



## Reducing computation complexity of the image segmentation algorithm based on ESFCM

Zengqiang Ma\*, Sha Zhong, Xingxing Zou and Yacong Zheng

School of Electrical and Electronics Engineering, Shijiazhuang Tiedao University, No.17  
Northeast, Second Inner Ring, Shijiazhuang, China

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### ABSTRACT

Image segmentation algorithm based on ESFCM (Edge-based Semi-Fuzzy C-means Clustering Method) is a hotspot in the domain of image processing. But the original image segmentation algorithm based on ESFCM cannot meet the requirements in real time image detection, due to its heavy computational effort to work out the spatial distance between an edge pixel with every clustering center. In order to reduce the computation complexity, an improved algorithm is put forward based on a new spatial distance, in which fuzzy distance is used to replace the physical distance. The experiment results show that not only the calculation amount but also the parameters convergence rate of the improved ESFCM have been drastically decreased after the redefinition of spatial distance. In other words, the computation complexity of the improved algorithm has been reduced much more significantly than that of the original one

**Key words:** Algorithm computation complexity; image segmentation; Edge-based semi-fuzzy c-means cluster; spatial distance; Fuzzy distance

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### INTRODUCTION

Image segmentation based on cluster analysis method is extremely important and useful, in which the fuzzy C-means algorithm (Fuzzy C-Means, referred to as FCM) is most widely used in practice. FCM algorithm was put forward by Dunn [1] firstly and then improved by Bezdek [2] [3]. FCM iterative optimization algorithm that based on the principle of least square was given in the literature [1] and [2] and its convergence was proved in literature [3].

The calculation amount of general FCM algorithm is large because the membership degree of every pixel to each subclass needs to be calculated. In order to reduce the calculation amount, Zhang aihua has put forward an algorithm of ESFCM (Edge-based Semi-Fuzzy C-means Clustering Method)[4] under the guide of the edge information, which is named as original algorithm in the paper. In the original algorithm, an image was firstly divided into two parts: one part is the edge pixels gained by the edge detection algorithm, the other part is the subclass domains that circled by the edge pixels. During the course of fuzzy iteration, only the membership degree of every edge pixel to each subclass domain was necessary to be calculated. This is the main reason for the calculation amount of the original algorithm based on ESFCM has been drastically reduced comparing with that of the original FCM algorithm. Despite all this, the algorithm computation complexity of the original ESFCM is still too heavy to be used in practice.

In order to reduce the computation complexity of the original ESFCM, this article has proposed an improved ESFCM based on the new spatial distance, in which the original physical distance was replaced by the fuzzy distance. The comparison of simulation results show that not only the calculation amount but also the parameters convergence rate of the improved ESFCM have been drastically reduced, comparing with that of the original one.

### Principle of original image segmentation algorithm based on ESFCM

Assuming  $F(x, y)$  is the grayscale images to be processed,  $E(x, y)$  is the binary image after edge detection. According to the spatial location of the correspondence between pixels, all the pixels in grayscale image can be divided into two sets according to the binary image  $E(x, y)$ , that is  $F(x, y) = F_0 + F_1$ , where the space position of  $F_0$  is corresponded to all non-edge points where  $E(x, y) = 0$ , the space position of  $F_1$  is corresponded to all non-edge points when  $E(x, y) = 1$ . The space position of  $F_1$  is corresponded to all edge points when  $F_1 = \{f_k | k=1, 2 \dots n_1\}$ , where  $n_1$  represents the number of edge pixels,  $f_k$  represents the gray of the pixels of the  $k^{\text{th}}$  edge point. In the grayscale image, non-edge parts of  $F_0$  are divided into mutual non-connected region by edge part  $F_1$ . That is  $F_0 = X_1 \cup X_2 \cup \dots \cup X_c$ , so  $X_1, X_2, \dots, X_c$  is the initial subclass of the ESFCM segmentation algorithm, and the average gray value of each sub-class is  $V^{(0)} = (v_1^{(0)}, v_2^{(0)}, \dots, v_c^{(0)})$ . Several steps are needed in original ESFCM segmentation algorithm, which is shown in Fig.1.

