Available online www.jocpr.com

Journal of Chemical and Pharmaceutical Research, 2015, 7(3):1191-1196

Research Article

ISSN : 0975-7384
CODEN(USA) : JCPRC5

Research and application of the rough multi-attribute lattice order decision-making

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ABSTRACT

According to the shortages of optional plan and optimal solution in decision-making system, we put forward a new rough lattice order decision-making model, which is based on the rough decision-making analysis method of dominant relation. The method of rough decision-making analysis, based on dominance relation, is discussed in the article, also provide new method of selection, establish partial order relation based on P-set and present construction method of optional plan lattice structure, which is based on dominance relations, thus we can choose optimal scheme reasonably. Finally, an application instance will be given.

Key words: Dominance relation; rough set; lattice order decision-making

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INTRODUCTION

Rough set theory proposed by Pol and scientist Pawlak in 1982 is a theoretical method studying incomplete, uncertain knowledge and data expression, learning and concluding. It is a new mathematical tool that deals with the fuzzy and uncertain knowledge. Its main idea is that, decision or classification rules, under the premise of keeping the classification function unchanged, induced through the reduction of knowledge. However, because of the effects of the environment that is complex and uncertain and the preference of the decision maker, information incompletion, information uncertainty and preference information have become significant features of the actual decision system. The multi-attribute decision-making problem with preference of incomplete information has attracted more and more attentions. Dominance relation rough set theory proposed by Greco has provided a new idea to deal with the multi-attribute decision-making problem of preference information. In Greco's theory, dominance relation is used to signify the preference information of the decision maker in the form of knowledge. It is good for dealing with the multi-attribute decision-making problems with preference information.

The lattice is a kind of order structure which enjoys wide application in the treatment of preference relation. The lattice is able to make the fuzzy and indistinct preference of decision maker's decisions ordered and structured. In this paper, the authors apply rough set theory in multi-attribute lattice order decision. Dominance relation is used to replace the indiscernible relation of classical rough sets to work out rough approximations of knowledge, and the lattice order theory is used to obtain decision-making information or decision-making results in order to achieve better classification and determine the preference orderings of the schemes faster.

THE ANALYSIS METHOD OF ROUGH DECISION BASED ON DOMINANCE RELATION

There are some basic concepts of the information system based on dominance relation.

Definition 1: The given decision-making system $S=<U,A,V,f>$, $\forall x,y\in U$, set $P\subseteq A$, for $q\in P$, if $yS_q x$ represents x inferior to y $xS_q y$ and $xS_q y$ represents x non-inferior to y, then $\bigcap_{q\in P} S_q$ is called the dominance relation of set P, written as $D_P=\bigcap_{q\in P} S_q$.

Obviously, the dominance relation is partial ordering relation, presenting reflexivity and transitivity.

Definition 2: For given set $P \subseteq A, \forall x, y \in U$, under the dominance relation, the P partial set of x is defined as $D_P^+(x) = \{y \in U : yD_P x\}$ and the P of x is defined as $D_P^-(x) = \{y \in U : xD_P y\}$ by partial set.

Suppose that the object set U is divided as the decision class of limited number according to decision-making preference attribute, and set $Cl_u = \{Cl_t^{\geq}, t \in 1,2,\cdots,n\}$. If $\{x \in Cl_r y \in Cl_s, 0 \leq s \leq r \leq n\} \Rightarrow \{xS_q y \wedge \neg yS_q x\}$, written as $Cl_r \succ Cl_s$. The upward and downward unions of these categories are respectively represented as: $Cl_t^{\geq} = \bigcup_{s \geq t} Cl_s$, $Cl_t^{\leq} = \bigcup_{s \leq t} Cl_s$ among which $t, s \in \{1,2,\cdots,n\}$. $x \in Cl_t^{\geq}$ means that x at least belongs to category Cl_t ; $x \in Cl_t^{\leq}$ means that x at most belongs to category Cl_t . Obviously, each object of $Cl_t^{\geq}$ is better than or at least equal to each object of $Cl_t^{\leq}$. $Cl_1^{\geq} = Cl_n^{\leq} = U$, $Cl_n^{\geq} = Cl_n$, $Cl_1^{\leq} = Cl_1$, $Cl_{t-1}^{\leq} = U - Cl_t^{\geq}$, $t = 2,3,\cdots,n$. That is to say, the decision maker has assigned objects in the decision table to categories in accordance with the following comprehensive evaluation. The worst object belongs to category Cl_1 , the best belongs to category Cl_n , and other objects belong to the remaining category Cl_t . According to this principle, the higher the value of t , the better the category Cl_t will be.

