the present work, the problem of steady-state heat transfer, in the slip flow regime, through a rarefied gas between two coaxial cylinders is solved numerically based on the Navier-Stokes and Fourier equations. The numerical simulation was performed using the general purpose fluid dynamic computer code, Fluent [9]. The LPBS method proposed in Fluent for implementing slip conditions is used in the case of two coaxial cylinders. The influence of the mesh type on the solution has been examined by using the CFD code Fluent and the analytical solution.

## EXPERIMENTAL SECTION

**Problem Definition**

We consider a long cylinder with an axial tungsten wire having a diameter (d) of 1.58 mm and a length of 23.5 mm, the wire is modeled as a solid cylinder. The external cylinder is defined to be 75 mm in diameter (D) and 250 mm in length. The filling rarefied gas is nitrogen ($N_2$). Boundary conditions are set equal to 3000 K and 1500 K for the filament or the inner cylinder and 300 K for the external cylinder. A filling pressure (10 Pa) is used to maintain a slip regime flow and therefore a conduction heat transfer inside the cell. The transport properties of the fluid (viscosity, thermal conductivity, and heat capacity) are defined as temperature dependent polynomials [10].

**Mathematical Modeling**

This work was performed by using the Navier-Stokes and Fourier equations and by an analytical solution:

**Analytical solution**

In the slip flow regime, the temperature distribution between two cylinders can be obtained from the energy conservation [11]

$$mnc_v \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(rk)\frac{\partial T}{\partial r} \quad (1)$$

Where $c_v$ is the annular specific isochoric heat capacity, $r$ is the radial coordinate between the cylinders, $t$ is the time, $k$ is the gas thermal conductivity, and $n$ is the gas number density.

In the steady state case $\frac{\partial T}{\partial t} = 0$ [11], and using the following heat flux equation (by applying Fourier's law)

$$q_r = -k\frac{dT}{dr} \quad (2)$$

The energy conservation equation is obtained as follows

$$\frac{\partial (rq_r)}{\partial r} = 0 \qquad \text{Where } q_r(r) = \frac{15}{8\delta_0}\frac{C}{r} \quad (3)$$

We can consider, the gas temperature next to the wall is equal to the wall temperature, so the Equation (3) can be solved analytically.

The dimensionless temperature profile is given as

$$T(r) = \left\{\frac{1}{2}\left[\left(T_{gA}^{\omega+1} + T_{gB}^{\omega+1}\right) + C(\omega+1)\left(\ln\left(\frac{R_A}{r}\right) + \ln\left(\frac{R_B}{r}\right)\right)\right]\right\}^{\frac{1}{\omega+1}} \quad (4)$$

Where

$$C = \frac{(T_{w_A}^{\omega+1} - T_{w_B}^{\omega+1})/(\omega+1)}{\ln\frac{R_A}{R_B} + \frac{\xi_A}{\delta_0}\frac{T_{w_A}^{\omega+1/2}}{R_A} + \frac{\xi_2}{\delta_0}\frac{T_{w_B}^{\omega+1/2}}{R_A}} \quad (5)$$

The gas temperature at the point vicinity of the wall

$$T_{g_A}^{\omega+1} = T_{w_A}^{\omega+1}[1-\epsilon_A(\omega+1)]$$

$$T_{g_B}^{\omega+1} = T_{w_B}^{\omega+1}[1-\epsilon_B(\omega+1)] \quad (6)$$

Where

$$\epsilon_A = \frac{\xi_A}{\delta_0}\frac{C}{R_A}\frac{1}{T_{w_A}^{1/2}} \quad (7)$$

$$\epsilon_B = -\frac{\xi_B}{\delta_0}\frac{C}{R_B}\frac{1}{T_{w_B}^{1/2}} \quad (8)$$

Here $T_{w_A}$, $T_{w_B}$ are dimensionless temperatures of the walls. $\xi_1$, $\xi_2$ are the temperature jump coefficients of the walls, their value can be taken as 1.95 in the complete accommodation [12]. ω is the viscosity index, which is equal to 0.5 for the Hard Sphere model and 1 for the Maxwell model.

