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Preventive maintenance model analysis on repairable components

Su Hong-Sheng*, Li Juan-Di and Kang Yong-Qiang

School of Automation and Electrical Engineering, Lanzhou Jiaotong University, Lanzhou, China

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ABSTRACT

In order to provide the scientific theoretical basis for component's maintenance and management, state transition diagram of repairable components is established, and state transition equations are achieved based on reliability mathematical theory in this paper. And then its transient process and stability is investigated. The three kinds of mathematical models are respectively established for the passive maintenance, and the preventive maintenance, and as well as the redundant maintenance, and their average maintenance cost rates are calculated for repairable components. The related investigation results show that the maintenance cost rate of preventive maintenance is the lowest, and is the optimal maintenance strategy.

Key words: State equation; transient; stability; preventive maintenance; average maintenance cost rate

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INTRODUCTION

Preventive maintenance is an important mean to improve component reliability. Since Barlow proposed minimal repairing theory, many periodic maintenance models with time-based have been established, and many servicing models to calculate fixing cycle T have already been reported [1-3]. But Khandelwal found it difficult to select the proper maintenance cycle T. For it can cause excessive maintenance so that maintenance cost increases to prolong or shorten the repairing interval[4]. Subsequently, Nguyen, Murthy and Nakagawa proposed a sequential policy where the preventive maintenance was performed in a time interval τ, but the τ varies with the age of the components[5-6]. Hence, the component maintenance models with state-based have been widely investigated since 1980s. In [7] a replacement maintenance strategy was proposed based on Markov decision process. Further, Love developed incomplete maintenance strategy where state space was component age and number of the failures [8]. However, it is difficult for these models to judge components recession correctly. In addition, the related simulation results also show certain errors compared with the actual state of components, and maintenance strategy is not for that optimal. Therefore, in this paper, the component state is divided into three groups: working state, storage state, and maintenance state. On the basis of it, the state equations are established, and its transient process and stability are analyzed. Moreover, the paper also compares repairable component's average maintenance cost rate among passive maintenance model, and preventive maintenance model, and redundant maintenance model. The related investigation results show that the preventive maintenance scheme is best, and redundant maintenance is worst.

1. MODEL DESCRIPTIONS

To establish the life cycle model for repairable component we need to do the following assumptions.

Hypothesis 1 Component maintenance and inspection only use a state to express.

Hypothesis 2 Whether component is in working state or storage state, it is not existed for the failure that can't be detected out.

Hypothesis 3 The probability of from state S_i at time t to state S_j at time $t+\Delta t$ is only proportional to the time interval

Δt, the transfer rate is a constant which does not depend on the time t and Δt.

Hypothesis 4 Maintenance will not change failure rate of the component.

Hypothesis 5 The used failure rate function is the bathtub curve with its first stage and the aging stage being ignored. And so the failure rate is the constant.

Hypothesis 6 Preventive maintenance is perfect, i.e., if there are faults detected out during preventive maintenance, and then it would be able to get a timely repair.

According to the assumptions mentioned above, the state transition diagram of the repairable component can be drawn out as shown in Fig. 1.

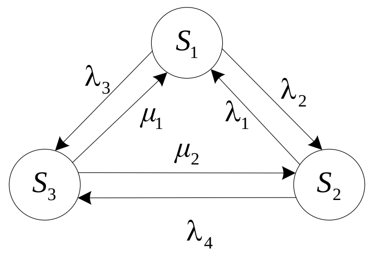


Fig. 1: State transition diagram of repairable component

In Fig.1, S_1 denotes that component is being in storage state, and S_2 denotes that component is in working state, and S_3 denotes the combined state of preventive maintenance and corrective maintenance because preventive maintenance time is considered to be same with corrective maintenance time, here we assume that preventive maintenance can accomplish an eventual substitution before there is an upcoming failure or after a random damage happens, and λ_1 is the transition probability from working state to storage state, and λ_2 is the transition probability from storage state to working state, and $\lambda_3=v_1+u_1$, where v_1 and u_1 denote respectively the failure rate and the checking rate of the stored component, and $\lambda_4=v_2+u_2$, where v_2 and u_2 denote respectively the failure rate and the checking rate of the working component, and μ_1 is the stored probability after one maintenance; μ_2 is the used probability after one maintenance.

