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Research Article

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Shooting competition hit rate influence factor research based on mechanics and laminar flow model

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ABSTRACT

For shooting, it generally divides into fixed target and clay-target two cases. Firstly this paper researches on fixed target, make force analysis of bullet, it can know that it takes constant motions in horizontal direction and constant decelerated motions in vertical direction. Make calculation by vertical direction displacement, displacement equation and horizontal displacement equation, get included angle between bullet speed and ground as well as relational expression of distance between front sight and fixed target. In fixed target shooting, shooters can decide the included angle size that formed by speed and ground according to target distance. The next for clay-target competition, this paper uses laminar flow model to make force analysis of clay-target, it can know that the clay-target takes constant deceleration motions both in horizontal and vertical directions. Clay-target bullet force status, speed changes following by time equation, as well as ballistic trajectory are the same as those in fixed target researches.

Key words: Oblique projectile motion, laminar flow mode, clay-target competition, movements' trajectory

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INTRODUCTION

Through shooting competition researching, establish projectile model combining with kinematics knowledge. For shooting competition, it can be generally supposed that bullet and clay-target make projectile movements. Projectile movement is object casts in the air with a certain initial speed and its movements only is made under gravity effects, whose initial speed is not 0, movement trajectory is parabola. Projectile movements is the important composition of mechanics.

As earlier as 2000 more years ago, human has already made research on projectile movements, the main representative personage are Aristotle, Galileo and others ,and gained certain research achievements. For example, Aristotle explained projectile movement as that reasons for projectile still moved after being out of hands is due to hands drove surrounding air to make movements when it made projectile movement, projectile moved forward by air driving after out of hand. His researches were mainly research on projectile movement causes aspect which was mostly speculation and deduction without real data verifying [1, 2]. With physical further development and science and technology further improvement, human has achieved new theory and research achievements on projectile movements [3, 4]. From which, Galileo put forward the concept of accelerated speed, the presentation of accelerated speed has overthrown Aristotle projectile natural origins , and based on that he came up with inertial principal under natural state, which was also the prototype of the three laws of Newton. Galileo first verified object movement trajectory was parabola, and made reasonable explanation of projectile movements' laws [5]. He had ever designed oblique plane experiment; put forward the famous law of falling body through oblique plane experimental theory and data analysis. In initial researching, Galileo thought that speed was in direct proportion to distance, but through subsequent researching and analyzing, he found that speed was in direct proportion to time. Based on which he put forward constant motion and constant acceleration motion theories. With these theory, it is nicely illustrated projectile motions every phenomenon [6].

Applications of projectile movement are very universal; the small field is like students' basketball shooting, and the big field as national satellite and missile launching. This paper utilizes projectile movement thoughts and theories, takes air resistance influence in shooting movements when analyzing and discussing and establishes projectile movement dynamical equation.

STANDARD SHOOTING MODEL ESTABLISHMENT

At first this paper provides standard shooting model as Figure 1 shows.

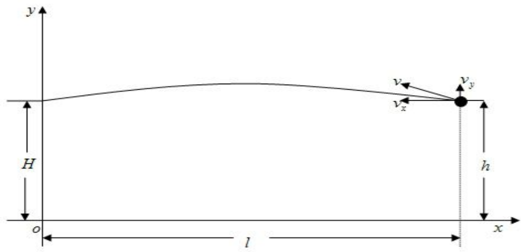

Figure 1: Standard shooting model

Carry out force analysis on bullet:

Bullet only affected by gravity, then its accelerated speed is $a = -g\, m \cdot s^{-2}$
According to above, it can get its ballistic trajectory, as Figure 1.
Dissolve initial speed v along axis x, y, so can get:

$$\begin{cases} v_x = v\cos\theta \\ v_y = v\sin\theta \end{cases} \tag{1}$$

Carry out force analysis on ballet; because resistance influence is neglected in hypothesis, bullet only is affected by gravity. In horizontal direction, it is constant movement at speed of v_x, in vertical direction it is $-g$ constant variable movement at accelerated speed of a.Assume that shot front sight and bull's eye have equal height, bullet vertical direction displacement s is:

$$s = H - h \tag{2}$$

According to above analysis can get. Speed equation is:

$$\begin{cases} v_x' = v_x \\ v_y' = v_y + at \end{cases} \tag{3}$$

Displacement equation is:

$$\begin{cases} l = v_x' t \\ s = v_y' t + \dfrac{1}{2} a t^2 \end{cases} \tag{4}$$