Definition 3: Suppose the decision system $S = < U, A, V, f >$, the given set $P \subseteq A, Cl_t^{\geq} \subseteq U$, under the dominance relation, the lower approximation, the upper approximation and the boundary region of $Cl_t^{\geq}$ are respectively defined as $\underline{P}(Cl_t^{\geq}) = \{x \in U : D_P^+(x) \subseteq Cl_t^{\geq}\}$, $\overline{P}(Cl_t^{\geq}) = \{x \in U : D_P^-(x) \cap Cl_t^{\geq} \neq \phi\}$ and $Cl_t^{\geq}$ is defined as $Bn_P(Cl_t^{\geq}) = \overline{P}(Cl_t^{\geq}) - \underline{P}(Cl_t^{\geq})$.

Similarly, those of are $\underline{P}(Cl_t^{\leq}) = \{x \in U : D_P^-(x) \subseteq Cl_t^{\leq}\}$, $\overline{P}(Cl_t^{\leq}) = \{x \in U : D_P^+(x) \cap Cl_t^{\leq} \neq \phi\}$ and $Bn_P(Cl_t^{\leq}) = \overline{P}(Cl_t^{\leq}) - \underline{P}(Cl_t^{\leq})$.

LATTICE ORDERED CHOICE BASED ON ROUGH SET FORMAT

Definition 4: Suppose decision system $S = < U, A, V, f >$, the given set $P \subseteq A, Cl_t^{\geq} \subseteq U$, $\underline{P}(Cl_t^{\geq})$ and $\underline{P}(Cl_t^{\leq})$ are respective the rough lower approximations of $Cl_t^{\geq}$ and $Cl_t^{\leq}$,then $U_t^{\geq} = \underline{P}(Cl_t^{\geq}) \cup Cl_t$ is the risk taking alternative scheme set and $U_t^{\leq} = \underline{P}(Cl_t^{\leq}) \cup Cl_t$ is the conservative alternative scheme set.

Definition 5: If the decision system $S = < U, A, V, f >$, $a_i \in U$, the given set $P \subseteq A$, there is $v_p[a_i] = a_{i1}x^{k-1} + a_{i2}x^{k-2} + \cdots + a_{i(k-1)}x + a_{ik}$, of which k is the number of elements in set P ; a_{ij} represents the corresponding element of item i in set U with respect to item j in set P , thus $v_p[a_i]$ is called P - assignment function, and $V_p = \{v_p[a_1], v_p[a_2], \cdots, v_p[a_n]\}$ is called P - assignment function set.

Definition 6: If the decision system $S = < U, A, V, f >, \cdots$, the given set $P \subseteq A, V_p$ is a P - assignment function set. $\forall v_p[a_i], v_p[a_j] \in V_p$, if there is $v_p[a_i] = a_{i1}x^{k-1} + a_{i2}x^{k-2} + \cdots + a_{ik}$ and $v_p[a_j] = a_{j1}x^{k-1} + a_{j2}x^{k-2} + \cdots + a_{jk}$, there will be:

(1) If $a_{i1} = a_{j1}, a_{i2} = a_{j2}, \cdots, a_{ik} = a_{jk}$, then $v_p[a_i]$ is called equal to $v_p[a_j]$, written as $v_p[a_i] = v_p[a_j]$;

