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Research on the influence of mapping choice on equilibrium manifold expansion for nonlinear system

Wang Laizhi* and Yang Xun

Chongqing City Management College, Chongqing, China

ABSTRACT

Based on the equilibrium manifold of nonlinear system, the influence of mapping choice on equilibrium manifold expansion results for a class of nonlinear systems which parametric dynamic matrix rank is not full is discussed. Considering different mapping choices with different equilibrium manifold expansion model errors, the conditions of equilibrium manifold expansion without form error for nonlinear system are given. Compared with single input system, the mapping choices which make the equilibrium manifold expansion meaningless for multi-input system are discussed. The simulation results show that the above mentioned theories are effective.

Key words: nonlinear system; equilibrium manifold expansion; mapping choice

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INTRODUCTION

Thermal objects generally have the characteristics of nonlinear and as conditions change system parameters change. Thermal parameters of systems may also appear very dramatic changes in the process of the complex operation mode transformation, therefore, ensure the thermal objects in each operating point can be safe, stable and efficient operation is very necessary. Seeking a simple and practical control strategy for nonlinear, parameter time-varying system is an effective way. As a kind of nonlinear control method, gain scheduling control has been applied in engineering. Belong to the category of multi-model control, gain scheduling control by using the mature theory of linear system to solve the control problem of nonlinear system [1]. As the two major categories of gain scheduling control method, whether the classical gain scheduling method [2] or the gain scheduling method based on linear parameter-varying (LPV) model [3], are needed to establish a linear parameter-varying model for the nonlinear system in the design of control system. Therefore, structure effective local linearization model is one of the core issues to realize the system gain scheduling control.

Different from the usually consideration that only linear approximation in the vicinity of one equilibrium point, the linearization family [4-7] consider different nonzero inputs and outputs of the nonlinear system, and have a family of equilibrium operating points. The linearized model with an adjustable parameter, suitable for different operating points, and the gain scheduling can be easily applied on the basis of linearization family [8-9]. The above family of equilibrium operating points constitute system equilibrium manifold, then the equilibrium manifold expansion model of nonlinear system is a nonlinear model constructed by the linearized model, and has the same equilibrium manifold with the original nonlinear system, in the whole equilibrium manifold neighborhood has good approximation capability on the dynamic characteristics of the original system.

Research and application on the equilibrium manifold expansion model has made a series of progress [10-16]. Some representatives are as follows: Yu, et al. proposed expansion model based on equilibrium manifold for real-time calculation, provided a method to determine the dynamic parameters and the scheduling variable of the model, and verified with minimum error of orthogonal expansion algorithm for single input turbine engine based on the case study of uniaxial turbojet engine simulation calculation [11]. Hu, et al. developed a simplified real-time simulation model for a turbofan engine based on the nonlinear system equilibrium manifold theory. The results obtained through the simulation calculation were accordant with the engine test results, which showed the model has good accuracy and real-time computing performance [12]. Zhao, et al. gave the theoretical basis of obtaining linear models of a nonlinear plant from its equilibrium manifold expansion (EME) model through the EME model construction of aero engines, and verified the validity of linear model obtained by a larger perturbation signal used to identify the EME model [13]. All have in common is that the engine was chosen for the study of nonlinear system, determined the input variation for one dimensional, emphasis lied in the identification method, real-time simulation and linear modeling.

Mapping choice is the only difference between different equilibrium manifold expansion models of the same nonlinear system, then resulting in the different model errors. At present, the research on equilibrium manifold expansion for nonlinear system mainly lies in the determination methods of mapping choice, and the new problems of the determined mapping choice to the equilibrium manifold expansion need to be further discussed. In view of this, the paper analyses the influence of mapping choice on equilibrium manifold expansion results for a class of nonlinear systems which parametric dynamic matrix rank is not full. This paper is organized as follows: Section 2 is devoted to describe the equilibrium manifold expansion model. In Section 3, the influence of mapping choice on equilibrium manifold expansion is explained and the expansion model based on equilibrium manifold for machine furnace coordinate system is described in Section 4. At last some conclusions are stated in Section5.

