



Research Article

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Research on the production function to investment behaviour

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ABSTRACT

The purpose of the business existence is to make profit maximization. Production function is relational expression that a certain kind of factor inputs can bring a maximum output under determine the conditions and the technical level. In this paper, we get the enterprise expansion path by deducing the production function. And find investment behavior model based on expansion path. By building a model and regress analysis, it is exact to reproduce and interpret appropriate investment behavior of Chinese macro economy the model comprises labor income, corporate earnings and government spending.

Keywords: Investment behavior; Dynamic optimization; Corporate earnings; Government spending index

INTRODUCTION

Since the 1930s, it was a period of rapid development of Western investment theory under Keynesian ideology, and reached its peak in the 1960s. After the Keynesian revolution, investment theory began as a relatively complete system in western economics. During this period, Post Keynesian, neo-Keynesians and neoclassical synthesis have made outstanding contributions to the development of Keynesian investment theory. From the research point of view, the theory of this period continued to invest in both directions along the "macro investment" and "micro-investment" development, and changed research tendencies of real macro investment, began to pay attention micro-macro investment theory research, developed a variety of dynamic investment models, technical economic theory and decision-making problems of indirect investment. For example, Jorgenson out of investment research framework of Keynesian theory, developed the neoclassical theory of investment. Jorgenson (1963)[1] overcome defects that investment theory analysis only focused on macro issues and thought people should study subject of micro-economic investment behavior - Enterprise to determine the level of investment by maximizing the present value of the production function. The basic principles of Jorgenson theory are similar to Q theory. Q theory was first proposed by Tobin (1969) [2], also called Tobin-Q. And it became the 20th century 70s to 80s mainstream investment theory after Abel (1979) [3], Yoshikawa (1980) [4] and Hayashi (1982) [5] developed it. Q theory is clearly more general than the neoclassical theory of investment. In fact, the neo-classical theory of investment is just a special case of q. It is noteworthy that, q theory is about assessing the valuation of the stock market and financial expectations of future earnings linked to the internal rate of return to compensate for expected future revenue streams arrangements. But in essence, Jorgenson and Tobin NPV principles still belong to Keynes's investment theory, in other words, the unit value of capital is exactly equal to its cost.

Production function is relational expression that a certain kind of factor inputs can bring a maximum output by the expression under determine the conditions and the technical level (Samuelson and Nordhaus,2009)[6]. Production functions mainly include CD production function, CES production function, VES production function beyond the production function and so on. Here, we have adopted an implicit form of the production function that is $y = f(k, l)$. And there are only two factors of production of physical capital k and labor l, companies need to pay interest in order to get physical capital, wages in order to obtain labors.

THEORETICAL MODEL DERIVATION

The purpose of the business existence is to make profit maximization. Under conditions of profit maximization, the corporate objective function of long-term profitability (time domain τ , the cycle is represented by t):

$$\pi_t = p_t \cdot y_t(I_t, k_t) - w_t \cdot I_t - \delta \cdot p_{k^-} \cdot k_t - r_t \cdot p_{kt} \cdot i_t \quad (1)$$

In the above formula, π is profit, p is the price of output, y for the production function, l and k , respectively, for the labor and physical capital, i is investment, w is the nominal wage, δ capital depreciation rate, r is the interest of financing, p_{k^-} and p_{kt} , respectively, for a capital and current prices.

And relationship between capital stock k and investment i :

$$\dot{k}_t = i_t - \delta \cdot k_t, \quad \dot{k} \geq 0 \quad (2)$$

Where k_t is the state variable and i_t as control variables.

If the makers have a different preference on earnings for each period, we are required to consider discounted present $e^{-\xi \cdot t}$ when calculating the discounted present value of earnings based on 0-cycle, ξ for the discount rate of time preference:

$$\frac{e^{-\xi \cdot t}}{e^{-\xi \cdot t}} = -\xi \leq 0, \quad \lim_{t \rightarrow 0} e^{-\xi \cdot t} = 1, \quad \lim_{t \rightarrow \infty} e^{-\xi \cdot t} = 0 \quad (3)$$

In the planning period T , the present value of the total profits of the enterprise:

$$\pi = \int_T [p_t \cdot y_t(I_t, k_t) - w_t \cdot I_t - \delta_t \cdot p_{k^-} \cdot k_t - r_t \cdot p_{kt} \cdot i_t] \cdot e^{-\xi \cdot t} dt \quad (4)$$

