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Research Article

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Research on portfolio model based on information entropy theory

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ABSTRACT

Entropy is a measure of the uncertainty. Entropy-based optimization model can help investors to make decision in the imperfect securities market. This paper improved the existing entropy optimization model by adding the securities transaction costs, and analyzed the maximization of the investment portfolio gains in an uncertain environment.

Key words: Entropy; portfolio; transaction costs

INTRODUCTION

There are some uncertainties in the securities industry. In order to avoid risks arising from investment securities, it is necessary to conduct research on the securities portfolio for investors in the process of investment securities. Markowitz's portfolio model laid a solid foundation for the quantitative study of portfolio investment. [1] In the Classical Markowitz mean-variance model, Investors minimized the variance as a risk function under the certain expected return. However, due to the difficulty of computing the covariance, application of Markowitz's mean-variance model greatly reduced.

Some scholars such as Li hua, Li Xingsi, LIANG Changyong, Han Miao, who established a series of entropy optimization model, and these models can solve the computational problems which the Markowitz mean-variance model encountered. However, the instability of the securities industry market has brought difficulties to the research of portfolio. Therefore, all costs arising from the transaction process becomes an important factor, and this article is based on the above analysis and explore the portfolio optimization model.

1. MARKOWITZ'S MEAN-VARIANCE MODEL

Markowitz assumed that investors are risk averse and they always want to get the maximum expected return under certain conditions or the minimum investment risk under certain conditions the expected revenue.

Markowitz's mean - variance model is expressed as follows.

$$\begin{aligned} &\min \ X^T C X \\ &s.t. \ \sum_{i=1}^{n} x_i r_i \geq c \\ &\qquad \sum_{i=1}^{n} x_i = 1 \ , i = 1, 2, 3, \cdots, n \end{aligned} \tag{1}$$

Where C used to represent the covariance matrix of random vector r , it usually means the investment risk matrix.

$X = (x_1, x_2, \cdots, x_n)^T$ is right weight matrix, x_i represents the ratio of investment securities i.
$r = (r_1, r_2, \cdots, r_n)^T$ is the expected rate of return investors matrix, c represents the expected return of portfolio investment.

2. PORTFOLIO OPTIMIZATION MODEL BASED ON INFORMATION ENTROPY

Generally, the gain of the securities x_i (i = 1, 2, … , n) is different at different time intervals, time intervals different gains, so it can be divided into T different intervals.

We may assume that π_t is a proportion of the gain of the T-th time interval in total revenue, r_{it} is yield of securities i in T-th time period, then

$$\bar{r}_i = \sum_{t=1}^{T} \pi_t r_{it} \qquad i=1,2,\cdots,n \tag{2}$$

From equation (2) can be drawn, B derived from the statistics in the past, so T th period income portfolio invested can be expressed as,

$$R_t = \sum_{i=1}^{n} r_{it} x_i \tag{3}$$

Average income securities $\bar{R} = \sum_{i=1}^{n} \bar{r}_i x_i$ may be drawn therefrom portfolio.

Mean - entropy model is as follows under certain constraints.

$$\begin{aligned} &\mathrm{Max} \quad -\sum_{t=1}^{T} \pi_t R_t \ln(\pi_t R_t) \\ &S.t. \quad \sum_{t=1}^{T} \pi_t R_t = c \end{aligned} \tag{4}$$

$R_t = \sum_{i=1}^{n} x_i r_{it}$ is substituted into the model (4).

$$\begin{aligned} &\mathrm{Max} \quad -\sum_{t=1}^{T} \pi_t \left(\sum_{i=1}^{n} x_i r_{it} \right) \ln \left(\pi_t \sum_{i=1}^{n} x_i r_{it} \right) \\ &S.t. \begin{cases} \sum_{t=1}^{T} \pi_t \sum_{i=1}^{n} x_i r_{it} = c \\ \sum_{i=1}^{n} x_i = 1, \quad x_i \geq 0, i = 1, 2, \cdots, n \end{cases} \end{aligned} \tag{5}$$