THE EQUILIBRIUM MANIFOLD EXPANSION MODEL

General description of the nonlinear system equilibrium manifold expansion structure

Consider the n order smooth nonlinear system

$$\begin{cases} \dot{x} = f(x,u) \\ y = g(x,u) \end{cases} \quad (1)$$

where $x = [x_1, x_2, \cdots, x_n]^T$, $u = [u_1, u_2, \cdots, u_m]^T$ and $y = [y_1, y_2, \cdots, y_s]^T$ are the state, input and output of the system respectively, $f = [f_1, f_2, \cdots, f_n]^T$ and the function $g = [g_1, g_2, \cdots, g_s]^T$ are all smooth nonlinear functions.

The equilibrium manifold of the system described by Eq.(1) is a set defined as

$$\left\{(x_e, u_e, y_e) \middle| f(x_e, u_e) = 0, y_e = g(x_e, u_e)\right\} \quad (2)$$

For the controllable nonlinear system, the equilibrium manifold can be parameterized by the scheduling variable α, which has the same dimension of the input variables, and can be represented as:

$$\begin{cases} x_e = x_e(\alpha) \\ u_e = u_e(\alpha) \\ y_e = y_e(\alpha) \end{cases} \quad (3)$$

So, the equilibrium manifold expansion model of the Eq. (1) can be written by

$$\begin{cases} \dot{X} = A(\alpha)(X - X_e(\alpha)) + B(\alpha)(U - U_e(\alpha)) \\ Y = Y_e(\alpha) + C(\alpha)(X - X_e(\alpha)) + D(\alpha)(U - U_e(\alpha)) \end{cases} \quad (4)$$

where $X = [x_1, x_2, \cdots, x_n]^T$, $U = [u_1, u_2, \cdots, u_m]^T$, and $Y = [y_1, y_2, \cdots, y_s]^T$.

We can get the Jacobian matrix $A(\alpha)$ by Taylor expansion as

$$A(\alpha)=\begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix} \quad (5)$$

The parameterized dynamic matrix $B(\alpha)$, $C(\alpha)$ and $D(\alpha)$ have the same structure, and obviously the matrix variables are functions of scheduling variable α.

By formula (4), the relationship between the current working point and Taylor expansion point need to be determined in the process of constructing equilibrium manifold expansion model, namely the choice of $X_e(\alpha)$ and $U_e(\alpha)$. Actually, as the determined $X_e(\alpha)$ and $U_e(\alpha)$ are the relationship between the scheduling variable α and state variable x and input variable u, the related literature behind Eq.(4) additional $\alpha = p(x,u)$ as a full expression of the equilibrium manifold expansion model. The overall determination situation of scheduling variable α without changing in the paper can be divided into two parts, the determination of parameter α called as the choice of scheduling variable and the relationship between the current working point and the Taylor expansion point called as mapping choice.

The choice of scheduling variable and mapping

As mentioned above, the equilibrium manifold expansion model for a nonlinear system needs to determine its scheduling variable and the choice of mapping. Different methods have been proposed to account for the choice of scheduling variable in previous literatures. It should be noted that, for the multidimensional input nonlinear system, the equilibrium manifold is no longer a space curve, and at the same time, to keep the same dimension of input variables, scheduling variable components $\alpha_1, \alpha_2, \ldots, \alpha_m$ can be defined as the scheduling variable composition, that is $\alpha = [\alpha_1, \alpha_2, \cdots, \alpha_m]^T$. Due to the diversity of the selection of scheduling variable, and for the sake of convenience, we can choose the controllable input variables as the scheduling variable:

$$\alpha = [\alpha_1, \alpha_2, \cdots, \alpha_m]^T = [u_{1e}, u_{2e}, \cdots, u_{me}]^T$$

After determining the scheduling variable α, the relationship between current working point (x,u,y) and the Taylor expansion point $(x_e(\alpha), u_e(\alpha), y_e(\alpha))$ need to be determined, that is the choice of mapping. The ways of mapping choice also have diversity, the literatures [10-11] have identified two possible selection methods as follows: One is to map the current working point to the nearest point on the equilibrium manifold, namely the orthogonal expansion method mentioned in the literature; Another is that, choose the equations arbitrarily from the equations consisted by $x_{ie}(\alpha) = x_i$, $u_{je}(\alpha) = u_j$. Considering the multidimensional nature of the nonlinear system input, the latter method is often selected as the mapping choice to reduce the complexity of the problems.

