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Productivity analysis and benchmark selection of X mining company by DEA

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ABSTRACT

A comprehensive productivity evaluation index system suitable for mining enterprises is constructed. The production efficiency is calculated with the C^2R model $(D_0)_\varepsilon$; the pure technical efficiency is calculated with the C^2GS^2 model (D_1); the returns to scale are analyzed with the C^2GS^2 model (F_1) and the C^2R model $(H_0)_\varepsilon$; the efficiency benchmark is selected with the SE-DEA model. Finally, pertinent policy recommendations are proposed to the enterprises that don't reach the optimal efficiency according to their input redundancy and output deficiency.

Key words: Data envelopment analysis; super efficiency DEA; production efficiency; pure technical efficiency; returns to scale

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INTRODUCTION

Productivity is an important indicator to measure the developmental level of production systems. It reflects the utilization degree of various kinds of production factors such us resource, environment, capital, technology and energy sources [1]. The study of the productivity of mines has enormous economic and practical significance to fully understand and objectively evaluate the economic growth mode of mines, accelerate the development of mines, and take the road of intensive management [3]. Currently, the extensive economic growth pattern of mining enterprises has not yet a substantial change, technological innovation and productivity improvement are extremely urgent. Mines are essentially multi-input multi-output complex systems, and data envelope analysis (DEA) is especially applicable to evaluate the same type of mines with multiple inputs and multiple outputs. DEA models have the distinctive advantages in evaluating mine productivity, because they have eliminated the scale differences of decision making units (DMUs) by comparing their relative efficiencies of output and inputs [5]. This paper chooses the reasonable numbers of input/output indexes and mines, analyzes these mines' production efficiencies, pure technical efficiencies and scale efficiencies with C^2R and C^2GS^2 models, sorts these mines' with SE-DEA model to select inner benchmark.

PRODUCTIVITY EVALUATION MODELS FOR MINE ENTERPRISE

Data Envelopment Analysis Models

In mine production, input control is much easier than output control, so input oriented DEA models are used to measure the productivity, and output oriented DEA models are used to analysis the returns to scale. Let m, s be the numbers of input indexes and output indexes. There are *n* DMUs, $J = \{1, 2, \dots, n\}$, DMU_k's input index vector is $X_k = (x_{1k}, x_{2k}, \cdots, x_{mk})^T$, its output index vector is $Y_k = (y_{1k}, y_{2k}, \cdots, y_{sk})^T$. The following three models are employed:

(1) Input oriented C^2R model $(D_0)_\varepsilon$

Two issues are concerned in the productivity evaluation on mines: 1) To calculate DMU's production efficiency, is to get the optimal value θ^* of C^2R model (D_0). 2) To decide whether a DMU is DEA efficient is to judge whether the optimal solution of the additive DEA model (G_0) is 0. To solve these two questions by one linear programming, the model $(D_0)_\varepsilon$ of formula (1) are constructed by merging (D_0) and (G_0) with non-Archimedes infinitesimal ε.

In formula (1), λ_1, λ_2, …, λ_n, θ are decision variables, the slack variables $S^- = (s_1^-, s_2^-, \cdots, s_m^-)^T$, $S^+ = (s_1^+, s_2^+, \cdots, s_s^+)^T$, s_i^- and s_j^+ indicate input redundancy of x_{i0} and output deficiency of y_{j0} respectively. In following calculations, ε takes 10^{-10}.

$$
\begin{aligned}
&\min\ a = [\theta - \varepsilon(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+)] \\
&s.t.\ \sum_{j\in J} X_j\lambda_j - \theta X_0 + S^- = 0 \\
&\quad -\sum_{j\in J} Y_j\lambda_j + S^+ = -Y_0 \\
&\quad \lambda_j \geq 0,\ j\in J, S^- \geq 0, S^+ \geq 0
\end{aligned}
\tag{1}
$$

(2) Input oriented C^2GS^2 model (D_1)
Let $DMU_0=(X_0, Y_0)$, the model (D_1) is mainly used to calculate the pure technical efficiency δ^0 of DMU_0.

$$
\begin{aligned}
&\min\ \delta \\
&s.t.\ \sum_{j\in J} X_j\lambda_j - \delta X_0 + S^- = 0 \\
&\quad -\sum_{j\in J} Y_j\lambda_j + S^+ = -Y_0 \\
&\quad \sum_{j\in J} \lambda_j = 1 \\
&\quad \lambda_j \geq 0,\ j\in J, S^- \geq 0, S^+ \geq 0
\end{aligned}
$$
(2)