Hamilton function constituted by formula (4) and (2):

$$\pi = \int_T [p_t \cdot y_t(I_t, k_t) - w_t \cdot I_t - \delta_t \cdot p_{k^-} \cdot k_t - r_t \cdot p_{kt} \cdot i_t] \cdot e^{-\xi \cdot t} dt + \mu_t \cdot i_t \quad (5)$$

In the above formula, Kuhn-Tucker multiplier unit μ_t represents the opportunity cost of capital stock adjustment. according to the principle of maximizing:

$$\frac{\partial H}{\partial I_t} = (p_t \cdot y_{I_t} - w_t) \cdot e^{-\xi \cdot t} = 0$$

$$p_t \cdot y_{I_t} = w_t \quad (6)$$

$$\frac{\partial H}{\partial i_t} = -r_t \cdot p_{kt} \cdot e^{-\xi \cdot t} + \mu_t = 0$$

$$\mu_t = r_t \cdot p_{kt} \cdot e^{-\xi \cdot t} \quad (7)$$

And the Euler equation:

$$-\dot{\mu}_t = \frac{\partial H}{\partial k_t} = (p_t \cdot y_{k_t} - \delta \cdot p_{k^-}) \cdot e^{-\xi \cdot t} \quad (8)$$

According to (6), (7), (8), can be obtained:

$$\frac{y_{I_t}}{y_{k_t}} = \frac{w_t}{\delta \cdot p_{k^-} + r_t \cdot (\xi - g_{r_t} - g_{pk})} \quad (9)$$

$$\text{Therein, } g_{r_t} = \frac{\dot{r}_t}{r_t}, \quad g_{pk} = \frac{\dot{p}_{kt}}{p_{kt}}$$

Then (9) obtained is company expansion path of dynamic optimization. But the form of Implicit function used in the production function in the form of derivation process. And Because China's national economy runs in line with CD production function type (Feng Xiao, Zhu Yan Yuan, 2012)[8], the output elasticity of labor and capital are respectively α and β , then,(9) can be further expressed as:

$$\frac{y_{It}}{y_{kt}} = \frac{\alpha}{\beta} \cdot \frac{k}{I} = \frac{w_t}{\delta \cdot p_{k^-} + r \cdot p_{kt} \cdot (\xi - g_r - g_{pk})} \quad (10)$$

By finishing (10), we obtain:

$$w \cdot I = \frac{\alpha}{\beta} \cdot \delta \cdot p_{k^-} \cdot k + \frac{\alpha}{\beta} \cdot r \cdot p_{kt} \cdot (\xi - g_r - g_{pk}) \cdot k \quad (11)$$

The formula (11) is used to approximate the expansion path of the national economy, alternative labor costs $w \cdot I$ with labor income Y_L , capital price by the previous depreciation D calculated according to the last price p_{k^-} and capital stock of K calculated according to the present price P_{kt} , we have:

$$Y_L = \frac{\alpha}{\beta} \cdot D + \frac{\alpha}{\beta} \cdot (\xi - g_r - g_{pk}) \cdot r \cdot K \quad (12)$$

Producers' time preference ξ set as exogenous variables, g_r and g_{pk} reflect the producers' expectation to finance rate changes and investment goods price changes (\bar{g}_r and \bar{g}_{pk}), D is approximately equal to $\delta \cdot K$, then:

$$Y_L = \frac{\alpha}{\beta} \cdot \delta \cdot K + \frac{\alpha}{\beta} \cdot (\xi - \bar{g}_r - \bar{g}_{pk}) \cdot r \cdot K \quad (13)$$

Formula (13) as a Taylor series expansion, take the constant term and the first term approximation expression, you can create a differential equation that is as follows:

$$\Delta Y_L = \frac{\alpha}{\beta} \cdot \delta \cdot I + \frac{\alpha}{\beta} \cdot (\xi - \bar{g}_r - \bar{g}_{pk}) \cdot r \cdot I - \frac{\alpha}{\beta} \cdot \Delta \bar{g}_{pk} \cdot r \cdot K \quad (14)$$

ANALYSIS OF THE ACTUAL MODEL

In the above formula, $\Delta K \approx I$ as the current incremental physical capital (investment), $r \cdot I$ reflecting financing cost F , $r \cdot K$ and corporate earnings Y_K positively correlated $r \cdot K \approx \gamma \cdot Y_K$. Because Chinese commercial bank lending rate is regulated by the central bank by the statutory interest rate, r , and then r and \bar{g}_r set up as the exogenous variables.

