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Risk contagion and model of cascading bankruptcy: A case from Chinese chemical industrial network

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ABSTRACT

The paper proposed a theoretical model of cascading bankruptcy process in the context of financial contagion that is tightly linked to systematic risk of industrial complex network. And the paper further offers a theoretical methodology to measure systematic risk of industrial complex network. To test its validity, an experimental analysis is conducted using network dataset from Zhejiang chemical industrial network. And this simulation results is fit to realistic performances of Zhejiang chemical industrial network. Therefore, the paper proposes a useful method to evaluate cascading effect and further measure systematic risk.

Key words: Risk contagion; cascading bankruptcy; systematic risk; industrial complex network

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INTRODUCTION

Risk contagion is tightly connected with systematic risk in complex business network. Industrial complex network is a complex network of companies that are interconnected with financial bonds and industrial bonds. Industrial clusters that are popular in many developing countries are one kind of typical industrial complex network. Financial contagion is important feature in the interdependent industrial complex network, which would lead to systematic risk. Idiosyncratic risk of bankruptcy of an important company would be likely to trigger cascading bankruptcy that finally leads to systematic risk. Cascading bankruptcy in such industrial complex network occurs more frequently after global financial crisis, and receives concerns of policy-makers. Literatures argue financial contagion is a critical step leading to system risk, and provides many in-depth insights about financial contagion [2][3][4][5][6][7]. Cascading bankruptcy is intensive financial contagion, and has adversely effects on stability and development of industrial complex network. And more efforts are needed to shed some light on cascading bankruptcy. In this paper, the author tries to propose a model of risk contagion and cascading bankruptcy. And the paper also proposes a theoretical method to measure systematic risk of industrial complex network.

The paper is organized as follows. In the second section, the paper modeled network background of financial contagion in the context of complex industrial network. And the paper proposed a theatrical model of cascading bankruptcy in the context of complex industrial network, and further discusses how to measuring systematic risk of complex industrial network in the context of industrial network. In the third section, the paper illustrates an experimental analysis in the background of a Chinese Chemical industrial network. In this section, contagion effects and systematic risk are analyzed by methodology proposed in the paper and discussed in details, along with this methodology proposed in the paper is validated.

1. RISK CONTAGION AND MODEL OF CASCADING BANKRUPTCY

2.1 Network Background of Financial Contagion

Before going into the details of the model, the paper describe industrial complex network and relevant definitions.

Denotes a industrial complex network as a graph $G = (N, R)$, in which N companies are interconnected with undirected financial bonds R. In mathematical form, the industrial complex network is also described as an adjacency matrix $M = [M_{ij}]$. where $M_{ij} = 0$ represents there is not financial flow from firm i to firm j. And $M_{ij} = 1$ means there is some form of financial flow from firm i to firm j, for example firm i lent some money to firm j. and the paper introduces a weight matrix $H = [H_{ij}]$ to describe the exposure of i to j relative to its portfolio of all exposures. The matrix H is defined as a row-stochastic matrix, and subjects to $\sum_j H_{ij} = 1$. And denote a financial robustness indicator β_i for each node i, to measure its distance from actual default or to indicate its state of liability reputation. If $\beta_i = 0$, the node i has been on the situation of default or bankruptcy. And if $\beta_i = 1$, the node i has no any risk of default or bankruptcy. To describe the two mechanisms of interdependence and financial accelerator that is stressed by financial contagion literatures [4][7][8], the paper characterizes financial contagion process with stochastic differential equation (SDE) in following form:

$$d\beta_i = L(\beta(t), \beta(t'))dt + \sigma d\varepsilon_i \tag{1}$$

In which, $L(\beta(t), \beta(t'))$ is a drift term that depends on the past realizations of β, σ is the variance of idiosyncratic shocks that hit firm i, and $d\varepsilon_i$ denotes the Wiener stochastic process. And more specific, the paper assumes:

$$L(\beta(t), \beta(t - dt)) = \begin{cases} -\theta & \text{when } \beta_i(t) - \beta_i(t - dt) < -\epsilon \frac{\sigma}{\sqrt{D}} dt \\ 0 & \text{when } \beta_i(t) - \beta_i(t - dt) \geq -\epsilon \frac{\sigma}{\sqrt{D}} dt \end{cases} \tag{2}$$