(2) If $a_{i1} \leq a_{j1}, a_{i2} \leq a_{j2}, \cdots, a_{ik} \leq a_{jk}$, $v_p[a_j]$ is called prior to $v_p[a_i]$, written as $v_p[a_i] \leq v_p[a_j]$;

(3) If $a_{i1} < a_{j1}, a_{i2} < a_{j2}, \cdots, a_{ik} < a_{jk}$, $v_p[a_j]$ is called strictly prior to $v_p[a_i]$, written as $v_p[a_i] < v_p[a_j]$.

According to the **definition 6**, the dominance relation of P - assignment function set V_p presents the following natures:
Character 1: Reflexivity. For any $v_p[a_i]$, there will be $v_p[a_i] = v_p[a_i]$.
Character 2: Antisymmetry, if $v_p[a_i] \le v_p[a_j]$ and $v_p[a_j] \le v_p[a_i]$, then $v_p[a_i] = v_p[a_j]$.
Character 3: Transitivity. If $v_p[a_i] \le v_p[a_j]$ and $v_p[a_j] \le v_p[a_k]$, then $v_p[a_i] < v_p[a_k]$.

In conclusion, under dominance relation of "$\le$", set V_p makes up partial order set.
As for the partial order set $(V_p, \le)$, suppose x and y are two arbitrary element so of the partial set V_p , if $x \le y$ or $y \le x$, x and y are regarded as comparable, otherwise x and y are not comparable, written as $x \| y$.

Definition 7: Suppose the decision system $S =< U, A, V, f >, (V_p, \le)$ is a partial order set. If any two elements have both supremum and infimum, V_p can make up a lattice in regard to partial order "$\le$"and "$\le$" is called a lattice order of V_p .

In the partial set $(V_p, \le)$, the given set $P \subseteq A$, there will be $v_p[a_i], v_p[a_j], v_p[a_l] \in V_p$:
(1) If $a_{i1} < a_{j1}$, and $a_{i1} < a_{l1}, a_{i2} < a_{j2}$, and $a_{i2} < a_{l2}, \cdots, a_{ik} < a_{jk}$ and $a_{ik} < a_{lk}$ (k is the number of the elements in set P), $v_p[a_i]$ is then called the infimum of $v_p[a_j]$ and $v_p[a_j]$, written as $v_p[a_i] = \inf(v_p[a_j], v_p[a_l])$.

(2) If $a_{i1} > a_{j1}$, and $a_{i1} > a_{l1}, a_{i2} > a_{j2}$, and $a_{i2} > a_{l2}, \cdots, a_{ik} > a_{jk}$ and $a_{ik} > a_{lk}$ (k is the number of the elements in set P), $v_p[a_i]$ is then called the supremum of $v_p[a_j]$ and $v_p[a_j]$, written as $v_p[a_i] = \sup(v_p[a_j], v_p[a_l])$.

For the limited partial order set $(V_p, \le)$, if each element in V_p is represented with a small circle on the same plane: if $a \prec b$, there is c, and $a \prec c \prec b$ is met; putting b above a and linking a and b with a licne, the graph worked out is called the Hasse graph of the partial order set $(V_p, \le)$. Hasse graph can intuitively and vividly presents the relationship among the various elements in partial order sets.