Where, let c be excepted profit for investor, these optimization models can be transformed into the form of multi-objective optimization.

$$
\begin{aligned}
&\text{Max} \quad \sum_{t=1}^{T} \pi_t \sum_{i=1}^{n} x_i r_{it} \\
&\text{Max} \quad -\sum_{t=1}^{T} \pi_t \left(\sum_{i=1}^{n} x_i r_{it} \right) \ln \left(\pi_t \sum_{i=1}^{n} x_i r_{it} \right) \\
&S.t. \quad \sum_{i=1}^{n} x_i = 1, \quad x_i \geq 0, i = 1, 2, \cdots, n
\end{aligned}
\tag{6}
$$

The linear weighting method can be usually used to solve the multi-objective planning problem. Therefore, the problem is expressed as follow:

$$
\begin{aligned}
&\text{Max} \quad \omega_1 \sum_{t=1}^{T} \pi_t \sum_{i=1}^{n} x_i r_{it} - \omega_2 \sum_{t=1}^{T} \pi_t \left(\sum_{i=1}^{n} x_i r_{it} \right) \ln \left(\pi_t \sum_{i=1}^{n} x_i r_{it} \right) \\
&S.t. \quad \sum_{i=1}^{n} x_i = 1, \quad x_i \geq 0, i = 1, 2, \cdots, n
\end{aligned}
\tag{7}
$$

3. IMPROVED PORTFOLIO OPTIMIZATION MODEL BASED ON INFORMATION ENTROPY

In the process of securities investment, in order to effectively avoid risks, investors often use the method of portfolio investment. Information entropy optimization model is introduced in this article can help investors make better optimization decisions in the case of asymmetric information. But in actual operation, investors will encounter more risk. For example, payment transaction costs will affect the overall revenue in the entire transaction process. Therefore, it is necessary to analyze the transaction costs of the whole process. Transaction costs were added to the model, so that the results would be more realistic.

Securities transaction costs include four categories: commissions, transfer fees, stamp duty and other expenses, the cost of each part are different for different stock exchanges. Therefore, it is necessary to remove transaction costs from the benefits of investment portfolio.

We assume that the transaction costs of buying and selling is the same as α times of the stock price in the investment process, then you can consider improving the above linear programming model.

Let ω_{it} is the price of the securities i during the t-th time interval, $\omega_{i(t+1)}$ is the price of the securities i during the (t+1)-th time interval, then investment rate of return by trading stocks i can be expressed as follow in the entire transaction process.

$$
r_i = \frac{[\omega_{\mathrm{i(t+1)}} - \alpha\omega_{\mathrm{i(t+1)}}] - (\omega_{\mathrm{it}} + \alpha\omega_{\mathrm{it}})}{\omega_{\mathrm{it}} + \alpha\omega_{\mathrm{it}}}
\tag{8}
$$

Since $r_i = \dfrac{\omega_{\mathrm{i(t+1)}} - \omega_{\mathrm{it}}}{\omega_{\mathrm{it}}}$ is the yield when transaction costs are not considered, so the yields can be rewritten as follow.

$$
r_i^{'} = \frac{1-\alpha}{1+\alpha} r_i - \frac{2\alpha}{1+\alpha}
\tag{9}
$$

Therefore, the above model (7) can be rewritten as follow.

$$\text{Max}\ \ \omega_1\sum_{t=1}^{T}\pi_t\sum_{i=1}^{n}x_ir_i^{'}-\omega_2\sum_{t=1}^{T}\pi_t\left(\sum_{i=1}^{n}x_ir_i^{'}\right)\ln\left(\pi_t\sum_{i=1}^{n}x_ir_i^{'}\right)$$
$$S.t.\ \begin{cases} r_i^{'}=\dfrac{1-\alpha}{1+\alpha}r_i-\dfrac{2\alpha}{1+\alpha} \\ \sum_{i=1}^{n}x_i=1,\quad x_i\geq 0, i=1,2,\cdots,n \end{cases} \tag{10}$$

But in real life, transaction costs of buying and selling the same securities are always different in a different time, even at different time intervals. Therefore, it is necessary to consider whether the model is adapted to the general case.