In which, ϵ represents sensitivity of her neighbors to temporary changes of financial robustness of node i, $\sigma/\sqrt{D}$ is the standard deviation of idiosyncratic shocks. Then, this implies that her neighbors react only when risk shock to node i is larger than ϵ times the standard deviation. In equation (2), the amplitude of adjustment of financial robustness is described as θ. In this way, the model proposed in the paper is able to accommodate financial interdependence and financial accelerator by adding draft term in the SDE equation.

2.1 Modeling Cascading Bankruptcy in Context of Industrial Network

Cascading bankruptcy is typical form of intensive financial contagion, and is easily to evolving into systematic risk. And such contagion process is characterized with its rapid propagation and destructive effects. To clarify complex process of risk contagion and crisis evolution, the paper combines inter-organization risk propagation and random walk of individual financial robustness in the same model. And the paper proposes to separate higher-intensity cascading bankruptcy process from lower-intensity risk contagion process. For simplicity, the paper assumes financial robustness will be adjusted with a very short time delay ($t' = t - dt$) in the process of cascading bankruptcy. Suppose one or several companies in industrial complex network go into bankruptcy, all nodes will experience a serial of adjustment of financial robustness as a result of risk propagation in time delay $[t, t + dt]$. And the paper denotes the time delay as a discrete process $\tau = 1, \cdots, n_\tau$. In each discrete step, financial robustness of all nodes will be adjusted according to following rules:

$$\beta_i(\tau) = \beta_i(t) - \frac{a}{D} \sum_j H_{ij} \omega_j(\tau) \tag{3}$$

In which, $\omega_j(\tau)$ is a discriminating function to distinguish which node j is in verge of bankruptcy. And its specific form is

$$\omega_j(\tau) = \begin{cases} 1 & \text{when } \beta_j(\tau') < 0, \exists \tau' \leq \tau \\ 0 & \text{when } \beta_j(\tau') \geq 0, \exists \tau' \leq \tau \end{cases} \tag{4}$$

When there is no new company is involved in bankruptcy crisis in the discrete dynamic process, the cascading bankruptcy is suspended and financial robustness of all nodes are adjusted as $\beta_i(t) = \beta_i(\tau = n_\tau)$. Then the dynamic process is shift into lower-intensity contagion process that is characterized with risk propagation and risk accelerator. In this process, infected companies will experience a serial of financial distress as a result of surviving from cascading bankruptcy. Stochastic dynamics are introduced into this propagation and recovering phase. The industrial complex network is likely to evolve into new financial status. If a company is hit into bankruptcy in this process, a new round of cascading bankruptcy process is triggered.

The adjustment mechanism of financial robustness is essentially calculating deduction in all node's financial robustness that caused by bankrupt companies. The parameter a represents damage caused by bankruptcy of firm j. For example, $a = 1$ means all assets are deducted after its bankruptcy. For the sake of financial interdependence, all damage will be shared among its financial partners (its network neighbors). And this leads to deduction of financial robustness of all these neighbor nodes. Loop deductions will be calculated in not more than N-1 steps. Considering small world characteristics of real industrial complex network, the loop deduction is roughly about $\log(N)$ step. The model stress positive self-feedback to one node's financial robustness from its own financial status of past periods in the general contagion process. Financial interdependence and interactions among different nodes are talk about in the cascading bankruptcy process. The destruction of cascading bankruptcy process depends, in large part, on financial networking in the industrial complex network.

2.3Further Discussion of Measuring of Systematic Risk of Industrial Network

After clarifying cascading bankruptcy process and general contagion process, the paper proposes a method to measure systematic risk. Define P^{μ} is probability of cascading bankruptcy crisis that involves at least a fixed fraction, $\mu \in [0,\ 1]$, of the industrial complex network. And the probability is suitable index to measure systematic risk of industrial complex network.