2. MATHEMATICAL MODEL ANALYSIS

According to Fig. 1, and reliability theory[9, 10], we can get

$$\left.\begin{aligned}
&\frac{\mathrm{d}x_1(t)}{dt} = -(\lambda_2+\lambda_3)x_1(t)+\lambda_1 x_2(t)+\mu_1 x_3(t)\\
&\frac{\mathrm{d}x_2(t)}{dt} = \lambda_2 x_1(t)-(\lambda_1+\lambda_4)x_2(t)+\mu_2 x_3(t)\\
&\frac{\mathrm{d}x_3(t)}{dt} = \lambda_3 x_1(t)+\lambda_4 x_2(t)-(\mu_1+\mu_2)x_3(t)\\
&x_1(t)+x_2(t)+x_3(t)=1,\ t\ge 0.
\end{aligned}\right\} \tag{1}$$

where $x_1(t)$ denotes the probability of being in the state S_1 at time t, and $x_2(t)$ denotes the probability in S_2, and $x_3(t)$ is the probability S_3. The above differential equations can be written as

$$\mathrm{d}\boldsymbol{x}(t)/\mathrm{d}t = \boldsymbol{A}_1\boldsymbol{x}(t)+\boldsymbol{\mu} \tag{2}$$

$$\boldsymbol{A}_1=\begin{bmatrix}-d_1 & a\\ b & -d_2\end{bmatrix},\quad \boldsymbol{x}(t)=\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix},\quad \boldsymbol{\mu}=\begin{bmatrix}\mu_1\\ \mu_2\end{bmatrix}.\quad \begin{matrix}d_1=\lambda_2+\lambda_3+\mu_1, a=\lambda_1-\mu_1\\ b=\lambda_2-\mu_2, d_2=\lambda_1+\lambda_4+\mu_2\end{matrix}.$$

$$x_1(t)+x_2(t)\le 1,\quad t\ge 0;\quad 0\le\lambda_i<\infty,\quad (i=1,2,3,4);\ 0\le\mu_i<\infty;\quad (i=1,2)$$

According to linear systems theory[11], if the matrix $\boldsymbol{A}_1$ satisfies $d_1d_2-ab>0$, the system (2) would be stable. When t tends to infinity, the steady state value of system (2) can be written as

$$\boldsymbol{x}(\infty) = -\boldsymbol{A}_1^{-1}\boldsymbol{\mu} = \begin{bmatrix} x_1(\infty) \\ x_2(\infty) \end{bmatrix}$$

Thus, the solution of the linear system (2) can be written as

$$\boldsymbol{x}(t) = \exp(\boldsymbol{A}_1\boldsymbol{t})[\boldsymbol{x}(0) - \boldsymbol{x}(\infty)] + \boldsymbol{x}(\infty) \tag{3}$$

Where $\boldsymbol{x}(0)$ is the initial value. From (3), the transient process of $\boldsymbol{x}(t)$ is mainly decided by matrix exponent function $\exp(\boldsymbol{A}_1 t)$. In other words, it is determined by the eigenvalues of $\boldsymbol{A}_1$. The eigenvalues of the matrix $\boldsymbol{A}_1$ can be resolved by

$$s_1 = -(d_1 + d_2)/2 - \sqrt{\Delta}/2$$
$$s_2 = -(d_1 + d_2)/2 + \sqrt{\Delta}/2$$

Where $\sqrt{\Delta} = \sqrt{(d_1 - d_2)^2 + 4ab} = s_2 - s_1$

As $\Delta>0$, the matrix has two different negative eigenvalues. It is easy to known that the $\boldsymbol{A}_1$ has similarity matrix, i.e., there exists an invertible matrix $\boldsymbol{P}$ meeting $\boldsymbol{P}^{-1}\boldsymbol{A}_1\boldsymbol{P} = \boldsymbol{B}$ which is formed by eigenvectors corresponding eigenvalues of $\boldsymbol{A}_1$, and so $\exp(\boldsymbol{A}_1 t)=\boldsymbol{P}\exp(\boldsymbol{B}t)\boldsymbol{P}^{-1}$[12], where $\boldsymbol{B}$ is a diagonal matrix of its diagonal elements being s_1 and s_2. Then we have

$$\exp(\boldsymbol{A}_1\boldsymbol{t}) = \frac{1}{\sqrt{s_2 - s_1}}\begin{bmatrix} (d_1+s_2)e^{s_1 t} - (d_1+s_1)e^{s_2 t} & a(e^{s_2 t} - e^{s_1 t}) \\ b(e^{s_2 t} - e^{s_1 t}) & (d_2+s_2)e^{s_1 t} - (d_2+s_1)e^{s_2 t} \end{bmatrix} = \begin{bmatrix} a_{11}(t) & a_{12}(t) \\ a_{21}(t) & a_{22}(t) \end{bmatrix}$$

The motion track of $a_{12}(t)$ and $a_{21}(t)$ are mainly determined by the e^{s2t}-e^{s1t}. Its motion curves can be shown as Fig.2.