INFLUENCE OF MAPPING CHOICE ON EQUILIBRIUM MANIFOLD EXPANSION

Mapping choice is the only difference between different equilibrium manifold expansion models of the same nonlinear system, then resulting in the different model errors. The scope of application and results of the determined mapping choice to the equilibrium manifold expansion need to be further discussed.

Mapping choice which let the expansion point over defined

The rank $R(A)$ of the parameterized dynamic matrix $A(\alpha)$ reflects an important characteristic of nonlinear system. When $R(A) = n$, that is the matrixfull rank, we can choose $[u_1, u_2, \ldots, u_m]^T = [u_{1e}, u_{2e}, \ldots, u_{me}]^T$ as the mapping and get the certainty $x_{1e}, x_{2e}, \ldots, x_{ne}$ by $u_{1e}, u_{2e}, \ldots, u_{me}$, then the obtained EME model without problems. But when $R(A) < n$, the expansion point may be over defined according to the above mapping choice method.

For a class of nonlinear system which state variable $\dot{x}_i$ without self-equilibrium ability, the corresponding parametric dynamic matrix $A(\alpha)$ is:

$$A(\alpha)=\begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_{i-1}} & 0 & \frac{\partial f_1}{\partial x_{i+1}} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \cdots & \frac{\partial f_2}{\partial x_{i-1}} & 0 & \frac{\partial f_2}{\partial x_{i+1}} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_{i-1}} & 0 & \frac{\partial f_n}{\partial x_{i+1}} & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix}$$

Obviously $R(A) < n$. Analysis of this matrix, we can know that the set of equilibrium manifold for nonlinear system has $n+m-1$ variables included in the equations.

If $\left[u_1, u_2, \cdots, u_m\right]^T = \left[u_{1e}(\alpha), u_{2e}(\alpha), \cdots, u_{me}(\alpha)\right]^T$ is considered as the mapping choice based on the above mapping choice method, then the set of equilibrium manifold has n equations, $n-1$ unknown variables, and the expansion point can't be uniquely determined. The problem can be analyzed from two aspects:

(1) Select the $m-1$ other equations in addition to the j-th ($1 \le j \le n$) equation, and the unknown variables $x_{1e}(\alpha), \ldots,, x_{(i-1)e}(\alpha), x_{(i+1)e}(\alpha), \ldots, x_{ne}(\alpha)$ can be determined in combination with the given mapping. But the determined x_e and u_e may occur the case of $f_j(x_e, u_e) \neq 0$, therefore the equilibrium manifold expansion for $\dot{x}_j$ based on the above determined expansion point will go wrong.

(2) Select the other $m-1$ mappings in addition to the j-th ($1 \le j \le m$) mapping $u_j = u_{je}(\alpha)$, and the variables $x_{1e}(\alpha), \ldots, x_{(i-1)e}(\alpha), x_{(i+1)e}(\alpha), \ldots, x_{ne}(\alpha), u'_{je}(\alpha)$ can be determined in combination with the n equilibrium manifold equations. But the determined $u'_{je}(\alpha)$ may occur the case of $u'_{je}(\alpha) \neq u_{je}(\alpha)$, therefore the equilibrium manifold expansion for $\dot{x}_j$ which include the input variable u_j based on the above determined expansion point will go wrong.

The main reason for the problem is that, when $R(A) < n$, the equilibrium manifold expansion point over defined according to the mapping determined by the dimension of input variables. To avoid this situation, an effective method for the mapping choice is dimension reduction. Therefore, change above selected mapping into $\left[u_1, u_2, \cdots, u_{m-1}\right]^T = \left[u_{1e}(\alpha), u_{2e}(\alpha), \cdots, u_{(m-1)e}(\alpha)\right]^T$ when the parameterized dynamic matrix rank $R(A) = n-1$, and there are m kinds of selection methods if the mapping choice based on input variables.