**Numerical simulation**

By using the CFD code Fluent (Fluent 6.3), the numerical simulation was carried out by solving the Navier-Stokes and Fourier equations. Rarefied gas flow simulation is directly attainable in Fluent using the "Low Pressure Boundary Slip" (LPBS) option in the "Viscous Model" panel; for the slip boundary conditions the Maxwell's first order model is used with an expression of the mean free path adjustable by means of the value of the Lennard-Jones length. The mean free path, λ, is calculated as follows [9].

$$\lambda = \frac{k_B T}{\sqrt{2}\pi {\sigma_d}^2 P} \quad (9)$$

Where $K_B$ is the Boltzmann constant, T the temperature, P the pressure and σ the Lennard-Jones characteristic length of the gas.

The frontier which defines the region of application of the Navier-Stokes equation is defined by the Knudsen number, which identifies the degree of rarefaction of a gas:

$$Kn = \frac{\lambda}{L_c} \quad (10)$$

Where $L_c$ is a characteristic device length

In our case $L_c = (D - d)$ which represents the radius (r) in our calculations.

Where D is the outer cylinder diameter and d is the filament diameter.

In the slip flow regime, the gas temperature at the surface differs from the wall temperature. The boundar conditions used by the Maxwell model in the Fluent code are as follows

The boundary condition of temperature jump

$$T_\omega - T_g = 2\left(\frac{2-\alpha_T}{\alpha_T}\right) Kn \frac{\partial T}{\partial n} \approx 2\left(\frac{2-\alpha_T}{\alpha_T}\right)\frac{\lambda}{\delta}\left(T_g - T_c\right) \quad (11)$$

Or equivalently

$$T_g = \frac{T_\omega + \beta T_c}{1+\beta} \quad (12)$$

Where

$$\beta = \frac{2(2-\alpha_T)}{\alpha_T \delta} \quad (13)$$

$\boldsymbol{\alpha_T}$ is the thermal accommodation coefficient of the gas mixture and is calculated as:

$$\alpha_T = \sum Y_i \alpha_{T,i} \quad (14)$$

The subscripts *g*, **w** and *c* indicate gas, wall and cell-center velocities. **δ** is the distance from the cell center of the wall.

The numerical simulations are performed using finite volume method for a nitrogen gas flow. A non-uniform circular grid of quadrilateral cells is constructed. Double precision calculations are done with a second order discretization scheme for a better accuracy. Convergence is considered to be reached when the normalized residuals have fallen below $10^{-5}$. The accommodation has been assumed totally diffuse [13], [14], [15]. Temperature, density and pressure fields are obtained.

## RESULTS AND DISCUSSION

A non-uniform grid, is constructed with 38000 nodal points on the basis of many tests. This computational grid is illustrated in Figure1.

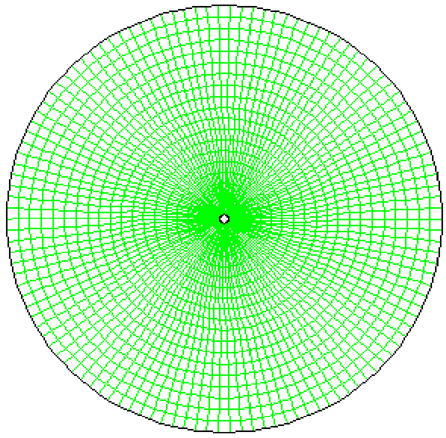

**Figure1. Geometry and mesh**

The accuracy of the results depends also on the type of mesh and finesse. We examined the mesh influence on the solution. To this effect, we computed numerically and analytically the temperature distribution in the vertical displacement along the radius.

Four numerical grids have been used to estimate the effect of the mesh type on the results. Grids structure is as follows: the first mesh is normal (a), the second one is progressive (b) where the density of mesh decreases from the filament to the external cylinder, the third and the fourth grids are consistent with a more (c) or less (d) mesh density near the filament.