Fig. 2: Motion curves of non-diagonal elements

For $a_{11}(t)$ and $a_{22}(t)$, if $ab>0((d_1-d_2)^2+4ab>0)$, $a_{11}(t)$ and $a_{22}(t)$ are all monotonically decreasing, its motion curves can be shown as Fig.3. Otherwise, if $ab<0(|d_1 - d_2| > 2\sqrt{-ab}$), both of them are not monotonically decreasing, as specified in Tab.1.

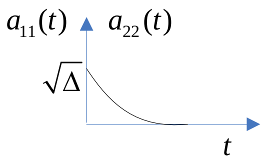
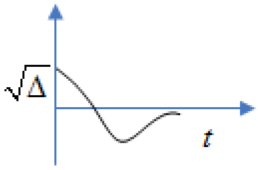
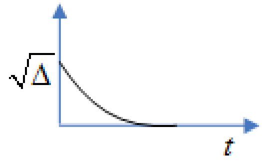
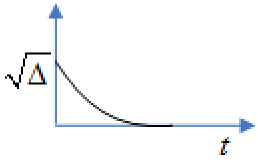
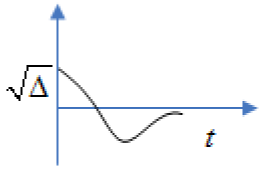


Fig. 3: Motion curves diagonal elements(*ab*>0)

Table 1 Motion curves of diagonal elements(ab<0)

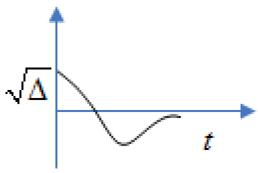
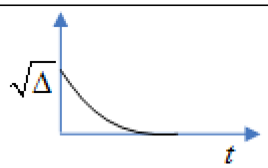
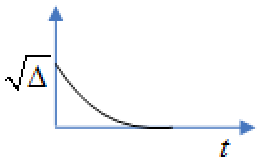
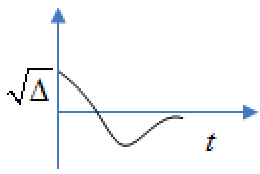
	$a_{11}(t)$	$a_{22}(t)$
$d_1 > d_2 + 2\sqrt{-ab}$		
$d_2 > d_1 + 2\sqrt{-ab}$		

As Δ=0($\left|d_1 - d_2\right| = 2\sqrt{-ab}$), the $\boldsymbol{A}_1$ has two same negative real eigenvalues, that is s=s_1=s_2= $-(d_1 + d_2)/2$. Clearly, $d_1 = d_2 \pm 2\sqrt{-ab}$, and then we have

$$\exp(\boldsymbol{A}_1 t) = \begin{bmatrix} e^{st}[1-(d_1-d_2)t/2] & ate^{st} \\ bte^{st} & e^{st}[1-(d_2-d_1)t/2] \end{bmatrix} = \begin{bmatrix} b_{11}(t) & b_{12}(t) \\ b_{21}(t) & b_{22}(t) \end{bmatrix}$$

For $b_{12}(t)$ and $b_{21}(t)$, their motion curves are mainly determined by the te^{st}, and is similar as Fig.2. For $b_{11}(t)$ and $b_{22}(t)$, its motion curves are specified as the Tab. 2.

Table 2 Motion curves of diagonal elements

	$b_{11}(t)$	$b_{22}(t)$
$d_1 = d_2 + 2\sqrt{-ab}$		
$d_2 = d_1 + 2\sqrt{-ab}$		

As Δ<0($\left|d_1 - d_2\right| < 2\sqrt{-ab}$), the matrix's eigenvalues are two conjugate imaginary roots, i.e.

$$s_1 = -(d_1 + d_2)/2 - \mathrm{j}\sqrt{-\Delta}/2 = \alpha - \mathrm{j}\beta$$
$$s_2 = -(d_1 + d_2)/2 + \mathrm{j}\sqrt{-\Delta}/2 = \alpha + \mathrm{j}\beta$$

Thus we have exp($\boldsymbol{A}_1$t)= $\boldsymbol{P}_1$ exp($\boldsymbol{B}_1 t$) $\boldsymbol{P}_1^{-1}$.