Fixed target is far away from shooter distance, only considering bullet crossing parabola symmetry axis to fixed target corresponding time here. Make discussion on vertical direction displacement s according to situation, determine time t :

$$\begin{cases} t = -2\dfrac{v_y}{a} & H = h \\ t = \dfrac{-v_y + \sqrt{v_y^2 + 2a(H-h)}}{a} & H \neq h \end{cases} \tag{5}$$

Input formula (5) into formula (4), can get:

$$\begin{cases} l = -2\dfrac{v_y}{a} \times v_x^{'} & H = h \\ l = \dfrac{-v_y + \sqrt{v_y^2 + 2a(H-h)}}{a} & H \neq h \end{cases} \tag{6}$$

Simultaneous (1) (2) (3) three formulas can get relationships between θ and l :

$$\begin{cases} l = \dfrac{v^2}{g}\sin(2\theta) & H = h \\ l = v\cos\theta\dfrac{[\sqrt{v^2\sin^2\theta - 2g(H-h)} + v\sin\theta]}{g} & H \neq h \end{cases} \tag{7}$$

In formula (7), front sight height h, bullet initial speed v, fixed target height H, can know from measurement that all these are known. Only given front sight to fixed target distance l then can get correct shooting angle θ.

Clay-target shooting model as Figure 3 shows.

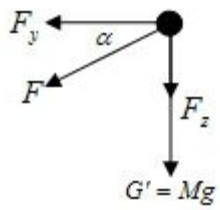

Figure 2: Force analysis

As Figure 2, make force analysis on clay-target.

It bears gravity $G^{'}$ and resistance F .For resistance F, it adopts laminar flow mode, which is remain linear air resistance, therefore:

$$F = Av^{'}r \tag{8}$$

$A = 6\pi\eta$ η is air viscosity coefficient (give $\eta = 1.8097 \times 10^{-5} Pa \cdot s$), $v^{'}$ is clay-target speed, r is clay-target radius. The direction of F is always in the line with the direction of $v^{'}$ .Dissolve resistance F along axis of x, z ,while its gravity is vertical and downwards, so can get:

$$\begin{cases} Ma_z = -\left(G^{'} + F\sin\alpha\right) \\ Ma_y = -F\cos\alpha \end{cases} \tag{9}$$

Therefore, its accelerated speed is:

$$\begin{cases} a_z = -\dfrac{G^{'} + F \sin \alpha}{M} \\ a_y = -\dfrac{F \cos \alpha}{M} \end{cases} \tag{10}$$

It can know that its movement trajectory is a parabola, as Figure 3 yoz plane parabola. Clay-target throwing time is earlier than bullet shooting time, in first question assume bullet shooting time as t, then clay-target throwing time is $t + \Delta t$, Δt is the difference between clay-target throwing time and bullet shooting time.

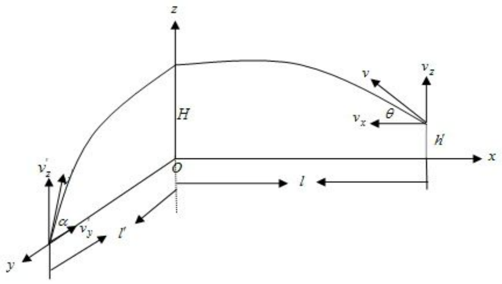

Figure 3: Clay-target shooting model

Dissolve $v^{'}$:

$$\begin{cases} v_y^{'} = v^{'} \cos\alpha \\ v_z^{'} = v^{'} \sin\alpha \end{cases} \tag{11}$$

Its speed equation is:

$$\begin{cases} v_y^{''} = v_y^{'} + a_y (t + \Delta t) \\ v_z^{''} = v_z^{'} + a_z (t + \Delta t) \end{cases} \tag{12}$$

Its displacement equation is:

$$\begin{cases} l^{'} = v_y^{'} (t + \Delta t) + \dfrac{1}{2} a_y (t + \Delta t)^2 \\ H = v_z^{'} (t + \Delta t) + \dfrac{1}{2} a_z (t + \Delta t)^2 \end{cases} \tag{13}$$

In formula (13), H is equal to formula (2) H, simultaneous formula (4) can get:

$$\left(\frac{a_z}{2} - \frac{a}{2} \right) t^2 + (a_z \Delta t + v_z^{'} - v_y) t + \frac{a_z}{2} (\Delta t)^2 - h = 0 \tag{14}$$