(3) Output oriented C^2GS^2 model (F_1)
Let $DMU_0=(X_0, Y_0)$, the model (F_1) is mainly used to judge the returns to scale of DMU_0.

$$
\begin{aligned}
&\max\ \alpha \\
&s.t.\ \sum_{j\in J} X_j\lambda_j + S^- = X_0 \\
&\quad -\sum_{j\in J} Y_j\lambda_j + \alpha Y_0 + S^+ = 0 \\
&\quad \sum_{j\in J} \lambda_j = 1 \\
&\quad \lambda_j \geq 0,\ j\in J, S^- \geq 0, S^+ \geq 0
\end{aligned}
$$
(3)

(4) Super efficiency DEA model
The model $(D_0)_\varepsilon$ can sort non-valid DMUs according to their θ values, but unable to compare effective DMUs whose θ are all 1. When there are many input and output variables, there may be a number of efficient DMUs, from which it is difficult to choose benchmark. So the improved DEA model, super efficiency DEA (SE-DEA), can be used to sort all DMUs and select the optimal as efficiency benchmark. To evaluate DMU_0, formula (4) compares DMU_0 with the linear combination of all the other DMUs except DMU_0, while formula (1) includes DMU_0. The optimal value ω (super efficiency) of formula (4) may be greater than 1.[1, 2, 6]

$$\min\ [\omega - \varepsilon(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^-)]$$
$$s.t.\ \sum_{j\in J, j\neq k} X_j\lambda_j - \omega X_0 + S^- = 0$$
$$-\sum_{j\in J, j\neq k} Y_j\lambda_j + S^+ = -Y_0$$
$$\lambda_j \geq 0,\ j\in J, S^- \geq 0, S^+ \geq 0 \tag{4}$$

DEA Models' Efficiencies and Economic Connotations

Productivity evaluation on mines by DEA mainly involves three evaluation factors: production efficiency, pure technical efficiency, and scale efficiency. The production efficiency mainly measures whether the inputs and outputs achieve the best, and whether there are input redundancy and output deficiency. The pure technical efficiency reflects the ability of consuming minimum inputs when the outputs are fixed. The scale efficiency said output changes caused by input changes. The methods of calculating these three evaluation factors are as follows.

(1) Production efficiency

Let $DMU_0=(X_0, Y_0)$, solve the optimal solutions θ^0, S^{0-}, S^{0+} of the model $(D_0)_\varepsilon$, θ^0 is the production efficiency of DMU_0. 1) When $\theta^0=1$, $S^{0-}=0$ and $S^{0+}=0$, DMU_0 is effective both in technology and in scale. In the system of n DMUs, resources X_0 are made full use of, and the output Y_0 has reached the optimum. 2) When $\theta^0=1$, and there at least exist $s_i^{0-}>0$ or $s_r^{0+}>0$, DMU_0 is weak effective. There are structural problems of DMU_0, input index x_{i0} or output index y_{r0} should be adjusted according to s_i^{0-} or s_r^{0+}. 3) When $\theta^0<1$, there must be $s_i^{0-}>0$ or $s_r^{0+}>0$, neither DMU_0's technology is effective, nor its scale is effective. 4) Let Δx_{i0} is the redundancy of input x_{i0}, Δy_{r0} is the deficiency of output y_{r0}, so the redundancy rate of x_{i0} is

$$\frac{\Delta x_{i0}}{x_{i0}} = \frac{(1-\theta^0)x_{i0} + s_i^{0-}}{x_{i0}}, i = 1, 2, ..., m \tag{5}$$

The deficiency rate of y_{r0} is

$$\frac{\Delta y_{r0}}{y_{r0}} = \frac{s_r^{0+}}{y_{r0}},\ \ r = 1,\ 2,\ ...,\ s \tag{6}$$

To make DMU_0 effective, the input x_{i0} should be reduced Δx_{i0}, and the output y_{r0} should be increased Δy_{r0}.

(2) Technical efficiency and scale efficiency

Let $DMU_0=(X_0, Y_0)$, the optimal solution δ^0 of the model (D_1) is the pure technical efficiency of DMU_0 [7]. If $\delta^0=1$ and $S^{0-}=S^{0+}=0$, DMU_0 has the optimum technical efficiency; if $\delta^0=1$ and $S^{0-}\neq 0$ or $S^{0+}\neq 0$, DMU_0 is weak effective because of its input redundancies or output deficiencies; if $\delta^0<1$, DMU_0 is inefficient, all its inputs can be compressed by δ^0. The scale efficiency σ^0 of DMU_0 can be got by θ^0 and δ^0 as follows

$$\sigma^0 = \theta^0/\delta^0 \tag{7}$$

If $\sigma^0=1$, DMU_0 gets constant returns to its scale; if $\sigma^0<1$, DMU_0 has either increasing or decreasing returns to its scale.