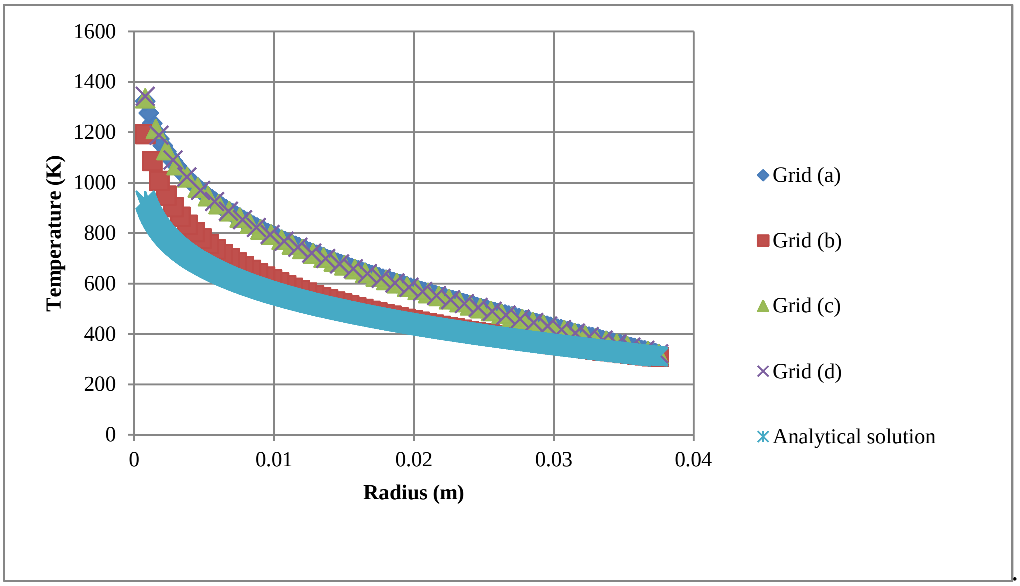


**Figure 2. Effect of mesh type**

Figure 2 shows the temperature profiles in the slip flow for different meshes a, b, c and d, where the temperature distribution in the vertical displacement along the radius is calculated, by the Fluent code and compared with the analytical solution (section 3.1). It is found that the results of the grid (b) which structured with a progressive refinement mesh, is more accurate than the grids a, c and d.

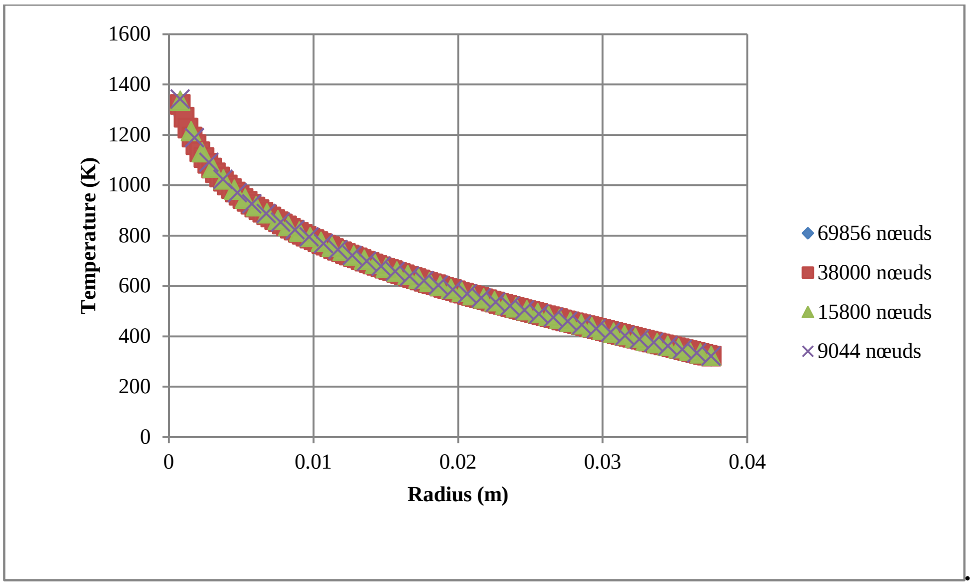


**Figure 3. Sensitivity to mesh density**

After testing the type of mesh, we also examined the influence of the number of nodes on the solution. We considered for this purpose four grids (Figure 3) ranging from 9044 to 69856 nodes. It is found that the four grids showed similar behavior, but from the number of 15800 nodes, it can be noted that the continuity of the points in the filament region in the temperature distribution is not ensured (the points become more spaced), which risks losing important information in this filament region and therefore the Knudsen layer. The grids with 69856 nodes and 38000 nodes give similar results and so the latter mesh is used throughout this study. The problem case described above is then submitted for computation and a converged solution is obtained after performing 700 iterations. Figure 4 shows the calculated temperature, and density contours.

