Available online www.jocpr.com

Journal of Chemical and Pharmaceutical Research, 2014, 6(5):1615-1620

Research Article

ISSN : 0975-7384
CODEN(USA) : JCPRC5

Pricing research for loans to small and medium-sized enterprises based on credit metrics model

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ABSTRACT

Pricing of guarantee fee reasonable is the key to solve the problem of small and medium-sized enterprises credit. This paper considers the influence of the macro-economic and the micro-factors to the Credit Metrics Model. Then adjust the credit transfer matrix of the enterprise after loan from the bank in order to measure the credit risk of the enterprise and the loss of the guarantee company dynamically. It also gives the method of dynamic pricing .Through the empirical analysis, explaining that the rate of guarantee fee from this method is more reasonable than current using.

Keywords: Credit Guarantee; Credit Metrics Model; Guarantee Rate; Dynamic Pricing

More and more countries carry out financing guarantee business to the small and medium-sized enterprises in order to solve their financing difficulty . China has alredy preliminary explore out a common development way about policy and commercial credit guarantee institutions .But there still exist many problems and insufficiencies credit guarantee system for the small and medium-sized enterprises. For example, most guarantee agencies have small scale and lack of funds compensation and mechanism. Most guarantee agencies have less capacity to defend the risk, especially lack of valid quantify model to solve risk sharing ratio and guarantee rate pricing . If the guarantee fee is high, it will increase the cost of financing of the enterprice. If the guarantee fee is low, the return and risk of the guarantee company is asymmetric. It influence enthusiasm of the credit guarantee institutions seriously. How to measure the enterprices' credit risk and compute the guarantee fee dynamicly is necessary.

1. The Research Status Domestic And Overseas

Since Samuelson[1]put forward the guarantee pricing model, domestic and overseas have carried out the reseach of the guarantee pricing model. Credit Metrics model is a risk management product that can use math to measurement. J. P. Mogan invented Credit Metrics model in 1997. Chan-June Kuo ·Chin-Ming Chen·Chao-Hsien Sung construct guarantee pricing model for the small and medium-sized enterprises and gives a reasonable method to compute the enterprise assets liquidation rate.

In the domestic,Gao Lijun(2011) gives a guarantee pricing model based on the VaR model[2].Chen Xiaohong, Chen Jian(2007) consider about the macro-economic influence of the credit transfer matrix[3]. Then construct a dynamic guarantee pricing method based on the Credit Metrics Model.Hu Yadong,Yang Guanghua[4] in their study about controling commercial bank credit risk gives a method to compute the VaR which though ensure the credit risk of a single loan or the loan portfolio. Qi Haifeng[5] considers about the macro-economic influence of the credit transfer matrix and gives a method to value the long-term value of assets. Zhang Songshan[6] use risk fuzzy comprehensive evaluation to evaluate the systemic risk of an enterprise.

Domestic and overseas are used to applying Credit Metrics Model. But this model not conclude the macro-economic influence of the credit transfer matrix. This paper will conclude the macro-economic and the micro-factors to the Credit Metrics Model. It will obtain the dynamic guarantee pricing method that can more conform to the actual.

2. The Basic Framework of Credit Metrics Model

To caculate the VaR based on Credit Metrics Model

①Aascertain the credit transfer matrix

②Ascertain the credit risk premium and the present value of the loan on each credit rating. This paper use Credit Metrics model to compute the present value.

$$V_p = L \times i + \sum_{m=1}^{n-1} \frac{L \times i}{(1 + r_m + s_m)^i} + \frac{L \times i + L}{(1 + r_n + s_n)^n} \tag{1}$$

In the fomula , $L \times i$ is the enterprise fixed annual interest. r_i is risk-free interest rate. S_m is the credit risk premium in the m year of the p credit rating. V_p is present value of the loan in the m year of the p credit rating.

③To caculate the VaR of this loan.