Formula (12) the following conversion:

$$I = \frac{\beta}{\alpha \cdot \delta} \cdot \Delta Y_L + \frac{\bar{g}_r + \bar{g}_{pk} - \xi}{\delta} \cdot F + \frac{\gamma}{\delta} \cdot \Delta \bar{g}_{pk} \cdot Y_K \quad (15)$$

The economic implications are:

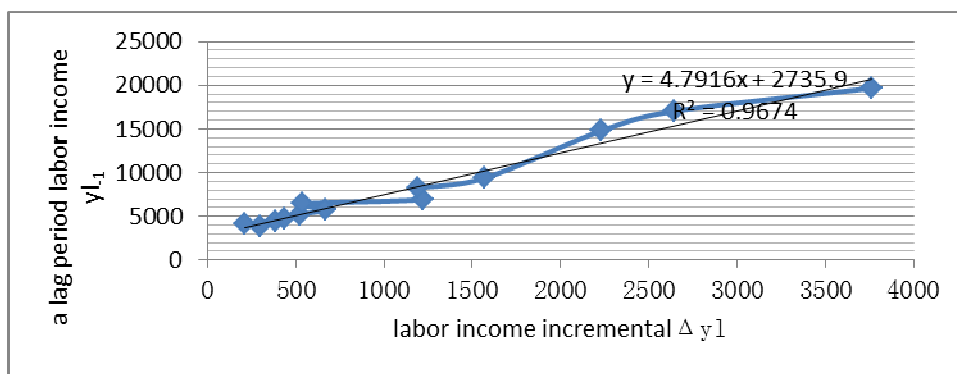
I was positively correlated with investment capital output elasticity β , and negative correlation with output elasticity α of labor;

With output and employment growth, increasing labor income ΔY_L is positively correlated with capital accumulation; Considering the financing costs F , then the pursuit of short-term profit (reflected by ξ) will reduce the current investment, expectation about financing interest rates and investment goods prices rising (reflecting by $\bar{g}_r + \bar{g}_{pk}$) will increase the current investment;

Corporate earnings Y_K will encourage investment, but also improve the ability of internal financing; Expected investment goods prices accelerated, the current investment will be expanded accordingly. Based on the formula (15), econometric models can be constructed according to the following steps:

1) Let marginal effect ΔY_L on I is constant c_1 ;

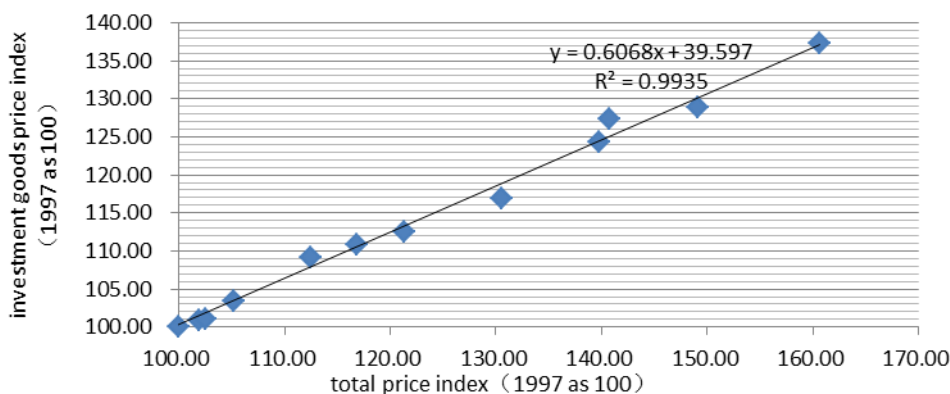
2) Here, labor income $y_{l,1}$ in the last year as an alternative to a variable increment Δy_l of labor income in the present year. There is strong correlation between a lag period labor income $y_{l,1}$ and labor income incremental Δy_l , shown in Figure 1:



Source: "China Statistical Yearbook" and the author's calculation

Figure 1: The correlation between lag labor income and labor income incremental

- 3) Impact of financing costs on the I and F is relevant with $\bar{g}_r + \bar{g}_{pk}$ and ξ ;
- 4) Due to the direct control of the central bank, financing rate is not really a market price signals, and it is not obvious that interest rate r regulate investment and financing, but the price level p_y actually played a role in regulating investment behavior (Feng Xiao, 2013)[8], Therefore, growth expectations \bar{g}_{py} of price level as alternative for \bar{g}_r ;