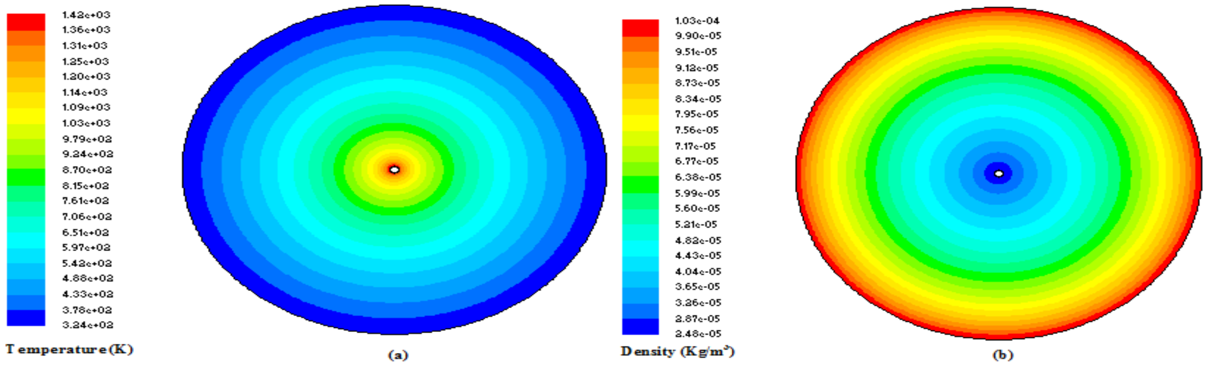


**Figure 4. Temperature (a) and Density (b) Contours**

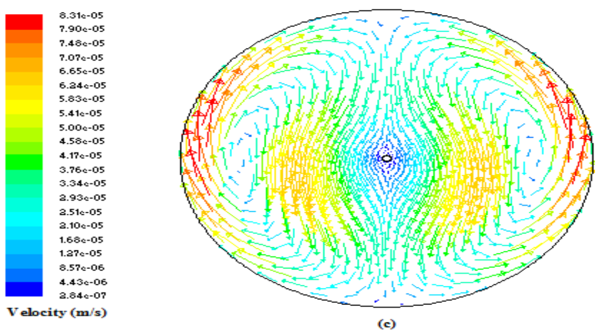


**Figure 5. Velocity vectors**

The contours and distributions of temperature, density and velocity in terms of radius (r) and two filament temperatures ($T_f$ =3000K and 1500K) are presented for Knudsen number (Kn) equal to 0.02.

Figure 4 and Figure 5 shows the contours of temperature and density and the velocity vectors of the wire temperature of 3000K, it can be seen that the temperature (Figure 4(a)) at its maximum in the surrounding area of the wire and gradually decrease to attain its minimum at the external cylinder. The distribution of the density (Figure 4(b)) is opposite of that of the temperature, the heating of the internal cylinder causes a decrease in the density in the proximity of the inner hot cylinder which generates a local variation in the density and consequently in the mass of gas from the internal to the external wall.

The velocity vectors (Figure 5) illustrate that even though the magnitude of the velocity is low within the enclosure; its maximum is attained at the external cylinder ($8.31.10^{-5}$); it shows that there is a macroscopic movement of the gas that was created, from the inner to the outer cylinder. The amplitude of the oscillation increases with the temperature wire (Figure 8).

By comparing the temperature profiles (Figure 6) for both wire temperatures ($T_f$ =3000K and 1500K), it is clearly seen that the temperature jump rises with the wire temperature, but the qualitative behavior is identical for both temperatures. Figure 7 shows the density behavior profiles, it can be seen that the density increases monotonically from the internal hot wall to the external cold wall. This is in good agreement with the temperature results.