3. Ascertain the dynamic credit transfer matrix

3.1. ascertain the economic cycle indicator

Set the impact factor z_t of the macro-economic. Z is the average annual transition probability of the matrix[8]. It can be decomposed in two parts: $Z_t = \sqrt{1-r}Y_t + \sqrt{r}z_t$ (2)

Z and Y $_t$ obeys the normal distribution. z $_t$ and Y $_t$ are dependent each other. z_t means the systemic risk and Y $_t$ explains the unsystematic risk. Use P (G, g, z $_t$) explains the average annual credit transition probability.

The formula:

$$P(G, g, z_t) = \Phi\left(\frac{Z_{g+1}^G - \sqrt{r}z_t}{\sqrt{1-r}}\right) - \Phi\left(\frac{Z_g^G - \sqrt{r}z_t}{\sqrt{1-r}}\right) \tag{3}$$

$$\min \sum_{G=CCC}^{AAA} \sum_{g=D}^{AAA} n_{t,G}[\mathrm{P}_t(G, g) - \mathrm{P}(G, g, z_t)]^2 \tag{4}$$

Among them, Z_{g+1}^G and Z_g^G are credit rating transition matrix threshold. Then use formula (3) and (4) to solve z $_t$. P$_t$ (G, g) is the real credit matrix transition probability at time t. n $_{t, g}$ is the number of the enterprises rating of G at time t. If we can get r , then we can use the least square method to solve z $_t$. From the formula (2):

$$Var(\mathrm{Z}) = (1-r)Var(\mathrm{Y}) + rVar(\mathrm{z}) \tag{5}$$

The variance of z , Z and Y is 1. The best value of r can make the variance of z_t is 1. Then do the linear regression of z $_t$. Formula (4) can forecast the factor z $_t$. Then get the credit rating transition matrix in the future.

3.2. Credit Rating Transition Matrix Adjustment From the Systemic Risk of Individual Enterprise

This paper use fuzzy comprehensive evaluation to assessment the systemic risk of individual enterprise:

①Determine the evaluation index set. The first grade assessment indicator is X=(x_1, x_2, x_3, x_4, x_5). The secondary assessment indicator is x $_{k i}$=(x_{k1},x_{k2},…,x $_{k n}$).

②Determine the comment set. Y= (y_1, y_2, y_3, y_4, y_5) is the comment set of five ratings. y_1, y_2 , y_3 ,y_4, y_5 express high security, security, generally, dangerous, very dangerous respectively.

③Determine the weight.[9]. This paper use A.L. Satty scaling to get weight. Construct judgment matrix according to the judge of bank managers and relevant experts. Then get the normalized weight.

④Determine the membership matrix of evaluation. This paper use expert evaluation method.

⑤Comprehensive evaluation. Set the weight of x_{ij} to x_i is a_{ij} , and $A_i = (a_{i1}, a_{i2}, \cdots a_{in})$, $B_i = A_i \otimes R_i$, $\otimes$ is synthetic operator symbol. Then get the membership matrix: $B = A \otimes R$.

⑥Deal with the evaluation result. The elements in set can be quantitative as 100,85,70,55,40.

⑦Adjustment credit rating transition matrix. if the score M>70,the possibility of the enterprise's rating to higher rating will increase $\frac{M-70}{100-70}$. The probability to lower rating will decrease $\frac{M-70}{100-70}$. The probability stay the same level is 1-(sum of transition probability). If M<70,we can do the corresponding process.

4.Construction of Dynamic Guarantee Rate Pricing Model

4.1. Six basic assumptions of guarantee rate pricing model based on VaR

①Suppose reimbursement T times a year. Suppose a corporate has a bank loan for n years.
②Suppose the corporate refund a at the end of each time period.

$$\frac{a}{1+i/T}+\frac{a}{(1+i/T)^2}+\cdots\frac{a}{(1+i/T)^{nT}}=L \tag{6}$$

③Suppose net asset of an enterprise is Y. The debt of this enterprise is M.
④Suppose the capital of the credit guarantee institution is C. The guarantee fund magnification is S and the fund settlement discount rate is c.
⑤Suppose the collateral ratio is b.