Let ω_{it} is the price of the securities i during the t-th time interval, $\omega_{i(t+1)}$ is the price of the securities i during the (t+1)-th time interval, Commission is α times of the stock price, transfer fee is β times of the stock price, Stamp duty is γ times of the stock price, Other costs are δ. So if you buy a unit cost of securities at time t, you will spend $\omega_{\text{it}}+\omega_{\text{it}}*\alpha+\omega_{\text{it}}*\beta+\omega_{\text{it}}*\gamma+\delta$, Similarly you will gain $\omega_{i(t+1)}-\omega_{i(t+1)}*\alpha-\omega_{i(t+1)}*\beta-\omega_{i(t+1)}*\gamma-\delta$ by selling the securities i at time t +1.

In the course of this transaction, the investment rate of return becomes

$$r_i^{''}=\frac{-(\omega_{\text{it}}+\omega_{\text{it}}*\alpha+\omega_{\text{it}}*\beta+\omega_{\text{it}}*\gamma+\delta)+\omega_{i(t+1)}-\omega_{i(t+1)}*\alpha-\omega_{i(t+1)}*\beta-\omega_{i(t+1)}*\gamma-\delta}{\omega_{\text{it}}+\omega_{\text{it}}*\alpha+\omega_{\text{it}}*\beta+\omega_{\text{it}}*\gamma+\delta} \tag{11}$$

Therefore, the model can be transformed into

$$\text{Max}\ \ \omega_1\sum_{\text{t}=1}^{T}\pi_t\sum_{i=1}^{n}x_ir_i^{''}-\omega_2\sum_{\text{t}=1}^{T}\pi_t\left(\sum_{i=1}^{n}x_ir_i^{''}\right)\ln\left(\pi_t\sum_{i=1}^{n}x_ir_i^{''}\right)$$
$$S.t.\ \ \sum_{i=1}^{n}x_i=1,\quad x_i\geq 0, i=1,2,\cdots,n \tag{12}$$

4. ANALYSIS OF THE MODEL

According to the preferences of investors, you can select the relevant data of several stocks in the stock exchange for the above model (12) empirical research, and these stocks may be in a better performance of the industry sector, or growing ability of certain stocks.

Firstly, combined with the standard fee charged by stock exchanges, each branch securities yields is obtained by the model (11), then used a uniform distribution (you can also consider other distribution) to calculate the value of π_{t}, which represents the proportion of income in the t-th time interval of total revenue.

ω_1 and ω_2 are weight coefficients, investors can appropriately adjusted coefficient according to their degree of risk aversion. Finally, investors can conduct empirical research based on the model (12).

The problem are involved in this model is constrained nonlinear programming problems, investors can consider using MATLAB Optimization toolbox fmincon function to solve the above problems. Thus, depending on the investor's choice to take a different weighting factor, choose a different and corresponding stock exchange, the results obtained will gain a greater change. This paper only provides the corresponding empirical research methods.

CONCLUSION

Due to the complexity and instability of the stock market, there are some deficiencies in Markowitz classic portfolio model. The portfolio model by the above analysis shows that the optimal model combination with transaction costs based on information entropy can combine the characteristics of information entropy with the uncertainty of the securities trading market together. To some extent, it makes the application of the model becomes more realistic, but also be able to adapt to the real stock market. But the stock market is more complex, any model has its limitations, this model requires that the securities market is relatively stable. In empirical research, you need to remove much of the volatility of individual raw data, so as to make the model more realistic.

Thus, this model can provide useful lessons for investment decisions, and investors choose the appropriate weighting factor based on the actual situation and their own preferences to maximize revenue as much as possible in certain risks.

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