Similarly, for such a class of nonlinear systems, the selection of scheduling variable α can no longer keep the same dimension with the input variables. Then the dimension of scheduling variable can be taken as $m-n+R(A)$.

Mapping choice which let the equilibrium manifold expansion model without form error

Different mapping choices get different EME model errors as the equilibrium points are different, the model error brought by EME can be reduced or even avoid through a reasonable way of mapping choice. Then, under what circumstances does the EME model of a nonlinear system without error?

Consider the following state variable x_i, $\dot{x}_i = f_i(x_1, x_2, \ldots, x_s, u_1, u_2, \ldots, u_t)$,where $1 \le s \le n$,$1 \le t \le m$. The number of state variables and input variables f_i contained are denoted by v ($0 \le v \le n$)and w ($0 \le w \le m$) respectively, while r_j ($1 \le j \le v + w$)is the power of the corresponding variables (state variables and input variables).

If meet the following conditions: (1) $v + w \le m + 1$ (2) Existing $r_j = 1, 1 \le j \le v + w$. Then we can get a EME model of $\dot{x}_i$ without error through mapping choice.

With $v + w = m + 1$, $j \le v$ as an example, mapping choice with the same dimension of input variables as follows: $\left[x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_s, u_1, \ldots, u_t\right]^T = \left[x_{1e}, \ldots, x_{(j-1)e}, x_{(j+1)e}, \ldots, x_{se}, u_{1e}, \ldots, u_{te}\right]^T$. Then the equilibrium manifold expansion model of $\dot{x}_i$ is

$$\dot{x}_i = g_1(x_{1e}, \ldots, x_{(j-1)e}, x_{(j+1)e}, \ldots x_{se}, u_{1e}, \ldots, u_{te})(x_j - x_{je}) \quad (6)$$

From $\dot{x}_i = f_i(x_1, x_2, \ldots, x_s, u_1, u_2, \ldots, u_t) = 0$, we can get

$$x_{je} = g_2(x_{1e}, \ldots, x_{(j-1)e}, x_{(j+1)e}, \ldots x_{se}, u_{1e}, \ldots, u_{te}) \text{ ,where}$$
$$g_2 = \frac{g_3(x_{1e}, \ldots, x_{(j-1)e}, x_{(j+1)e}, \ldots x_{se}, u_{1e}, \ldots, u_{te})}{g_1(x_{1e}, \ldots, x_{(j-1)e}, x_{(j+1)e}, \ldots x_{se}, u_{1e}, \ldots, u_{te})}, \; g_3 = f_i(x_1, x_2, \ldots, x_s, u_1, u_2, \ldots, u_t) - g_1 x_j .$$

Substituting into the above Eq.(6), we can get the equilibrium manifold expansion model of $\dot{x}_i$

$$\dot{x}_i = f_i(x_1, x_2, \ldots, x_s, u_1, u_2, \ldots, u_t)$$

Note that getting an error-free equilibrium manifold expansion model of state variable $\dot{x}_i$ does not mean the response changes of the state variable compared with the original nonlinear model without error. Only the equilibrium manifold expansion model of other variables associated with the state variable $\dot{x}_i$ without error, the obtained EME model compared with the original nonlinear model can have no error.

Meanwhile, as a counter-example can be seen in the following nonlinear system:

$$\begin{cases} \dot{x}_1 = -4x_1^{\,2} + x_2^{\,2} + u^2 \\ \dot{x}_2 = -x_1^{\,2} + u^2 \end{cases}$$

Due to the above conditions are not satisfied, so there is not a mapping selected so that the equilibrium manifold expansion model of the nonlinear system without error in form.

Mapping choice which let the equilibrium manifold expansion without meaning

For nonlinear equations which the number of variables (input variables and state variables) contained is less than the number of system input variables, the equilibrium manifold expansion for nonlinear equation may create new problems when arbitrary choose m equations to determine the mapping from the equations consisted by $x_{ie}(\alpha) = x_i$ and $u_{je}(\alpha) = u_j$.