Suppose the fraction, S, is real part of bankrupt companies in the whole contagion process (including general contagion process and cascading bankruptcy). Assume the industrial complex network is a regular graph with degree D. And assume the initial distribution of financial robustness is subject to Gaussian distribution with a mean μ and a variance σ_{β}^{2}, and denote its cumulative probability distribution as $\emptyset(\mu, \sigma_{\beta})$. Denote S_0 the fraction of companies whose financial robustness is not more than zero at initial of the process. Equitation (8) can be expressed as following more convenient expression:

$$\beta_i(\tau + 1) = \beta_i(0) - \frac{a}{D_i}\sum_{j=1}^{D_i} \omega_j^s(\tau) \tag{5}$$

Where $\omega_j^s(\tau) = 1$, iff node j go to bankruptcy at any time in $[0, \tau]$. The cumulative fraction of nodes that have gone into bankruptcy in $[0, \tau]$ is expressed as following cumulative probability distribution:

$$S(\tau + 1) = \mathrm{Prob}\{\beta_i(\tau + 1) < \theta\} = \mathrm{Prob}\left\{\sum_{j=1}^{D_i} \omega_j^s(\tau) > \beta_i(0)\right\} \tag{6}$$

For a regular graph with degree D, whether a given company i goes bankruptcy by the time step depends on the number D_f of its financial partners who have already failed. There are D possible events at time step $\tau + 1$. These possible events are expressed in equitation (7).

$$D_f = \sum_{j=1}^{D_i} \omega_j^s(\tau) = 1,2,3 \cdots, D \tag{7}$$

In each event, the bankruptcy probability of a given company is depends on its initial financial status. Assume all bankruptcies are uncorrelated probability events across agents. The probability of each possible event is easily expressed in binomial distribution.

$$\mathrm{Prob}\{D_f \text{ bankruptcies among D partners}\} = \binom{D}{D_f} p^{D_f}(1 - p)^{D - D_f} \tag{8}$$

In which, p is the bankruptcy probability of a given company by the time step$\tau + 1$. If industrial complex network is large enough, the probability is approaching to $S(\tau)$. And the model assumes these bankrupt companies are not able to restore in observed time. So, $S(\tau)$ is a non-decreasing function. Therefore:

$$S(\tau+1) = \max\left\{S_0,\ \sum_{D_f=1}^{D}\binom{D}{D_f} p^{D_f}(1-p)^{D-D_f} * \text{Prob}\left\{\beta_i\left(\tau 1 \le \frac{aD_f}{D}\right)\right\}\right\} \tag{9}$$

Where S_0 is the initial fraction of bankrupt companies. Systematic risk is able to estimated in this recursive equation of $S(\tau+1) = F(S(\tau))$, which can be accomplished by querying the fixed points in $n = F(n)$. The model has assumed that β_i follows Gaussian distribution with with a mean μ and a variance σ_β^2. Therefore,

$$S(\tau+1) = \max\left\{S_0,\ \sum_{D_f=1}^{D}\binom{D}{D_f} p^{D_f}(1-p)^{D-D_f} * \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\frac{aD_f}{D}} \exp\left(-\frac{(u-\mu)^2}{2\sigma^2}\right)\right\} \tag{10}$$

In the model proposed in the paper, $F(S(\tau))$ is a non-decreasing function, there is $F(S(\tau)) \ge S_0$ if $\mu < \infty \text{ and } \sigma > 0$. The equation above can be solved numerically for any choice of the parameters. In following experiment analysis, the paper provides a simulation for estimating systematic risk.