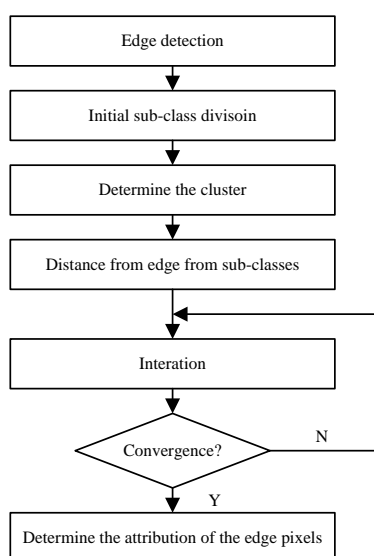


Fig.1 Flow chart of the original ESFCM algorithm

The original gray image is shown in Fig.2 (a). In original algorithm, edge detection is carried out with Sobel method [5] firstly. Then, an edge connection algorithm that based on active growth method and the close operation of mathematical morphology [6] is executed to connect the broken edge curves. During the connecting process of two edge curves, the endpoint position and direction of one edge curve was detected firstly. Then, an algorithm of first growth and degradation that is similar to close operation of the mathematical morphology is executed to connect the edge curve with the other one [7]. The subclass partition result is shown in Fig.2 (b).



(a)Original image

(b) Two edge fractures

Fig.2 Initial subclass partition after edge detection

### Computation complexity analysis of the original algorithm

Compared with original FCM, the calculation amount and iteration rate of the original ESFCM algorithm have been significantly improved, but its computation complexity is still too heavy to meet the real time applications.

### 1. Original algorithm principle

In the original ESFCM algorithm, an objective function is defined as followed:

$$J(U, V) = \sum_{k=1}^{n_1} \sum_{i=1}^c (u_{ik}^{(l)})^m d_{ik}^2 \quad (1)$$

In this formula,

$$d_{ik}^2 = (d_{ik}^G)^2 + D(f_k, X_i)^2 \quad (2)$$

$$d_{ik}^G = |f_k - v_i^{(l)}| \quad (3)$$

$$D(f_k, X_i) = \text{Min}\{d(f_k, p)\} \quad (4)$$

$$d(f_k, p) = \sqrt{(x_k - x_p)^2 + (y_k - y_p)^2} \quad (5)$$

$$v_i^{(l)} = \frac{\sum_{f \in X_i} f + \sum_{f_k \in F_1} (u_{ik}^{(l)})^m f_k}{\sum_{f \in X_i} 1 + \sum_{f_k \in F_1} (u_{ik}^{(l)})^m} \quad (6)$$

It is assumed that there are  $n_1$  edge pixels abstracted and its grayscale value is denoted as  $f_k$ . All of the pixels except edge ones in the image are encircled as  $c$  domains:  $X_1 \cup X_2 \cup \dots \cup X_c$ .  $X_i$  is one of the initial subclasses used in the ESFCM algorithm. In formula (1),  $J(U, V)$  is the square sum of the weighted distance from the pixels of each subclass to the cluster center.  $J(U, V)$  is an objective function and its value reflects the degree of consistency and compaction in  $X_i$ . Smaller the  $J(U, V)$  is, more compact the cluster and higher the quality division is. In formula (3),  $d_{ik}^G$  is defined as the gray difference between  $f_k$  and  $v_i^{(l)}$ , which is the grey value of the center pixel inside  $X_i$  at  $l^{\text{th}}$  iteration. The computing formula of  $v_i^{(l)}$  is shown in formula (6). In formula (4),  $D(f_k, X_i)$  presents the minimum physical distance from  $f_k$  to  $X_i$  and  $d(f_k, p)$  is the spatial distance from  $f_k$  to  $p$ , which is a non-edge pixel inside  $X_i$ . In formula (5),  $(x_k, y_k)$  and  $(x_p, y_p)$  are the coordinates of  $f_k$  and  $p$  respectively.

$[U_{ik}^{(l)}]$  is a membership matrix with  $c$  lines and  $n_1$  columns. Lines number of  $c$  is corresponding to the initial subclasses of  $X_1, X_2, \dots, X_c$  and  $n_1$  is the number of the edge pixel of  $f_k$ . The matrix element of  $u_{ik}^{(l)}$ , whose computing formula is shown in formula (7), is the membership degree of  $f_k$  belonging to  $X_i$  and it meets the constraints of

$$\forall 1 \leq k \leq n_1, \quad \sum_{i=1}^c u_{ik}^{(l)} = 1.$$

### 2. Computation complexity analysis

The principle of original algorithm can be described as follows: every edge pixel of  $f_k$  should be redistributed to a most similar subclass of  $X_i$  according to the iterative results. Four detailed iterative steps were shown as follows:

(1) An original image is segmented into  $c$  disjoint domains of  $X_1^{(0)}, X_2^{(0)}, \dots, X_c^{(0)}$  and all edge pixels of  $f_k$  are distributed into one of the subclasses  $X_i^{(0)}$ .