Consider the following state variable x_i, $\dot{x}_i = f_i(x_1, x_2, \ldots, x_s, u_1, u_2, \ldots, u_t)$, where $1 \le s \le n$, $1 \le t \le m$. The number of state variables and input variables f_i contained are denoted by v ($0 \le v \le n$) and w ($0 \le w \le m$) respectively. When $v + w \le m$, if the mapping choice contains the following components:

$$\left[x_1, x_2, \ldots, x_s, u_1, \ldots, u_t\right]^T = \left[x_{1e}(\alpha), x_{2e}(\alpha), \ldots, x_{se}, u_{1e}(\alpha), \ldots, u_{te}(\alpha)\right]^T \quad (7)$$

Then such a mapping choice makes the equilibrium manifold expansion of $\dot{x}_i$ without meaning.

With $v + w = m$ as an example, consider equation (7) as the mapping choice. Because the $\dot{x}_i$ only concerned with x_1, $x_2, \ldots, x_s, u_1, \ldots, u_t$, and if let the current operating point as the expansion equilibrium point, then the variable $\dot{x}_i$ is always in a state of equilibrium, so the equilibrium manifold expansion of $\dot{x}_i$ without meaning. Compared with the single input nonlinear system, multi-input nonlinear system is more prone to this kind of situation due to the multidimensional mapping choice. Therefore, the arbitrariness of mapping choice is restricted for this kind of nonlinear system model. The mapping choice which makes the equilibrium manifold expansion without meaning for nonlinear variable should be avoid.

EXPANSION MODEL BASED ON EQUILIBRIUM MANIFOLD FOR MACHINE FURNACE COORDINATE SYSTEM

Simulation experiment model

In this paper and as a real case study, the multi-input nonlinear dynamic model of a boiler-turbine unit presented by Bell and Astrom is considered [17-18]. This practical model has been used in many of previous works, especially to investigate control aspects of the problem. Parameters of this model were estimated by data measurement from the Synvendska Kraft AB plant in Malmo, Sweden. Dynamics of this 160 MW oil-fired unit is given in the state space representation as:

$$\begin{cases} \dot{x}_1 = -0.0018 u_2 x_1^{9/8} + 0.9 u_1 - 0.15 u_3 \\ \dot{x}_2 = \left(0.073 u_2 - 0.016\right) x_1^{9/8} - 0.1 x_2 \\ \dot{x}_3 = \left(141 u_3 - (1.1 u_2 - 0.19) x_1\right) / 85 \\ y_1 = x_1 \\ y_2 = x_2 \\ y_3 = 0.05\left(0.13073 x_3 + 100 a_{cs} + q_e / 9 - 67.975\right) \end{cases}$$

where x_1 refers to the boiler pressure (MPa), x_2 for electric output (MW) and x_3 for the fluid density (kg/cm^3). Input variables are denoted by u_1, u_2 and for valves position of the fuel flow, steam control, and feed-water flow, respectively.

$$\begin{cases} a_{cs} = \dfrac{(1 - 0.001538 x_3)(0.8 x_1 - 25.6)}{x_3 (1.0394 - 0.0012304 x_1)} \\ q_e = (0.854 u_2 - 0.147) x_1 + 45.59 u_1 - 2.514 u_3 - 2.096 \end{cases}$$

The variables a_{cs} and q_e for steam quality and evaporation rate (kg/s), are determined by the above formula. The constraint conditions for input variables as follows:

$$\begin{cases} 0 \le u_1, u_2, u_3 \le 1, -0.007 \le \dot{u}_1 \le 0.007 \\ -2 \le \dot{u}_2 \le 0.02, -0.05 \le \dot{u}_3 \le 0.05 \end{cases}$$