SAMPLE ANALYSIS
The real estate database of a city has accumulated a lot of data information about the real estate transaction. There are ten alternative schemes, the sets of which are represented with $\{a_1, a_2, \cdots, a_{10}\}$. Before making decisions, the buyers need independently study these condition attributes including the regional environment c_1, the type of housing c_2, the size of area c_3 and the decision attribute is price d as shown in Table 1:

Table 1 Real estate transaction information data of a city

TABLE 1. Real estate transaction information data of a city

	Environment c_1	Style c_2	Size c_3	Price d
a_1	3	2	1	2
a_2	2	3	2	2
a_3	2	2	1	1
a_4	2	2	3	3
a_5	1	2	3	2
a_6	1	3	3	3
a_7	3	3	1	2
a_8	1	2	3	1
a_9	3	2	2	2
a_{10}	2	1	3	3

In this table,
$U = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}\}$ is the alternative scheme set. For regional environment c_1: if it belongs to the inner ring region, it is written as 3; if it is between the inner and the outer ring, it is written as 2; and if it is between the inner and the outer ring, it is written as 2; if it belongs to the outer ring region, it is written as 1. For the type of housing c_2 the high-rise is written as 3; the small high-rise is written as 2; and the multi-level is written as 1. For the size of area c_3, the large is written as 3; the medium is written as 2 and the small is written as 1. For the decision attributes d , if the price is high, it is written as 3; if the price is middle, it is written as 2; and if the price is low, it is written as 1.

Suppose the buyer pays more attention to the regional environment and the type of housing, setting $P = \{c_1, c_2\}$. According to the decision attribute, the samples are divided into three decision classes:

$$Cl_1 = \{a \in U_p,\ f(a,d) = lowprice\} = \{a_3, a_8\}$$
$$Cl_2 = \{a \in U_p,\ f(a,d) = middleprice\}$$
$$= \{a_1, a_2, a_5, a_7, a_9\}$$
$$Cl_3 = \{a \in U_p,\ f(a,d) = highprice\}$$
$$= \{a_4, a_6, a_{10}\}$$

Obviously:

$$Cl_1^{\geq} = Cl_1 \cup Cl_2 \cup Cl_3$$
$$= \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}\}$$
$$Cl_2^{\geq} = Cl_2 \cup Cl_3 = \{a_1, a_2, a_4, a_5, a_6, a_7, a_9, a_{10}\}$$
$$Cl_3^{\geq} = Cl_3 = \{a_4, a_6, a_{10}\}$$
$$Cl_2^{\leq} = Cl_1 \cup Cl_2 = \{a_1, a_2, a_3, a_5, a_7, a_8, a_9\}$$
$$Cl_3^{\leq} = Cl_1 \cup Cl_2 \cup Cl_3$$
$$= \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}\}$$

According to the economic conditions and intention of buyers, they tend to buy the middle priced houses. Then the result calculated by rough decision analysis method based on the dominance relation is as follows ($P=\{c_1,c_2\}$):

$D_p^+(a_1)=\{a_1,a_7,a_9\}$, $D_p^-(a_1)=\{a_1,a_3,a_4,a_5,a_8,a_9,a_{10}\}$;
$D_p^+(a_2)=\{a_2,a_7\}$, $D_p^-(a_2)=\{a_2,a_3,a_4,a_5,a_6,a_8,a_{10}\}$;
$D_p^+(a_3)=\{a_1,a_2,a_3,a_4,a_7,a_9\}$, $D_p^-(a_3)=\{a_3,a_4,a_5,a_8,a_{10}\}$;
$D_p^+(a_4)=\{a_1,a_2,a_3,a_4,a_7,a_9\}$, $D_p^-(a_4)=\{a_3,a_4,a_5,a_8,a_{10}\}$;
$D_p^+(a_5)=\{a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9\}$, $D_p^-(a_5)=\{a_5,a_8\}$;
$D_p^+(a_6)=\{a_2,a_6,a_7\}$, $D_p^-(a_6)=\{a_5,a_6,a_8\}$;
$D_p^+(a_7)=\{a_7\}$, $D_p^-(a_7)=\{a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10}\}$;
$D_p^+(a_8)=\{a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9\}$, $D_p^-(a_8)=\{a_5,a_8\}$;
$D_p^+(a_9)=\{a_1,a_7,a_9\}$, $D_p^-(a_9)=\{a_1,a_3,a_4,a_5,a_8,a_9,a_{10}\}$;
$D_p^+(a_{10})=\{a_1,a_2,a_3,a_4,a_7,a_9,a_{10}\}$, $D_p^-(a_{10})=\{a_{10}\}$;