Solve:

$$\begin{cases} t_1 = \dfrac{-(a_z \Delta t + v_z^{'} - v_y) - \sqrt{(a_z \Delta t + v_z^{'} - v_y)^2 - 2(a_z - a) \cdot \left(\dfrac{a_z}{2} \cdot (\Delta t)^2 - h \right)}}{a_z - a} \\ t_2 = \dfrac{-(a_z \Delta t + v_z^{'} - v_y) + \sqrt{(a_z \Delta t + v_z^{'} - v_y)^2 - 2(a_z - a) \cdot \left(\dfrac{a_z}{2} \cdot (\Delta t)^2 - h \right)}}{a_z - a} \end{cases} \tag{15}$$

Because bullet and clay-target movement trajectories are parabolas, both have two hitting height H (except for top point), therefore it will have 4 cases, as Figure 4, Figure 5, Figure 6 and Figure 7 show.

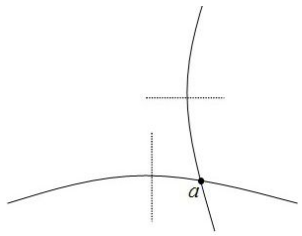

Figure 4: Symmetric line front half (on the right of symmetric line)

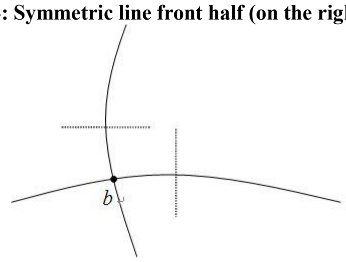

Figure 5: Symmetric line front half (on the left of symmetric line)

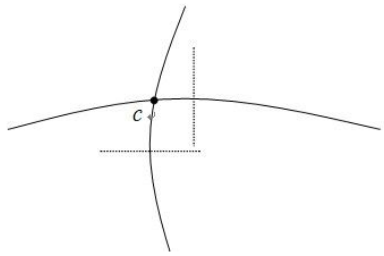

Figure 6: Symmetric line back half (on the right of symmetric line)

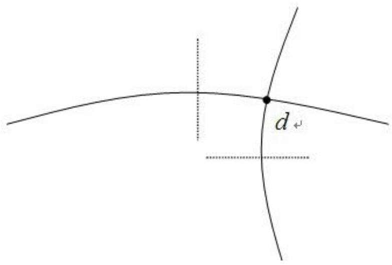

Figure 7: Symmetric line back half (on the left of symmetric line)

Can get each intersection point positions in two parabolas, as Table 1.

Table 1: Each intersection point positions in two parabolas

Item	a	b	c	d
Bullet parabola trajectory	On the right of symmetric line	On the left of symmetric line	On the right of symmetric line	On the left of symmetric line
Clay-target parabola trajectory	symmetric line front half	symmetric line front half	symmetric line back half	symmetric line back half

Through analysis, it is know when bullet initial speed v is larger than clay-target initial speed v', it will appear

a 、 c such two cases；when bullet initial speed v is smaller than clay-target initial speed v', it will appear b 、 d such two cases. The first case is conforming to actual, therefore can only discuss the first case, as following Figure 8, then $v > v'$.

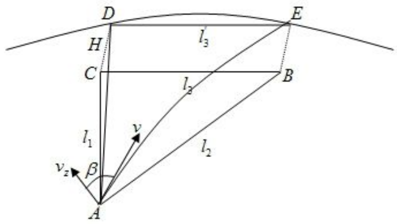

Figure 8: Analysis when bullet initial speed v larger than clay-target initial speed v'

A is shooter position, D, E are respectively shooting clay-target position at the time t_1, t_2, two points projection in ground are C, D, therefore $l_3' = l_3$. According to formula (12) can get:

$$l_3 = l_3' = l_{t_2}' - l_{t_1}' \tag{16}$$

ΔABC is right triangle, according to Pythagorean Theorem can get l_2:

$$l_2 = \sqrt{l_1^2 + l_3^2} \tag{17}$$

In figure, v_z is vertical to ground, the included angle with v is β, so v horizontal component is $v_{x1} = v \sin\beta$. Therefore：

$$l_2 = v_{x1} \cdot t \tag{18}$$

Respectively input (15) t_1, t_2 into formula (4) and (18), can get:

$$\begin{cases} l_1 = v_x \cdot \dfrac{-(a_z \Delta t + v_z' - v_y) - \sqrt{(a_z \Delta t + v_z' - v_y)^2 - 2(a_z - a) \cdot \left(\dfrac{a_z}{2} \cdot (\Delta t)^2 - h \right)}}{a_z - a} \\ l_2 = v_{x1} \cdot \dfrac{-(a_z \Delta t + v_z' - v_y) + \sqrt{(a_z \Delta t + v_z' - v_y)^2 - 2(a_z - a) \cdot \left(\dfrac{a_z}{2} \cdot (\Delta t)^2 - h \right)}}{a_z - a} \end{cases} \tag{19}$$

Simultaneous formula $(1), (2), (3), (4), (8), (10), (11), (12)$ can get:

$$\begin{cases} l_1 = \dfrac{\left(v\sin\theta - v^{'}\sin\alpha + \left(g + \dfrac{Av^{'}r\sin\alpha}{M}\right)\cdot\Delta t\right)}{\dfrac{Av^{'}r\sin\alpha}{M}} \times v\cos\theta - \\ \dfrac{\sqrt{\left(v\sin\theta - v^{'}\sin\alpha + \left(g + \dfrac{Av^{'}r\sin\alpha}{M}\right)\cdot\Delta t\right)^2 - \left(\dfrac{Av^{'}r\sin\alpha}{M}\right)\cdot\left(\left(g + \dfrac{Av^{'}r\sin\alpha}{M}\right)\cdot(\Delta t)^2 - 2h\right)}}{\dfrac{Av^{'}r\sin\alpha}{M\times v\cos\theta}} \end{cases} \tag{20}$$

$$\begin{cases} l_2 = \dfrac{\left(v\sin\theta - v^{'}\sin\alpha + \left(g + \dfrac{Av^{'}r\sin\alpha}{M}\right)\cdot\Delta t\right)}{\dfrac{Av^{'}r\sin\alpha}{M}} \times v\sin\beta + \\ \dfrac{\sqrt{\left(v\sin\theta - v^{'}\sin\alpha + \left(g + \dfrac{Av^{'}r\sin\alpha}{M}\right)\cdot\Delta t\right)^2 - \left(\dfrac{Av^{'}r\sin\alpha}{M}\right)\cdot\left(\left(g + \dfrac{Av^{'}r\sin\alpha}{M}\right)\cdot(\Delta t)^2 - 2h\right)}}{\dfrac{Av^{'}r\sin\alpha}{M\times v\sin\beta}} \end{cases} \tag{21}$$

In formula(20) (21), shooter height h', bullet initial speed v, clay-target initial speed $v^{'}$, clay-target quality M, clay-target throwing time that earlier than bullet Δt, all can get by measurement, all can be regarded as known items. According to clay-target initial speed $v^{'}$ and vertical direction included angle α （In normal situation, it has high throwing and low throwing, therefore α will have two values）, can get bullet shoot to D, E initial speed v and their included angles with ground θ, β.

CONCLUSION

This paper mainly researched shooting movement fixed target and clay-target two cases. To fixed target, through bullet force analysis only bearing gravity G can know that in horizontal directions it took constant movement, while in vertical direction it take constant decelerated movement. By vertical directions displacement s and displacement equation can determine time t.Input time t into horizontal displacement equation can get bullet speed and ground formed included angle θ as well as front sight and fixed target distance l expression. In fixed target shooting, shooter can decide the size of θ according to distance from target l.For clay-target competition, through force analysis of clay-target can know clay-target bearing air resistance F （air resistance F adopts laminar flow mode that keeps linear）and gravity $G^{'}$, it took constant decelerated movement both in horizontal and vertical directions. Clay-target bullet force status, speed changes following by time equation, as well as ballistic trajectory are the same as those in fixed target researches. Both bullet and clay-target's movement trajectories are parabolas, assume they came across in the point that height is H ; According to equal vertical displacement conditions, it can get time t_1, t_2 when bullet came across clay-target. According to bullet horizontal direction displacement equation, it can get bullet speed and its included angle (θ, β) with ground and clay-target throwing speed included angle α with ground、the expression that front sight distance from clay-target movement trajectory plane $l^{'}$. Shooter can shoot the clay-target by adjusting α and $l^{'}$ into θ or β at the time t_1 or t_2.

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