Simulation process and result analysis

According to the process of EME for nonlinear system described earlier, perform the EME of variables $\dot{x}_1$, $\dot{x}_2$ and $\dot{x}_3$. Choose $\alpha = [\alpha_1, \alpha_2, \alpha_3]^T = [u_{1e}, u_{2e}, u_{3e}]^T$ as the scheduling variable with the same dimension of input variables, then we can get the parameterized dynamic matrix as:

$$A(\alpha) = \begin{bmatrix} -\frac{9}{8}\sigma_1\alpha_2(\gamma_1\alpha_3/(\gamma_2\alpha_2-\gamma_3))^{1/8} & 0 & 0 \\ \frac{9}{8}(\beta_1\alpha_2-\beta_2)(\gamma_1\alpha_3/(\gamma_2\alpha_2-\gamma_3))^{1/8} & -\beta_3 & 0 \\ -\frac{\gamma_2}{\gamma_4}\alpha_2+\frac{\gamma_3}{\gamma_4} & 0 & 0 \end{bmatrix}$$

Apparently, $R(A) < n$. According to the analysis of mapping choice described earlier, there are two methods of mapping choice for this machine furnace coordinate system: (1) Keep the same dimension with input variables, then $[u_1, u_2, u_3]^T = [u_{1e}(\alpha), u_{2e}(\alpha), u_{3e}(\alpha)]^T$ is selected; (2) Reduce the dimension of mapping, then the mapping choice is $[u_2, u_3]^T = [u_{2e}(\alpha), u_{3e}(\alpha)]^T$. Perform the equilibrium manifold expansion for the system at the two mapping choice methods, and we can get the relative errors between EME model and the original nonlinear model which are shown in Figures 1 ~ 6.

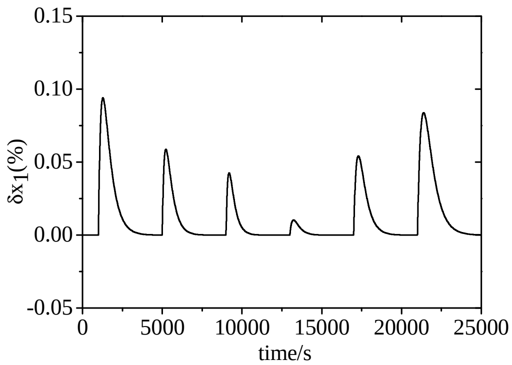

Fig. 1: Relative error of X_1 under mapping choice (1).

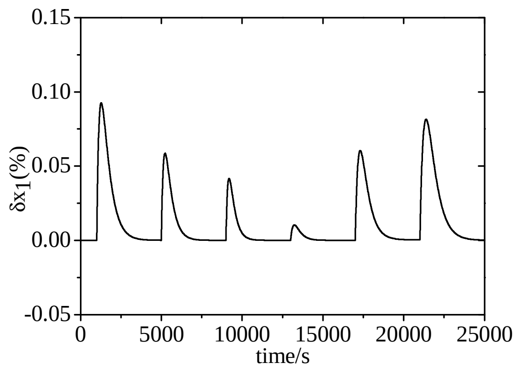

Fig. 2: Relative error of X_1 under mapping choice (2).

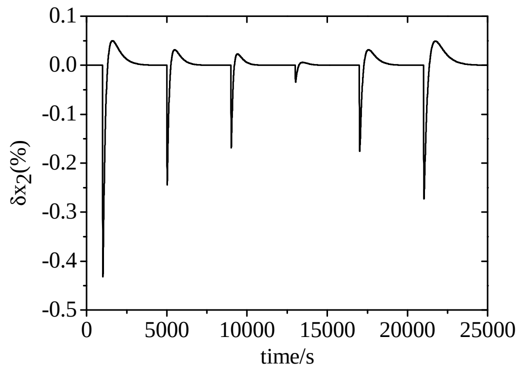

Fig. 3: Relative error of x_2 under mapping choice (1).

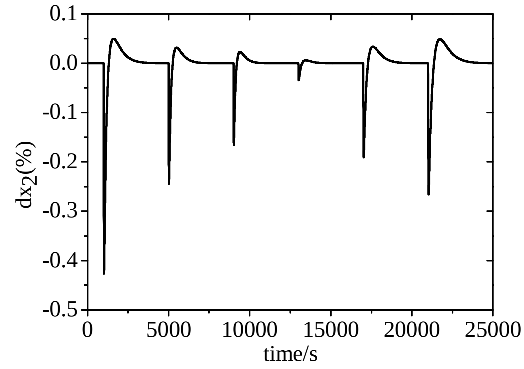

Fig. 4: Relative error of x_2 under mapping choice (2).