Data source: China Statistical Yearbook and author calculations

Figure 2: correlation between the general price level and investment goods price index

- 5) Time preference is reflected in the investment behavior, the time preference using $\xi = e^{\frac{y-i}{i}} - 1$ derived formulas, $\lim_{i \rightarrow y} e^{\frac{y-i}{i}} - 1 = 0$, $\lim_{i \rightarrow 0} e^{\frac{y-i}{i}} - 1 = \infty$. Taken together, the impact of financing costs F to I can be approximated by $c_2 \cdot (\bar{g}_{py} - \xi)$;
- 6) Under normal circumstances p_k capital goods prices should not continue to accelerate change, so $\Delta \bar{g}_{pk}$ is set to a constant; influence coefficient Y_K for I also is constant c_3 ;
- 7) a very important role in the mathematical model is able to predict, and labor income, price index, investment preferences and corporate earnings we are used to represent its expected value;
- 8) other random factors affecting investment can be summarized as \tilde{u} .

REGRESSION ANALYSIS

The various factors and variables discussed in the previous section into equation (15), we can get the regression equation (16):

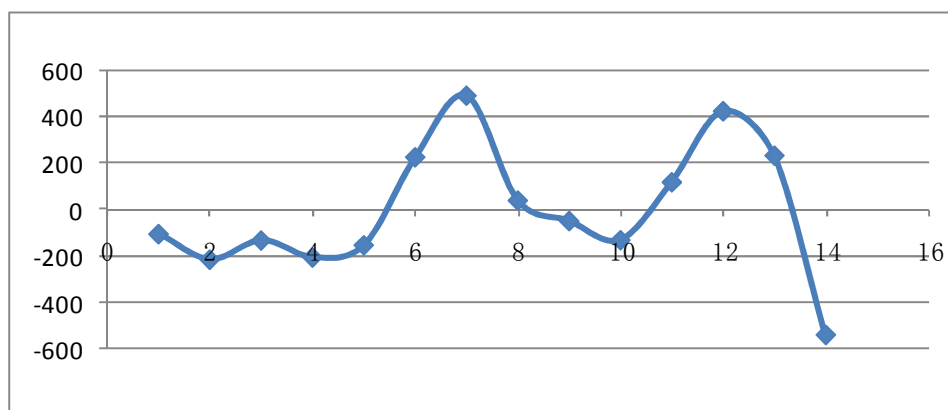
$$I = c_0 + c_1 \cdot y_{l-1} + c_2 \cdot (\bar{g}_{py} - \xi) \cdot \bar{F} + c_3 \cdot y_{K-1} + \tilde{u} \tag{16}$$

The formula (16) regression analysis by Chinese economic data, the results were as follows:

VARIABLES	I	I	I	I
Y_{l-1}	0.861*** (0.0570)	0.859*** (0.0652)		1.152*** (0.0373)
Y_{k-1}	0.527*** (0.0860)	0.534*** (0.112)	1.926*** (0.152)	
$(\bar{g}_{py} - \bar{\xi}) \cdot \bar{F}$		-0.00693 (0.0631)	-0.342 (0.236)	0.172* (0.0876)
Constant	-1,289*** (166.7)	-1,300*** (202.0)	-967.8 (818.1)	-1,127*** (342.6)
Observations	14	14	14	14
R-squared	0.998	0.998	0.962	0.993

Standard errors in parentheses
 *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

From the regression results of the formula (16), the results are not satisfactory, although the overall goodness of fit is high, are up more than 96%. But reflecting financing costs, variable investment preferences and price index at the 5% significance level is not significant. This may be because statutory interest, financing costs through interest rate to some extent represents the company's financing costs, but it does not reflect the real needs of enterprises for capital; corporate external financing capacity is limited, more to meet the demand for funds through internal surplus. Although weed out variables, lagged labor income, corporate earnings as explanatory variables to explain investment behavior, the two variables including the constant term in the 0.1% level of significance significant, the overall goodness of fit reached 99.8%, but By DW test, DW is 1.26, showing that there is a clear model of serial correlation, shown in Figure 3.