Where $\boldsymbol{P}_1 = \dfrac{2}{a\sqrt{-\Delta}} \begin{bmatrix} 0 & a \\ -\sqrt{-\Delta}/2 & (d_1 + d_2)/2 \end{bmatrix}$, $\boldsymbol{B}_1 = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}$, $\exp(\boldsymbol{B}_1 t) = e^{\alpha t} \begin{bmatrix} \cos\beta t & \sin\beta t \\ -\sin\beta t & \cos\beta t \end{bmatrix}$

And so

$$\exp(\boldsymbol{A}_1 t) = \frac{e^{\alpha t}}{\sqrt{-\Delta}} \begin{bmatrix} \sqrt{-\Delta}\cos\beta t - (d_1 - d_2)\sin\beta t & -2a\sin\beta t \\ -2b\sin\beta t & \sqrt{-\Delta}\cos\beta t + (d_1 - d_2)\sin\beta t \end{bmatrix} = \begin{bmatrix} c_{11}(t) & c_{12}(t) \\ c_{21}(t) & c_{22}(t) \end{bmatrix}$$

The motion track of $c_{12}(t)$ and $c_{21}(t)$ are mainly determined by the $e^{\alpha t}\sin\beta t$. Its motion curves can be shown as Fig.4.

Fig. 4: Motion curves of non-diagonal elements

For $c_{11}(t)$ and $c_{22}(t)$, they can be simplified as

$$c_{11}(t) = \frac{2e^{\alpha t}\sqrt{-ab}}{\sqrt{-\Delta}}\sin(\beta t + c), \quad \tan c = \frac{\sqrt{-\Delta}}{d_2 - d_1}$$

$$c_{22}(t) = \frac{2e^{\alpha t}\sqrt{-ab}}{\sqrt{-\Delta}}\sin(\beta t + c'), \quad \tan c' = \frac{\sqrt{-\Delta}}{d_1 - d_2}$$

Their motion tracks are similar to Fig.4, and the initial phases are different alone.

From what has been discussed above, as long as the eigenvalues of the matrix $\boldsymbol{A}_1$ possess negative real parts, then $\lim_{t\to\infty}\exp(\boldsymbol{A}_1 t) = \boldsymbol{0}$. In other words, the system (3) is stable.

3. STABILITY ANSLYSIS WITH TIME-VARIABLE SYSTEM

For system (2), if $\boldsymbol{A}_1$ is independent of time, and whose eigenvalues have negative real parts, the system is then stable. If $\boldsymbol{A}_1$ relies on time, its stability needs to make study, further. If the checking rates of matrix $\boldsymbol{A}_1$ change with time while others keep invariant, $\boldsymbol{A}_1$ can be then written as

$$\boldsymbol{A}_1 = \boldsymbol{A} + \boldsymbol{U}(t)$$

where $\boldsymbol{A} = \begin{bmatrix} -d_3 & a \\ b & -d_4 \end{bmatrix}$, $\begin{matrix} d_3 = \lambda_2 + v_1 + \mu_1, a = \lambda_1 - \mu_1 \\ b = \lambda_2 - \mu_2, d_4 = \lambda_1 + v_2 + \mu_2 \end{matrix}$. $\boldsymbol{U}(t) = \begin{bmatrix} -u_1(t) & 0 \\ 0 & -u_2(t) \end{bmatrix}$

For conveniently analysis, we first make the following definitions.

(1) The mark $\|\bullet\|$ stands for the norm of vectors or matrices.

(2) e^{At} can be expressed by $\varphi_{ij}(t)$, and $e^{A(t-\tau)}$ can be expressed by $\varphi_{ij}(t\text{-}\tau),\ i,j = 1,2$

Theorem 1. For linear time-variable system

$$\mathrm{d}\boldsymbol{x}(t)/\mathrm{d}t = \left(\boldsymbol{A} + \boldsymbol{U}(t)\right)\boldsymbol{x}(t) \tag{4}$$

having that $\int_0^\infty \left(u_1(t) + u_2(t)\right)\mathrm{d}t < \infty$. If the system $\mathrm{d}\boldsymbol{x}(t)/\mathrm{d}t = \boldsymbol{A}\boldsymbol{x}(t)$ is stable, the system (4) is then stable[13], and if $\mathrm{d}\boldsymbol{x}(t)/\mathrm{d}t=\boldsymbol{A}\boldsymbol{x}(t)$ is asymptotically stable, system (4) is asymptotically stable.