$\underline{P}(Cl_2^{\geq})=\{a_1,a_2,a_6,a_7,a_9\}$, $\overline{P}(Cl_2^{\geq})=\{a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10}\}=U$

$$\begin{aligned}Bn_P(Cl_2^{\geq})&=\overline{P}(Cl_2^{\geq})-\underline{P}(Cl_2^{\geq})\\&=\{a_3,a_4,a_5,a_8,a_{10}\}\end{aligned};$$

$\underline{P}(Cl_2^{\leq})=\{a_5,a_8\}$,

$\overline{P}(Cl_2^{\leq})=\{a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10}\}=U$

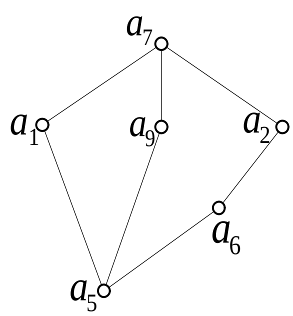
$$\begin{aligned}Bn_P(Cl_2^{\leq})&=\overline{P}(Cl_2^{\leq})-\underline{P}(Cl_2^{\leq})\\&=\{a_1,a_2,a_3,a_4,a_6,a_7,a_9,a_{10}\}\end{aligned}$$

According to the intention of the buyers to choose middle priced houses, if other condition attributes are satisfying, the high priced ones may also be accepted. There will thus be the risk taking alternative scheme set $U_2^{\geq}=\underline{P}(Cl_2^{\geq})\bigcup Cl_2=\{a_1,a_2,a_5,a_6,a_7,a_9\}$ and P - assignment function sets $V_p=\{v_p[a_1],v_p[a_2],v_p[a_5],v_p[a_6],v_p[a_7],v_p[a_9]\}$

According to the definition 6:
$v_p[a_1]=3x+2$, $v_p[a_2]=2x+3$, $v_p[a_5]=x+2$, $v_p[a_6]=x+3$, $v_p[a_7]=3x+3$, $v_p[a_9]=3x+2$

through calculation , a_7 is the optimal scheme.

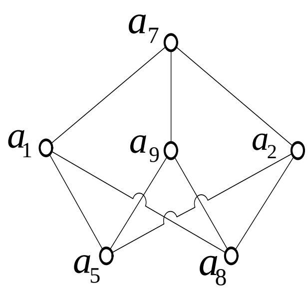
The Hasse graph is as follows:



According to the intention of the buyers to choose middle priced houses, whatever the other conditions, the middle priced is the limit. There will be the conservative alternative scheme set $U_2^{\leq} = \underline{P}(Cl_2^{\leq}) \bigcup Cl_2 = \{a_1, a_2, a_5, a_7, a_8, a_9\}$

P - assignment function set $V_p = \{v_p[a_1], v_p[a_2], v_p[a_5], v_p[a_7], v_p[a_8], v_p[a_9]\}$

According to the definition 6:
$v_p[a_1] = 3x + 2$, $v_p[a_2] = 2x + 3$, $v_p[a_5] = x + 2$, $v_p[a_7] = 3x + 3$, $v_p[a_8] = x + 2$, $v_p[a_9] = 3x + 2$
through calculation, a_7 is the optimal scheme. The Hasse graph is as follows:



CONCLUSION

Rough decision analysis based on dominance relation can make better generalization and practicability research on lattice order decision model. It is development and improvement of rational behaviour decision theory. In this paper, rough set theory is applied to lattice order decision theory; an insightful analysis of the lattice order natures of the alternative scheme set is made; and lattice structure is constructed in order to achieve better classification and determine the preference orderings of the schemes faster.

Acknowledgments
Supported by National Science Fund of China (NO.61370168), National Science Fund of Hebei Province China (NO.A2012209030)

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