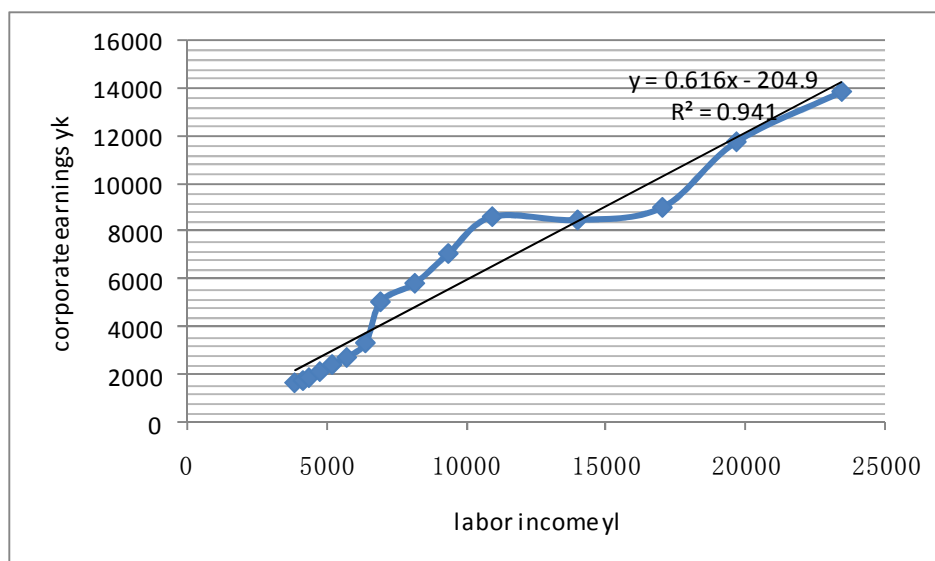


Source: "China Statistical Yearbook" and the author's calculation
Figure 3: The residual distribution model of investment behavior

In addition, although the presence of multicollinearity (Figure 4) between Y_L and Y_K , but the use of these two variables to explain investment behavior, their different economic implications. Omit any one of the explanatory variables, the accuracy of the regression equation have different degrees of decline:

VARIABLES	(1) I	(2) I
Y_{l-1}	1.195*** (0.0334)	
Y_{k-1}		1.771*** (0.112)
Constant	-1,463*** (330.4)	-248.6 (678.9)
Observations	14	14
R-squared	0.991	0.954

Standard errors in parentheses
 *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$



Source: "China Statistical Yearbook" and the author's calculation

Figure 4: multicollinearity between labor income and corporate earnings

If you are using labor income and corporate earnings into regression equation as explanatory variables, the regression results show that the constant standard deviation is too large. The reason may be that the investment is not only corporate behavior, and also about the government's fiscal expenditure G. Therefore, it is necessary to further observe the behavior of government consumption.

By regression analysis, it has significant positive correlation between government consumption G and gross domestic product Y:

VARIABLES	G
Y	0.128*** (0.00211)
Constant	210.2*** (50.51)
Observations	15
R-squared	0.996

Standard errors in parentheses
***p<0.01, **p<0.05, *p<0.1

However, government consumption G and gross domestic product Y resulting regression residuals u items show that there are some fluctuations (expansion or contraction) for each year of G, which may have an impact on investment.

For residuals u as conversion, and seek Index G⁺:

$$G^+ = \ln \frac{1}{\max(u, \forall u) - u + 1} \tag{17}$$

Seeking G⁺ aims to analyze the impact of fiscal policy on investment I ,based on the year of biggest G (relative to Y) .

Formula (18) Contains Index G+ and other variable shown on above:

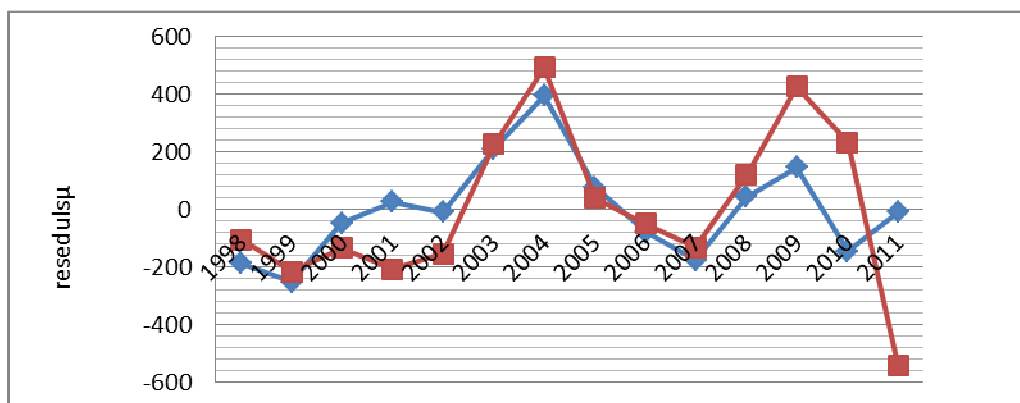
$$I = c_0 + c_1 \cdot Y_{L-1} + c_2 \cdot Y_{K-1} + c_3 \cdot G^+ + \tilde{u} \tag{18}$$