4.2. Dynamic Guarantee Rate Pricing Based on VaR Model

When an enterprise's liquidating value can't afford all the debt. The guarantee agencies should compensate.
At the time after the bank provides a loan. If the enterprise pays back a in the end of the first $\frac{1}{T}$ phrase, LaR_1 is:

$$[L(1+\frac{i}{T})^{nT}-a(1+\frac{i}{T})^{nT-1}+M-(Y-VaR+M+L\ (1+\frac{i}{T})^{nT}-a\ (1+\frac{i}{T})^{nT-1})\times c_1]\times\frac{b[L(1+\frac{1}{T})^{nT}-a(1+\frac{i}{T})^{nT-1}]}{M+[L(1+\frac{1}{T})^{nT}-a(1+\frac{i}{T})^{nT-1}]} \tag{7}$$

$\mathbf{C_1}$ meet the formula:

$$[L\ (1+\frac{i}{T})^{nT}-a(1+\frac{i}{T})^{nT-1}](1-P_2)+P_2(Y-Var+M+L\ (1+\frac{i}{T})^{nT}-a(1+\frac{i}{T})^{nT-1})\times c_1=[L-\frac{a}{1+i/T}]\prod_{m=1}^{n}(1+r_m) \tag{8}$$

It based on the theory that borrowers are risk-neutral to compute c[10]. Income of the bank has two parts: the interest of the loan if the enterprise didn't break a contract and the assets after paying off debts when the enterprise bankrupt. These two parts are the risk-free investment of the bank.

The probability surpass the biggest loss is α. Set LaR_1 represent the loss. The probability of loss in $(-\infty, LaR_1)$ is $1-\alpha$ and loss in $(LaR_1, b[L(1+\frac{i}{T})^{nT}-a(1+\frac{i}{T})^{nT-1})$ is α .Suppose probability of loss $[0, b[L(1+\frac{i}{T})^{nT}-a(1+\frac{i}{L})^{nT-1}]$ is λ_1. Then the probability of loss in $(0, LaR_1)$ is $\lambda_1-\alpha$. The probability of guarantee agent loss LaR_1 when need to compensate is $\lambda_1-\alpha$. The λ will change with LaR and the loss distribution. Guarantee fee is: $R_1=R_{11}+R_{12}$ (R_{11} is risk-return, R_{12} is risk-free premium)
Guarantee rate = Guarantee fee/amount guaranteed:

$$e_1=\frac{(\lambda_1-\alpha)\times LaR_1}{b[L(1+\frac{i}{T})^{nT}-a(1+\frac{i}{T})^{nT-1}]\prod_{m=1}^{n}(1+r_m)}+r_1/S$$

In the same way, at the end of the $\frac{1}{T}$ payment phrase $nT-1$.The guarantee rate is:

$$e_{nT-1}=\frac{(\lambda_{nT-1}-\alpha)LaR_{nT-1}}{b[L(1+\frac{i}{T})^{nT}-\sum_{j=1}^{nT-1}a(1+\frac{i}{T})^{nT-j}]\prod_{m=1}^{2}(1+r_m)}+r_n/S \tag{9}$$

5. The Empirical

In early January 2012, the net asset of rating for BB manufacturing enterprise is 10 million. Liabilities are 50 million. This enterprise loans from the ICBC 100 million. The loan interest rate is 6%. The deadline of the loan is one year and pay back same amount of money at the end of each quarter. In early January in 2013 can payback all the loan. Shenyang Tianlin guarantee agent provides the assurance and guarantee rate is 5.2%. Microeconomic indicators should be adjusted every quarter and the unsystematic risk should be adjusted every four months.

5.1. Guarantee rate table in a month without regard to macro-economic and the micro-factors

This enterprise should repay for 116.8 thousand each time. In this paper, we analyze credit ratings for (1999-2011) considering the credit transfer matrix of J.P.Morgan at the same time[11]. Then work out credit transfer matrix .