Proof. The solution of system (4) is

$$\boldsymbol{x}(t)=e^{\boldsymbol{A}(t-t_0)}\boldsymbol{x}_0+\int_{t_0}^{t}e^{\boldsymbol{A}(t-\tau)}\boldsymbol{U}(\tau)\boldsymbol{x}(\tau)d\tau$$

Hence, we have

$$\|\boldsymbol{x}(t)\|\le\left\|e^{\boldsymbol{A}(t-t_0)}\boldsymbol{x}_0\right\|+\int_{t_0}^{t}\left(\sum_{i、j=1}^{2}\left|\varphi_{ij}(t-\tau)\right|\right)\left(u_1(\tau)+u_2(\tau)\right)\|\boldsymbol{x}(\tau)\|d\tau \tag{5}$$

Since $\mathrm{d}\boldsymbol{x}(t)/\mathrm{d}t=\boldsymbol{A}\boldsymbol{x}(t)$ is stable, let $\left\|e^{\boldsymbol{A}(t-t_0)}\boldsymbol{x}_0\right\|\le K$, we have $\sum_{i、j=1}^{2}\left|\varphi_{ij}(t-\tau)\right|\le L\ \left(t_0\le\tau\le t<\infty\right)$. Let M=max(K, L), so we have

$$\|\boldsymbol{x}(t)\|\le M+M\int_{t_0}^{t}\left(u_1(\tau)+u_2(\tau)\right)\|\boldsymbol{x}(\tau)\|d\tau \tag{6}$$

Let $u(\tau)=u_1(\tau)+u_2(\tau)$, then substituting it into (6) , we have

$$\frac{\|\boldsymbol{x}(t)\|u(t)}{1+\int_{t_0}^{t}u(\tau)\|\boldsymbol{x}(\tau)\|d\tau}\le Mu(t)$$

Then integrating from t_0 to t, we have

$$\ln\left(1+\int_{t_0}^{t}u(\tau)\|\boldsymbol{x}(\tau)\|\mathrm{d}\tau\right)\le M\int_{t_0}^{t}u(\tau)d\tau \tag{7}$$

And so, $\|\boldsymbol{x}(t)\|$ can be written as

$$\|\boldsymbol{x}(t)\|\le M+M\int_{t_0}^{t}\left|u_1(\tau)+u_2(\tau)\right|\|\boldsymbol{x}(\tau)\|d\tau\le Me^{M\int_{t_0}^{t}u(\tau)d\tau} \tag{8}$$

Hence, if $\int_0^{\infty}\left(u_1(t)+u_2(t)\right)dt<\infty$, then $\|\boldsymbol{x}(t)\|$ is bounded, system (4) is stable. In another hand, if $\mathrm{d}\boldsymbol{x}(t)/\mathrm{d}t=\boldsymbol{A}\boldsymbol{x}(t)$ is asymptotically stable, then $\lim_{t\to\infty}\left\|e^{\boldsymbol{A}t}\boldsymbol{x}_0\right\|=0$, and $\lim_{t\to\infty}M\int_{t_0}^{t}u(\tau)\mathrm{d}\tau=$ $0\times\int_{t_0}^{\infty}u(\tau)\mathrm{d}\tau$ =0 ×bounded function =0. Thus, we obtain $\|\boldsymbol{x}(t)\|\le Me^{M\int_{t_0}^{t}u(\tau)d\tau}=0\times1=0$.

And therefore, system (4) is also asymptotically stable.