Regression analysis to formula (18), the results were as follows:

VARIABLES	t
$Y_{L,t}$	0.920*** (0.0400)
$Y_{K,t}$	0.465*** (0.0582)
G^+	-166.1*** (41.78)
Constant	-2,257*** (266.6)
Observations	14
R-squared	0.999

Standard errors in parentheses
 *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

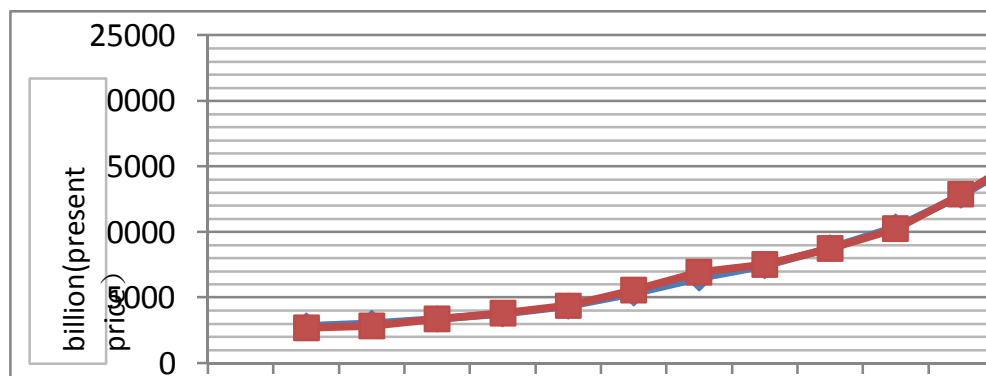
As can be seen from the regression results, the formula (18) in the regression coefficients were above 99% confidence level significantly, the goodness of fit of 99.9% compared with the accuracy of the model considering only labor income and corporate earnings regression results has been significantly improved (see Figure 5).



Source: Author's calculations

Figure 5: residuals distribution about government spending on investment behavior

Using the formula (18) is estimated from 1998 to 2011 China's investment trends, in addition to the estimated value in 2004 was a slight deviation from the actual value, the estimated value is quite consistent with the true value in other years (see Figure 6).



Source: Author's calculations

Figure 6: Investment estimates and the actual value comparison

But the formula (18) to explain investment behavior still faced the same problem with explaining investment behavior with a lag of labor income, corporate earnings as explanatory variables, it is not tested by DW, DW value is too low. With a lag of labor income, corporate earnings as explanatory variables to explain investment behavior model, DW is 1.26, while the formula (18) the results of the regression test DW, DW value of only 1.11. Although the formula (18) has a high goodness of fit, but the model is still the obvious serial correlation. Then we have the formula (18) be amended to eliminate the serial correlation.

CONCLUSION

We know collinearity between variables will cause serial correlation of model. To the labor income, corporate earnings and government spending, because we use the model in the process of regression with nominal amount

affected by the general price level, so we first eliminate the cause of the price level of linearity. The impact of the government consumption is reduced observably, after government consumption indexation. We primarily eliminate the linear price index between labor income and corporate earnings, we use the actual corporate earnings instead of the nominal amount of corporate earnings excluding price factors, regress formulation (18). The regression results are as follows.

VARIABLES	I
Y_{L-1}	1.001*** (0.0300)
Y_{K-1}	0.546*** (0.0669)
G^+	-140.9*** (41.97)
Constant	-2,712*** (250.4)
Observations	14
R-squared	0.999

Standard errors in parentheses
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

As can be seen from the regression results, after we modify the model, goodness of fit is still 99.9%, but DW value increased to 1.94, basically eliminating the serial correlation.

From formula (18), figure 5 and 6, the use comprising Labor income (interactive elements on employment and capital accumulation expansion path), Corporate earnings (earnings incentives and factors influencing investment enterprise financing) and government spending (fiscal policy factors on investment), regression model of three explanatory variables, it is exact to reproduce and interpret appropriate investment behavior of Chinese macro economy.

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