The above figure 1, figure 3 and figure 5 determine the equilibrium points under mapping choice (1) by equation $\dot{x}_1$ and $\dot{x}_2$. In this case, we can also determine $u'_{3e}(\alpha)$ by equation $\dot{x}_3$. Compared with the given mapping choice $u_{3e}(\alpha)$, $u_{3e}(\alpha) \neq u'_{3e}(\alpha)$, therefore, the expansion equilibrium points based on the above mapping choice (1) are problematic for the equilibrium manifold expansion of $\dot{x}_3$, this is also the main reason that x_3 has large error in figure 5. On the other hand, as the expansion equilibrium point certainty under mapping choice (2), so the same phenomenon do not occur in Figure 6.

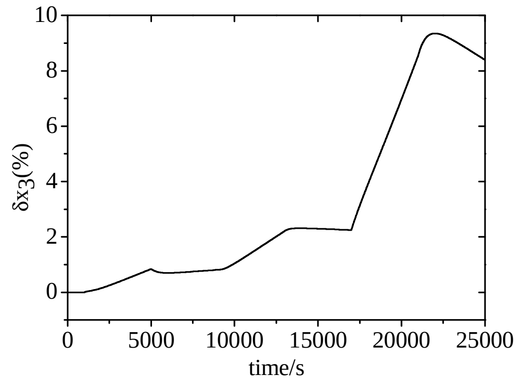

Fig. 5: Relative error of x_3 under mapping choice (1).

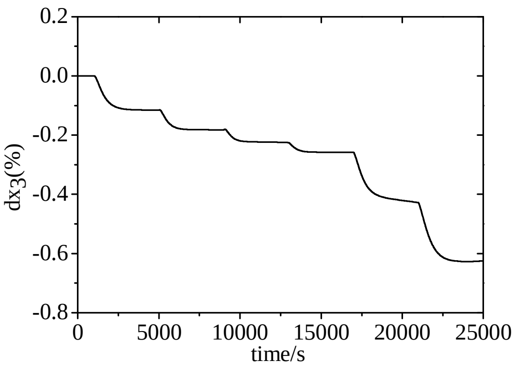

Fig. 6: Relative error of x_3 under mapping choice (2).

In fact, for the above boiler turbine coordinate system model, since it satisfies the nonlinear system without equilibrium manifold expansion model error conditions, so when the mapping choice is $[x_1, x_2]^T = [x_{1e}(\alpha), x_{2e}(\alpha)]^T$ or $[x_1, u_2]^T = [x_{1e}(\alpha), u_{2e}(\alpha)]^T$ the equilibrium manifold expansion model without error for the system can be obtained. At the same time, by the boiler turbine coordinate system model equation, when the mapping choice is $[x_1, x_2, u_2]^T = [x_{1e}(\alpha), x_{2e}(\alpha), u_{2e}(\alpha)]^T$ with the same dimension of input variables , the equilibrium manifold expansion of $\dot{x}_2$ become meaningless. The nonlinear variable $\dot{x}_3$ as variable $\dot{x}_2$ is, without meaning, when $[x_1, u_2, u_3]^T = [x_{1e}(\alpha), u_{2e}(\alpha), u_{3e}(\alpha)]^T$ as the mapping choice. So can also shows that by dimension reduction mapping choice can avoid the equilibrium manifold expansion of nonlinear variable without meaning.

CONCLUSION

(1) For a class of special nonlinear system, when the parameterized dynamic matrix rank $R(A) < n$, by the dimension reduction mapping choice can avoid problems caused by the different equilibrium points for each variable equation.

(2) We can reduce the error brought by EME model through the conditions of EME model without form error for nonlinear system.

(3) For multi-input nonlinear system, due to the mapping choice multidimensional, the arbitrariness of mapping choice is restricted in order to avoid make the equilibrium manifold expansion of some variables becomes meaningless.

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