Table 1. Credit transfer matrix of enterprises in China(%)

rating	AAA	AA	A	BBB	BB	B	CCC	default
AAA	90.32	7.23	1.68	0.77	0	0	0	0
AA	2.72	90.35	5.79	0.88	0.18	0.06	0.02	0
A	0.09	3.29	89.5	5.68	0.85	0.38	0.15	0.06
BBB	0.01	0.23	5.84	88.16	4.3	1.12	0.16	0.18
BB	0.03	0.14	0.57	6.73	81.53	8.78	1.16	1.06
B	0	0.14	0.23	0.45	6.48	81.23	4.27	7.2
CCC	0	0.25	0.36	1.48	2.92	8.24	64.96	21.79
default	0	0	0	0	0	0	0	100

We can compute the credit risk premium from the debt information network. And work out the loan discount rate due to the national debt treasury bonds return rate. As table 2:

Table 2 . Loan discount rate (%)

year	1	2	3	4
Discount rate	3.52	3.91	4.10	4.41

Then get the long-term value distribution of the loan belongs to the enterprise which rating BB at the end of 2012.

Table 3. The long-term value distribution of the loan belongs to the enterprise rating BB

Rating at the end of 2012	Transition probability(%)	long-term value
AAA	0.03	58.94
AA	0.14	58.93
A	0.57	58.91
BBB	6.73	58.86
BB	81.53	58.73
B	8.78	58.56
CCC	1.16	58.26
default	1.06	51.13

The average of the long-term value is $\overline{V} = \sum_{j=1}^{d} p_j V_j$ =58.64. Then set α =5%. VaR [12] is 1.72 at the confidence level of 95%. As the formula (7), work out c_4=0.34. Work out LaR_4=22.68. Suppose that amount guaranteed is three times of net asset. That means $b[L(1+i/T)^8 - \sum_{j=1}^{4} a(1+i/T)^4] = 30$

According to the definition and formula (9), we can work out $\lambda_4 = 11.1\%$. Guarantee fee is e_4=5.1%.

Table 4. Guarantee rate table in a month without regard to macro-economic and the micro-factors

month	1	2	3	4	5	6	7	8	9	10	11	12
Guarantee rate %	5.1	5.1	5.1	5.1	5.1	5.1	5.1	5.1	5.1	5.1	5.1	5.1

5.2.Guarantee rate table in a month considering the macro-economic factors

According to the standard & poor's actual credit transfer matrix in 1998-2011 can ensure the best value of r is 0.0279. Then evaluate predictive value of the macro-economic indicator Z_t.

Table 5. Predictive value of Z_t

T	Start of February in 2012	Start of April in 2012	Start of July in 2012	Start of October in 2012	2013
Z_t	1.3328	1.52635	1.7199	1.91345	2.1070

Use formula $P(G,g,z_t) = \Phi(\frac{Z_{g+1}^{G} - \sqrt{r}z_t}{\sqrt{1-r}}) - \Phi(\frac{Z_g^G - \sqrt{r}z_t}{\sqrt{1-r}})$ can get the four quarters credit transfer matrix in 2012-2013.

$C_{2012} = C(Z_{2012,1})(Z_{2012,4})(Z_{2012,7})(Z_{2012,10})$ ($Z_{2012,1}$ is the Credit transfer matrix in the first quarter)

Table 6. Credit transfer matrix at the end of 2012

Rating	AAA	AA	A	BBB	BB	B	CCC	default
AAA	0.8426	0.1387	0.0163	0.0014	0.0008	0.0002	0	0
AA	0.0416	0.8038	0.1379	0.0134	0.0012	0.0018	0.0003	0
A	0.0095	0.134	0.7908	0.0526	0.0097	0.0029	0.0003	0.0002
BBB	0.0077	0.0204	0.1217	0.7732	0.0628	0.0121	0.0013	0.0008
BB	0.0036	0.0221	0.1174	0.2071	0.5621	0.0612	0.0236	0.0029
B	0.0015	0.0049	0.1411	0.0754	0.2398	0.4899	0.0327	0.0147
CCC	0.006	0.0072	0.0922	0.0641	0.1293	0.2159	0.4555	0.0298