(3) Returns to scale

Let $DMU_0=(X_0, Y_0)$, the way to determine the returns to scale of DMU_0 is as follows: 1) Solve the optimal solution α_0 of the model (F_1) for (X_0, Y_0). $\alpha_0\geq 1$ and $(X_0, \alpha_0 Y_0)$ is weak efficient. 2) Solve the C^2R model $(H_0)_\varepsilon$ for $(X_0, \alpha_0 Y_0)$,

$$\max\ [\alpha + \varepsilon(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+)]$$
$$s.t.\ \sum_{j\in J} X_j\lambda_j + S^- = X_0$$
$$-\sum_{j\in J} Y_j\lambda_j + S^+ = -\alpha(\alpha_0 Y_0)$$
$$\lambda_j \geq 0,\ j\in J, S^- \geq 0, S^+ \geq 0 \tag{8}$$

Get the optimal solutions α^* and $\lambda_j^0 (j \in J)$, if α^*=1, the returns to scale of DMU_0 is unchanging; if α^*>1 and $\sum_{j\in J} \lambda_j^0 > 1$, DMU_0 gets decreasing returns to its scale, it's production scale may be reduced; if α^*>1 and $\sum_{j\in J} \lambda_j^0 < 1$, DMU_0 gets decreasing returns to its scale, it's production scale may be expanded.

(4) Super efficiency
Let $DMU_0=(X_0, Y_0)$, the optimal solution ω^0 of formula (4) is the super efficiency of DMU_0. If DMU_0 are ineffective, its super efficiency ω^0 is equal to its production efficiency θ^0 of formula (1), else if DMU_0 are effective, e.g. ω^0=1.5 means that DMU_0 remains relatively effective in all DMUs even if its inputs are increased by 50%. The greater ω^0 is, the more stable DMU_0 is. DMUs can be sorted according to their super efficiencies, the DMU with the maximum super efficiency is chosen to be benchmark.

INDEX SYSTEM FOR PRODUCTIVITY EVALUATION ON MINES

Index selection is one of the key steps of comprehensive evaluation. Because of the complexity of evaluated objects, multiple indexes are chosen for evaluation. However, it is not always true that more indexes are better for evaluation. Too many indicators may be repeated, and will interfere with each other. The variables are more, the calculation is more complicated; Too little index may miss some important factors representative of the evaluated object.

Mine's total stripping reflects its output level. This paper analyses the utilization efficiency of various inputs resources with respect to the total stripping. So the 5 indexes of power consumption, mining procedure net energy consumption, diesel consumption, explosives consumption and average number of mining employees are chosen to be DMU's input indexes, because they are strongly correlative with the number of borers, number of excavators, number of motor vehicles, etc. Productivity evaluation index system suitable for mining enterprise is constructed as Table 1.

Table 1 Productivity evaluation index system for mining enterprise

Index type	Index numbers	Technical and economic indexes
Input	1	The average number of employees in the mining
	2	Power consumption (KWh)
	3	Mining procedure net energy consumption (Kgce)
	4	Explosives consumption (kg)
	5	Diesel consumption (kg)
Output	1	Total stripping (Ten thousand tons)

PRODUCTIVITY EVALUATION AND BENCHMARK SELECTION OF X MINING COMPANY

Selecting Samples

The original data of productivity evaluation are derived from the production data of 5 mines of X Mining Company in 2012. DEA models demand that the sample amount is not less than two times of the sum of input-output indexes [6]. Six input-output indexes in table 1 and five mine samples in a month, do not meet the requirements of DEA models. To solve this problem, two solutions are put forward:

(1) Horizontal expansion. To objectively evaluate the productivity of each mine of X Mining Company, a nationwide comparison is also needed. Other mining company's mine samples can be chosen, so both the relative ranking of 5 mines of X Mining Company and the comparison of X Mining Company and other mining enterprises can be obtained. However, due to the difficulty of obtaining the data of other mining enterprise, this paper chooses the following method (2).

(2) Vertical expansion. 5 mines of X Mining Company use the production data from July to December in 2012, the total of samples are 30 meeting the requirements of DEA models. In the evaluation of 30 samples, both the development trends of each mine in 6 months can be analyzed, and the relative ranking of 5 mines in a certain month can be obtained.