2. EXPERIMENTAL ANALYSIS OF AN CHINESE CHEMICAL INDUSTRIAL NETWORK

3.1 Background, Data and Methodology

To examine the validity of theoretical model of cascading bankruptcy, the paper illustrates an experimental analysis. A MATLAB simulation program, based on the theoretical model in section 2, is formulated and applied in this experimental analysis. And the MATLAB program is run with the help of software package MATLAB 16.0. All data used in the simulation work is come from an investigation of a Chinese chemical industrial network, Zhejiang chemical industrial cluster. Zhejiang province is important industrial area of Chemical & Pharmaceutical industry, with total 72.7 billion RMB output. In Zhejiang Province, the industry is mainly layout in several major cities, such as Hangzhou city, Taizhou city, Shaoxing city and Jinhua city. The characteristics of Zhejiang Chemical &Pharmaceutical industrial network is illustrated in table 1.

TABLE 1 Network characteristics of Zhejiang Chemical Industrial Network

Network Characteristics	Value
Network centralization	6.417%
Network scale	229 (nodes)
Financial Bonds	288 (ties)
Average Degree	2.524
Network Density	0.012
Clustering Coefficient	0.192

And fig.1 offers a visualization of the industrial complex network. All 229 nodes represent 229 chemical companies in Zhejiang province, which is interconnected with financial bonds. In here, financial bonds include share-holding relationship, debtor-creditor relationship and mutually credit guarantee. Financial robustness is evaluated based on their asset data and other information that collected in the investigation.

3.2 Results

In the experimental analysis, cascading bankruptcy process is simulated in the context of financial contagion, and the simulation conditions approach realistic environment. The effect of cascading bankruptcy trigged by each node of Zhejiang chemical industrial network is calculated. The companies that are infected in the contagious process and finally go to bankruptcy in the process are counted as cascading effect in the experimental analysis. And the experimental analysis distinguish directly effects, which is calculated as the number of companies with directly financial bonds with the impacted node, and indirect cascading effect that is found in following cascading bankruptcy process. Figure 2 illustrates simulation results of cascading bankruptcy under three different financial conditions. The financial conditions are controlled by a parameter, which is similar to general liability ratio.

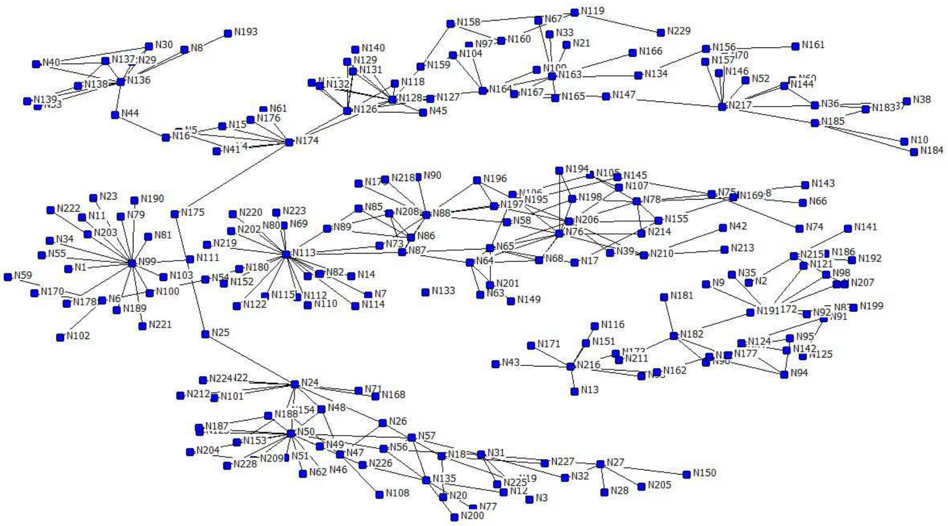
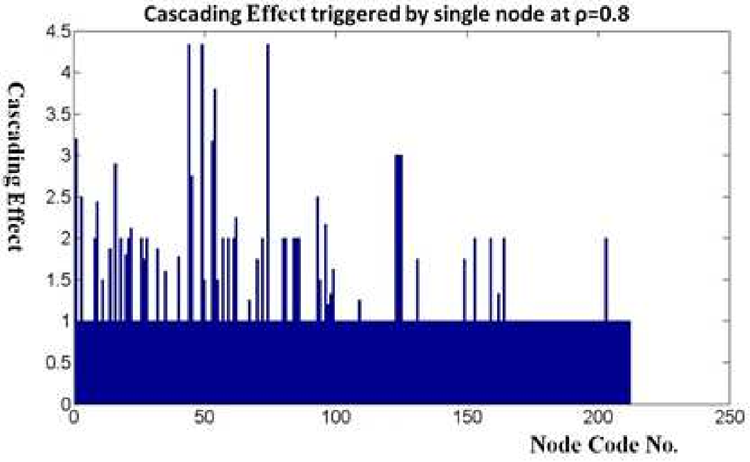
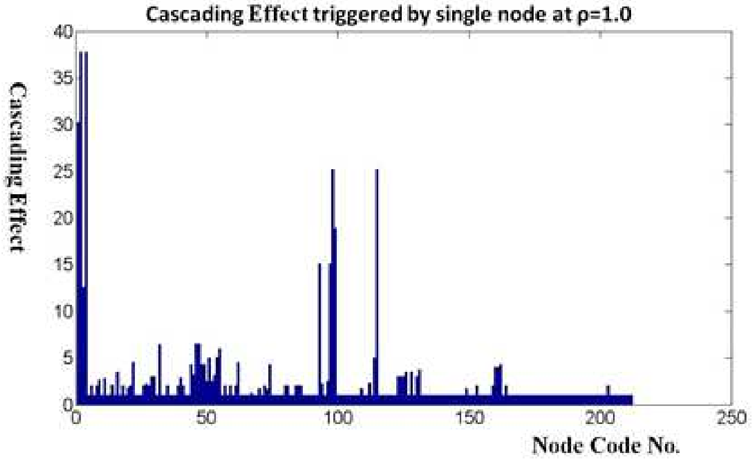