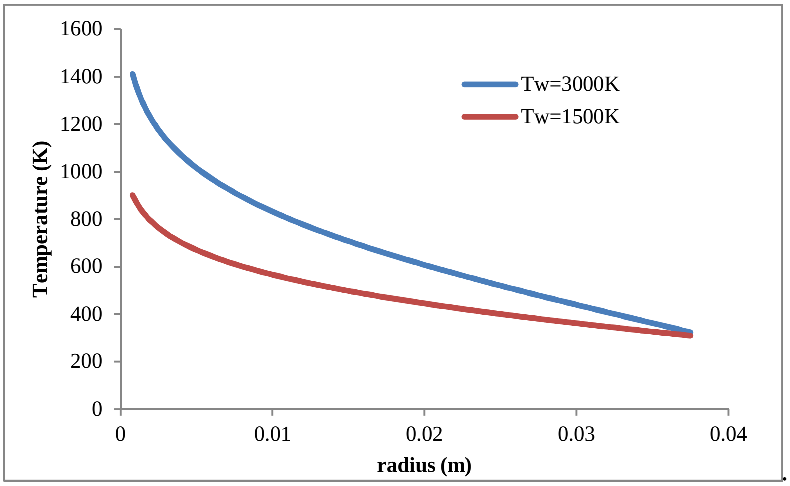


**Figure 6. Temperature distribution**

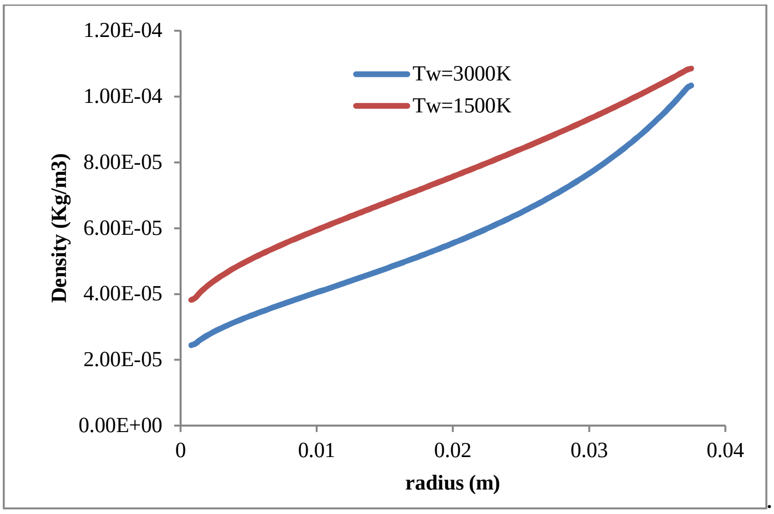


**Figure 7. Density distribution**

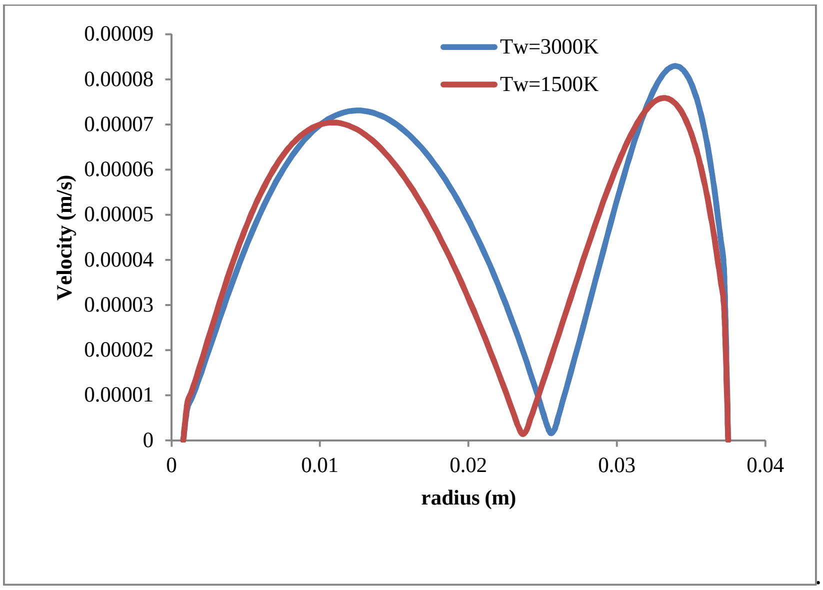


**Figure 8. Velocity distribution**

## CONCLUSION

The simulation of slip flow and heat transfer through a rarefied nitrogen gas, confined between concentric cylinders is presented on the basis of the Navier-Stokes-Fourier equations by using the CFD code Fluent. The calculations have been carried out in the slip regime for Knudsen number Kn=0.02 and two high temperatures of the filament.

The behavior of the temperature, density, and velocity are investigated. It is found that the temperature jump rises with the wire temperature, but the qualitative behavior is identical for both temperatures.

The influence of the mesh type on the solution has been examined by comparing the distributions of temperature calculated by using Navier-Stokes and Fourier equations and the analytical solution. It is found that the mesh structured with a progressive refinement is more accurate.

The LPBS method proposed in Fluent for implementing slip conditions is used in the case of concentric cylinders.

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