Table 2 Productivity evaluation results of X Mining Company from July to December in 2012

DMU	Ranking	Super efficiency	Production efficiency	Technical efficiency	Scale efficiency	Returns to scale
Mine1 11	13	0.9196	0.9196	1	0.9196	Increasing
Mine1 12	14	0.8802	0.8802	1	0.8802	Increasing
Mine2 11	25	0.6110	0.6110	0.7262	0.8413	Increasing

Mine2 12	17	0.7464	0.7464	0.8339	0.8950	Increasing
Mine3 11	16	0.8449	0.8449	0.9753	0.8663	Increasing
Mine3 12	18	0.7356	0.7356	0.9569	0.7687	Increasing
Mine4 09	1	27.9073	1	1	1	Constant
Mine4 11	3	1.0847	1	1	1	Constant
Mine4 12	12	0.9252	0.9252	0.9403	0.9839	Increasing
Mine5 11	30	0.5079	0.5079	0.6080	0.8354	Increasing
Mine5 12	28	0.5381	0.5381	0.6345	0.8480	Increasing

Calculation and Analysis

There are total 30 DMUs of 5 mines in 6 months from July to December in 2012. According to formulae (1) ~ (8), taking the indexes in Table 1 as the inputs and outputs of every DEA model, the productivity comparison results of 30 DMUs are shown in Table 2~4. Formulas (1) ~ (8) can be calculated with the statistical analysis software DEAP2.1. Due to the limited space, only the data of November and December are listed in Table 2 and Table 4.

(1) Production Efficiency: In table 2, Six DMUs of Mine3 in July. and Sept., Mine4 in July., Aug., Sept., and Nov. are effective both in scale and in technology, whose production efficiencies, technical efficiencies and scale efficiencies are all 1. The 6 DMUs reach the optimal. From Table 2, the production efficiency line chart of 5 mines in 6 months is available in figure 1. In the second half of 2012, the production efficiencies of Mine1, Mine2, Mine4 and Mine5 are relatively stable, while Mine3 had a relatively violent fluctuation of production efficiencies. The monthly average production efficiencies of 5 mines in 6 months of 2012 see Table 3.

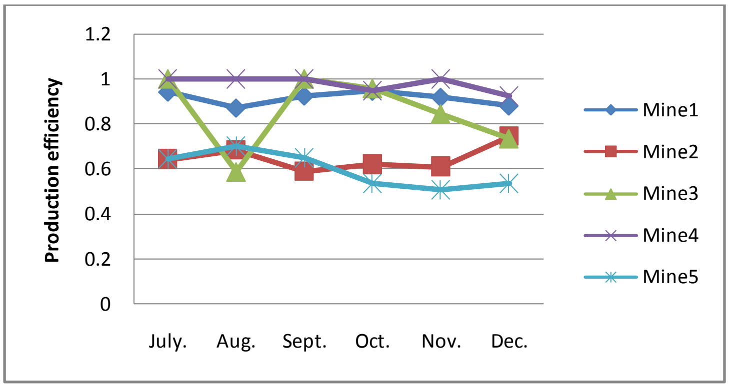

Figure 1 Production efficiencies of 5 mines in the second half of 2012

Table 3 The monthly average efficiencies of 5 mines in the second half of 2012

Mines	Mine1	Mine2	Mine3	Mine4	Mine5
The monthly average production efficiency	0.9144	0.6499	0.8545	0.9787	0.5970
The monthly average technical efficiency	1	0.7547	0.9843	0.9901	0.6767
The monthly average scale efficiency	0.9144	0.8600	0.8663	0.9885	0.8794

In Table 4, 24 ineffective DMUs (C^2R) all have input redundancies, namely excessive consumption of resources. E.g. the production efficiencies of Mine1 in 6 months are all less than 1, mainly because Mine1's excessive consumptions of mining procedure net energy, explosives, and diesel. E.g. Mine1 consumed explosives as much as 72.13% more in Dec.

Table 4 Input redundancy rates and output deficiency rates of X Mining Company from July to December in 2012

DMU	Production efficiency	Input redundancy rate					Output deficiency rate
		S_1^-	S_2^-	S_3^-	S_4^-	S_5^-	S_1^+
Mine1 11	0.9196	0	0.4435	0.4346	0.6049	0.5230	0
Mine1 12	0.8802	0	0.3719	0.4709	0.7213	0.4589	0
Mine2 11	0.6110	0	0	0.2066	0.3229	0.3462	0
Mine2 12	0.7464	0	0	0.2994	0.3764	0.4286	0
Mine3 11	0.8449	0	0.0085	0.1893	0.0618	0	0
Mine3 12	0.7356	0	0	0.2021	0.1805	0	0
Mine4 11	1	0	0	0	0	0	0
Mine4 12	0.9252	0.0358	0.1765	0	0.0043	0	0
Mine5 11	0.5079	0	0	0.3671	0.2731	0.0704	0
Mine5 12	0.5381	0	0	0.4017	0.3245	0.1742	0