(2) The matrix of  $U^{(0)}$  is initialized according to the following rules:  $u_{ik}^{(0)}=1$  as  $f_k \in X_i^{(0)}$ , otherwise  $u_{ik}^{(0)}=0$ . Its iterative calculation is executed according to formula (7), among which  $I_k = \{i \mid 1 \leq i \leq c; d_{ik} = 0\}$  and  $\#I_k$  is the set cardinal of  $I_k$ .

$$u_{ik}^{(l)} = \begin{cases} 1 / \sum_{j=1}^c (d_{ik} / d_{jk})^{2/(m-1)} & f_k \in F_1 \text{ and } I_k = \varnothing \\ 1 / \#I_k & f_k \in F_1, I_k \neq \varnothing \text{ and } i \in I_k \\ 0 & \text{others} \end{cases} \quad (7)$$

(3) The iteration process of membership matrix  $U^{(l)}$  will be stopped until its change is very small, viz,  $\|U^{(l)} - U^{(l+1)}\| < \varepsilon_L$  ( $\varepsilon_L$  is a positive number that is small enough and in this article  $\varepsilon_L = 2e-004$ ). Otherwise, the next random iteration of  $l=l+1$  will be continued.

(4) After the iterative procedure was terminated at last, formula (8) is used to judge which subclass the edge pixel of

$f_k$  should be redistributed to in the end.

$$i_k = \underset{1 \leq i \leq c}{\text{Arg}}(\max[u_{ik}^{(l)}]) \quad (8)$$

$u_{ik}^{(l)}$  is the element of the membership matrix of  $U^{(l)}$  in the last rand iteration, in which  $i=1,2,\dots, c$ ;  $k=1,2,\dots, K$ .  $\max_i(u_{ik}^{(l)})$  is the biggest element in  $k^{\text{th}}$  column of  $U^{(l)}$  and its row number is denoted as  $i_k$ . After the iterative procedure, every edge pixel of  $f_k$  in the original image will be totally redistributed to the subclass of  $X_{(i_k)}$ , which is most similar with the pixel.

According to the original ESFCM algorithm, the image segment result is shown in Fig.2 (b). Every initial subclass is surrounded by the edge pixels. The spatial distance from  $f_k$  to  $X_i$  was calculated according to the formula (2) (3) (4) and (5). In formula (4), the physical distance from every pixel of  $p$  inside  $X_i$  to the edge points of  $f_k$  needs to be worked out. So, the calculation amount of spatial distance is still quite large the computation complexity of the original algorithm should be further reduced to meet real-time requirements.

### Principle of the improved algorithm based on ESFCM

In order to further decrease the computation complexity of original algorithm, this paper puts forward a concept of fuzzy distance to replace the physical distance that is defined as formula (4). As shown in formula (9), the fuzzy distance of  $d_{ik}^S$  is defined as the difference between an edge pixel of  $f_k$  with a subclass center pixel of  $v_i$ . The fuzzy distance and the original grayscale difference constitute a new spatial distance. The new spatial distance can significantly decrease not only the calculation amount but also the parameters convergence rate.

$$d_{ik}^S = 1 - \frac{\sum_{f_k \in \eta_k} u_{ik}}{\sum_{i=1}^c \sum_{f_k \in \eta_k} u_{ik}} \quad (9)$$

In formula (9),  $\eta_k$  is a set including eight circumjacent pixels around  $f_k$ ,  $u_{ik}$  is the membership degree [8] that  $f_k$  affiliated to the subclass of  $X_i$ . The bigger  $\sum_{f_k \in \eta_k} u_{ik}$  is the smaller  $d_{ik}^S$  is, otherwise the bigger  $d_{ik}^S$  is.