Fig.1 Visualization of Zhejiang Chemical Industrial Network





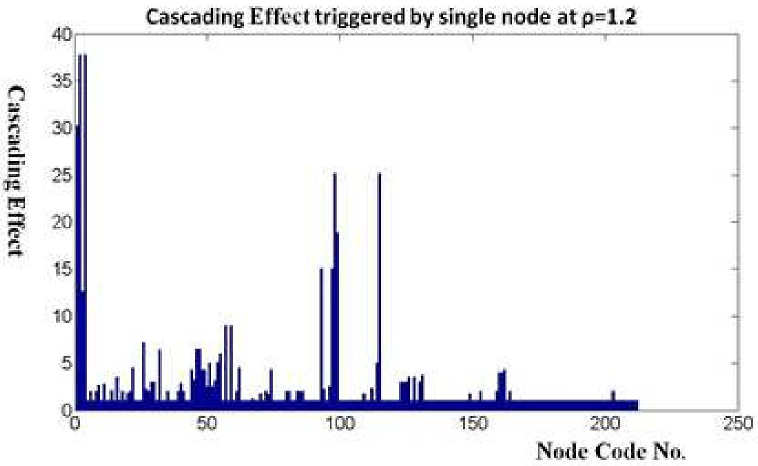


Fig.2 Cascading Effect in Simulation for Zhejiang Chemical Industrial Network

CONCLUSTION

The paper has constructed a theoretical model of cascading bankruptcy process in the context of financial contagion, and proposes a theoretical methodology to measure systematic risk of industrial complex network. And to test validity of the theoretical model that proposed in this paper, an experimental analysis is conducted using network dataset from Zhejiang chemical industrial network. And simulation results in the experimental analysis reflects "robust yet fragile " characteristics of industrial complex network in the context of cascading bankruptcy process. And this simulation results is fit to realistic performances of Zhejiang chemical industrial network [9][10][11]. Therefore, the paper proposes a useful method to evaluate cascading effect and further measure systematic risk.

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