(2) Pure technical efficiency: In table 2, Fourteen DMUs of Mine1 in July. ~ Dec., Mine3 in July, Sept. and Oct. and

Mine4 in July. ~ Nov. are technical effective, namely their production capacity and management level reached the best. From Table 2, the technical efficiency line chart of 5 mines in 6 months is available in figure 2. In the second half of 2012, the technical efficiencies of 5 mines are relatively stable. Mine1, Mine4 and Mine3 have higher pure technical efficiencies than Mine2 and Mine5. The monthly average technical efficiencies of 5 mines in 6 months see Table 3.

(3) Scale efficiency: In table 2, Six DMUs of Mine3 in July. and Sept., Mine4 in July., Aug., Sept., and Nov. are scale effective. Of the 24 scale ineffective DMUs, only the returns to scale of Mine5 in Aug. are decreasing, other DMUs' returns to scale are increasing, their input scales can be expanded to improve their scale efficiencies, then production efficiencies. From Table 2, the scale efficiency line chart of 5 mines in 6 months is available in figure 3. In the second half of 2012, the scale efficiencies of Mine1, Mine2, Mine4 and Mine5 are relatively stable, only Mine3 had a relatively violent fluctuation of scale efficiencies. The monthly average scale efficiencies of 5 mines in 6 months see Table 3.

(4) Super efficiency: All DMUs are sorted according to their super efficiencies as their ranking in Table 2. The super efficiency of Mine4 in September is the maximum, so the technical and economic indexes of Mine4 in September are chosen to be the productivity benchmark.

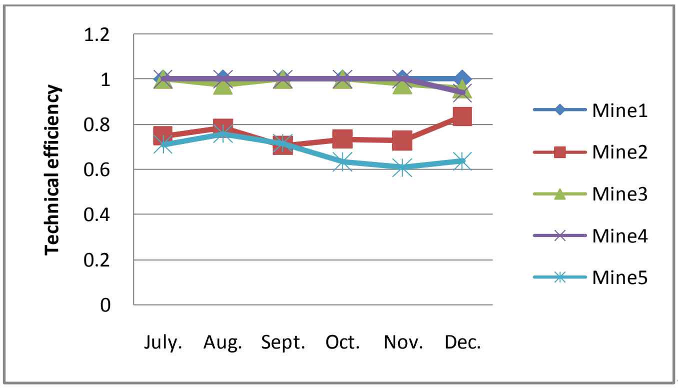

Figure 2 Technical efficiencies of 5 mines in the second half of 2012

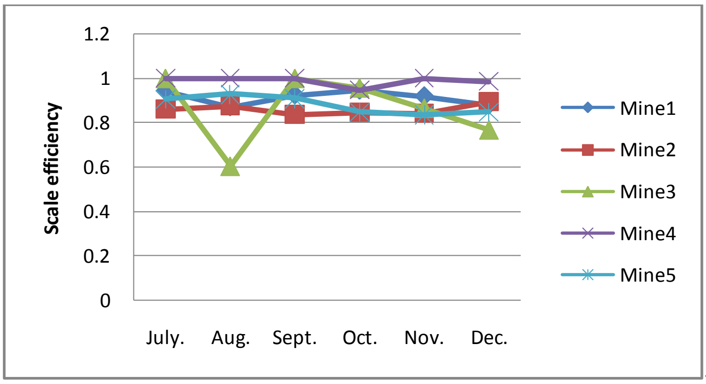

Figure 3 Scale efficiencies of 5 mines in the second half of 2012

CONCLUSION

Quantitative evaluation on productivity is scientific and objective avoiding some subjective factors. The evaluation results indicate that the productivities of most mines are acceptable, but there are large individual differences of the 5 mines, some mines still have much room for growth. To improve the productivity of mining enterprises, effective measures can be taken to enhance innovation, improve the technical, expand production scale and reduce energy consumption.

However, DEA is only a method of mutual comparison, can't reflect all problems of mining enterprises. There are limitations in selecting indexes because of the availability of data, some problems cannot be fully reflected, e.g. whether or not the location of mining enterprise is good, and whether or not the mineral of mining enterprise is abundant. The evaluation index system of this paper is representative, but not perfect because of the constraints of statistical data and model computing, which is needed to be improved in the future. Mine productivity evaluation has just started, and there are still not perfect evaluation models and evaluation methods. It's suggested to evaluate productivity with multiple models and methods from multiple angles, to make evaluation as objective as possible.

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