After the fuzzy distance of  $d_{ik}^S$  is worked out, an improved spatial distance of  $d_{ik}$  between  $f_k$  and  $X_i$  is gained according to formula (10), in which  $d_{ik}^G$  is not change as that of original algorithm in formula (3)

$$d_{ik}^2 = (d_{ik}^G)^2 + (d_{ik}^S)^2 \quad (10)$$



(a) Using original algorithm (b) Using improved algorithm

**Fig.3 Comparison of segmentation results between the original algorithm and the improved one**

Using of the original ESFCM algorithm and the improved one respectively, the original image of Fig.2 (a) is segmented and the results are shown in Fig.3. It can be seen that the district consecution and the segmentation

accuracy of each subclass are much better with the improved spatial distance replaced by the original one. In other words, the improved algorithm can not only reduce the algorithm computation complexity but also increase the image segment accuracy at the same time.

### Computation complexity comparisons between improved algorithm and original one

#### 1. Comparison of the algorithm calculation amount

Comparing formula (2) with (10), a conclusion can be drawn that the decrease of calculation amount in ESFCM is absolutely caused by the calculation amount difference between the fuzzy distance and the physical one. The execution numbers of difference mathematical operator (Add/Subtraction, Multiplication/Division, Evolution) can be used to evaluate the calculation amount quantitatively. In Table 1, the respective evaluation formulas are given and their application results to Fig.2 (a) are shown detailed.

There are 59200 pixels, 11594 edge pixels, 110 subclasses in Fig.2 (a), that is, total pixels number is  $n=59200$ , extracted edge pixels number is  $n_1=11594$  and the subclasses number is  $c=110$ . The fuzzy control coefficient of  $m$  in formula (1) is assigned as 2 in the light of empirical evidence.

According to the ratio value in Table 1, a conclusion can be drew that calculation amount of the fuzzy distance is drastically decreased compared with that of the physical one. It may also be further said that the calculation amount of improved algorithm is drastically decreased compared with that of the original one.

**Table 1. Calculation amount comparisons between fuzzy distance and physical distance**

Arithmetic type	Physical distance(a)	Fuzzy distance(b)	Ratio(b/a)
Add/Subtraction	$3 \times n_1 \times (n - n_1) = 1655831892$	$8 \times n_1 \times c = 10202720$	0.62%
Multiplication/Division	$2 \times n_1 \times (n - n_1) = 1103887928$	1	0.00000009%
Evolution	$n_1 \times (n - n_1) = 551943964$	0	0

#### 2. Comparison of the algorithm convergence rate

Convergence rate is another extremely important factor to evaluate the calculation amount of iterative algorithm. Increasing of the convergence rate implies decreasing of the calculation amount. As is shown in the chapter above, the end condition of the iteration process is  $abs(U^{(l)} - U^{(l+1)}) < \varepsilon_L$  (in this article  $\varepsilon_L = 2e-004$ ). Table 2 records the values of  $abs(U^{(l)} - U^{(l+1)})$  in each iterative cycle while the Fig.2 (a) is segmented using the original algorithm and the improved one. It takes twelve iterative cycles for original algorithm and six iterative cycles for improved algorithm to satisfy the end condition respectively. It may also be further said that the convergence rate of improved algorithm is much faster compared with the original one.

**Table 2. Parameters Convergence rate comparison between improved algorithm and the original one**

Iteration rounds	Original algorithm	Improved algorithm
1	0.001.	7.2268e-004
2	8.6809e-004	4.3567e-004
3	8.0401e-004	3.3471e-004
4	6.3739e-004	2.5598e-004
5	5.5789e-004	2.0047e-004
6	4.8915e-004	1.6116e-004
7	4.5066e-004	
8	3.9700e-004	
9	3.5820e-004	
10	2.8864e-004	
11	2.2093e-004	
12	1.8692e-004	
13		

## CONCLUSION

According to the computation complexity analysis of the original algorithm above, the physical distance between an edge pixel with every clustering center is the main reason for the heavy computation complexity. Then, an improved algorithm, in which the fuzzy distance is designed to replace the original physical distance to redefine the spatial distance, is put forward in the paper. Experimental results show that the calculation amount and the convergence rate are both reduced significantly in the image segment.

In the paper, two important problems of the improved algorithm had not been studied: one is how to assess the image segmentation accuracy with the application of the improved algorithm, the other is that if the improved

algorithm has an excellent anti-noise ability. So, quantitative assessment of the image segment and the anti-noise performance of the improved algorithm are the following tasks to study.

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