According to the Credit transfer matrix ,work out the guarantee rate table:

Table 7. Guarantee rate table in a month considering the macro-economic factors

month	1	2	3	4	5	6	7	8	9	10	11	12
Guarantee rate %	3.51	3.51	3.51	3.48	3.48	3.48	3.37	3.37	3.37	3.09	3.09	3.09

5.3. Guarantee rate table in a month considering the micro-economic factors

The method of fuzzy comprehensive of evaluation can be used to evaluate the unsystematic risk of the enterprise at the end of 2012. Then use A.L. satty scaling to construct judgment matrix.According to the matrix can conpute the weight coefficient:

$$A_1 = (0.53,0.47) \quad A_2 = (0.68,0.32) \quad A_3 = (0.47,0.32,0.21)$$

$$A_4 = (0.45,0.28,0.17,0.10) \quad A_5 = (0.38,0.32,0.30) \quad A = (0.26,0.31,0.18,0.11,0.14)$$

Ensure membership matrix of expert evaluations, Then make the comprehensive evaluation:

$$B_1 = A_1 \otimes R_1 = (0.094,0.194,0.306,0.253,0.153)$$

$$B_2 = (0.132,0.264,0.336,0.2,0.068) \qquad B_3 = (0.222,0.232,0.308,0.1,0.108)$$

$$B_4 = (0.106,0.256,0.224,0.262,0.152) \qquad B_5 = (0.222,0.232,0.308,0.1,0.108)$$

$$B = A \otimes R = (0.16749,0.18608,0.29728,0.18414,0.08712) \qquad y = BY^{\mathrm{T}} = 68.83$$

It can get three scores 75.8, 72.3, 68.83.The adjusted credit transfer matrix after one year.

Table 8. Credit transfer matrix considering the macro-economic factors and unsystematic risk (at the end of 2012)

Rating	AAA	AA	A	BBB	BB	B	CCC	default
AAA	0.8386	0.1423	0.0167	0.0014	0.0008	0.0002	0	0
AA	0.0405	0.80095	0.1415	0.0137	0.0012	0.0018	0.0003	0
A	0.0093	0.1305	0.7657	0.0540	0.0100	0.0300	0.0003	0.0002
BBB	0.0075	0.0199	0.1185	0.7752	0.0644	0.0124	0.0013	0.0008
BB	0.0035	0.0215	0.1143	0.2017	0.569	0.0628	0.0242	0.0030
B	0.0015	0.0048	0.1374	0.0734	0.2336	0.5006	0.0336	0.0151
CCC	0.0058	0.0070	0.0898	0.0624	0.1259	0.02109	0.6574	0.0306

According to the above matrix, we can get the guarantee rate is 3.33%. Table of the guarantee rate as follows:

Table 9. Guarantee rate table in a month considering the macro-economic factors and unsystematic risk (%)

month	1	2	3	4	5	6	7	8	9	10	11	12
Guarantee rate %	3.48	3.48	3.48	3.45	3.46	3.46	3.35	3.35	3.62	3.33	3.33	3.33

Because of the macro-economic indicators are getting better gradually in the empirical. Guarantee rate in table 7 is below in table 4. Guarantee rate in table 9 concludes the micro-economic factors. In the early four years, the credit of the enterprise is more than 70 points. So the guarantee rate in table is below in table 7. In 5-8 month, though the score of credit is more than 70 points, not exceed in the early four years. So the falling range will below the early four years. In 9-12 month, the credit of the enterprise is lower than 70 points. The guarantee rate in table 9 is higher than that in table 7 in these four months.

In the actual operation, considering the macro-economic factors and unsystematic risk guarantee rate should down Guarantee rate should be down according to the risk and return symmetry principle. Guarantee rate is also very high in reality. It will increase the enterprise financing costs and not conducive to its development. The Empirical improve that Guarantee rate pricing method in this essay is better than traditional method.

PROJECT SUPPORT

The research is a key project of humanities and social sciences of Beijing Education Committee and Beijing Philosophy and Social Science as well under Grant No. sz2012037016 .

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