4. MAINTENANCE MODEL ANALYSIS

In Fig.1, if u_1=0, u_2=0, then $\lambda_3=v_1$, $\lambda_4=v_2$, and then, the system (4) shows the passive maintenance. Then equation (2) can be written as

$$\mathrm{d}\boldsymbol{x}(t)/\mathrm{d}t=\boldsymbol{A}_{11}\boldsymbol{x}(t)+\boldsymbol{\mu} \tag{9}$$

Where $\boldsymbol{A}_{11} = \begin{bmatrix} -d_{11} & a \\ b & -d_{22} \end{bmatrix}, \boldsymbol{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \boldsymbol{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}.$ $\begin{array}{l} d_{11} = \lambda_2 + v_1 + \mu_1, a = \lambda_1 - \mu_1 \\ b = \lambda_2 - \mu_2, d_{22} = \lambda_1 + v_2 + \mu_2 \end{array}.$

As $t \to \infty$, the steady state value of the system is

$$\left.\begin{array}{l} x_1(\infty) = \dfrac{(\lambda_1 + v_2)\mu_1 + \lambda_1\mu_2}{(\lambda_1+\lambda_2)(\mu_1+\mu_2)+\lambda_1 v_1+\lambda_2 v_2+v_1 v_2+v_1\mu_2+v_2\mu_1} \\ x_2(\infty) = \dfrac{(\lambda_2 + v_1)\mu_2 + \lambda_2\mu_1}{(\lambda_1+\lambda_2)(\mu_1+\mu_2)+\lambda_1 v_1+\lambda_2 v_2+v_1 v_2+v_1\mu_2+v_2\mu_1} \end{array}\right\} \tag{10}$$

When the system being at steady-state, and in range of time T, let the time being in state S_1, S_2 respectively be T_1, T_2, and then

$$T_1 = Tx_1(\infty),\ T_2 = Tx_2(\infty) \tag{11}$$

Let C be a component cost, and C_{m1} is the component's desired storage cost per unit time, and N be the component's expected number of failures on the scope of time T, thus we have

$$N = \int_0^{T_1} v_1 dt + \int_0^{T_2} v_2 dt = T_1 v_1 + T_2 v_2 \tag{12}$$

If a component failure occurs it will be replaced using new one. So average maintenance cost rate during time T can be written as

$$f_1 = \frac{C_{m1}T_1 + CN}{T} = C_{m1}x_1(\infty) + C\left(x_1(\infty)v_1 + x_2(\infty)v_2\right) \tag{13}$$

If $u_1 \neq 0$, $u_2 \neq 0$, $\lambda_3 = u_1 + v_1$, $\lambda_4 = u_2 + v_2$, Fig.1 stands for the preventive maintenance model of component. When $t \to \infty$, the steady state value of the system is

$$\left.\begin{array}{l} x_1(\infty) = \dfrac{(\lambda_1 + \lambda_4)\mu_1 + \lambda_4\mu_2}{(\lambda_1+\lambda_2)(\mu_1+\mu_2)+\lambda_1\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4+\lambda_3\mu_2+\lambda_4\mu_1} \\ x_2(\infty) = \dfrac{(\lambda_2 + \lambda_3)\mu_2 + \lambda_2\mu_1}{(\lambda_1+\lambda_2)(\mu_1+\mu_2)+\lambda_1\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4+\lambda_3\mu_2+\lambda_4\mu_1} \\ x_3(\infty) = \dfrac{\lambda_1\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4}{(\lambda_1+\lambda_2)(\mu_1+\mu_2)+\lambda_1\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4+\lambda_3\mu_2+\lambda_4\mu_1} \end{array}\right\} \tag{14}$$

In the preventive maintenance period, if component is in failure, then we perform minimal repairs (repaired as old). At steady state, during time T, the time being in state S_1, S_2, and S_3 is $T_1^{'}$, $T_2^{'}$, and $T_3^{'}$, respectively. Therefore

$$T_1^{'} = Tx_1(\infty),\ T_2^{'} = Tx_2(\infty),\ T_3^{'} = Tx_3(\infty) \tag{15}$$

Let C_{m2} be the component's desired storage cost per unit time, and C_{r2} is the minimal repair cost for each time(ignoring minimal repairs' repairing time), and C_{f2} be the component's desired preventive maintenance cost per unit time, N'be the component's expected number of failures during time T, thus

$$N^{'} = \int_0^{T_1^{'}} v_1 dt + \int_0^{T_2^{'}} v_2 dt = T_1^{'} v_1 + T_2^{'} v_2 \tag{16}$$

So the average maintenance cost rate in T can be written as

$$f_2 = \frac{C_{m2}T_1^{'} + C_{r2}N^{'} + C_{f2}T_3^{'}}{T} = C_{m2}x_1(\infty) + C_{r2}\left(x_1(\infty)v_1 + x_2(\infty)v_2\right) + C_{f2}x_3(\infty) \tag{17}$$

When there is redundant spare working component, a state S_4 is increased as shown in Fig.5 [14]. At this moment, S_2 represents one component in the working state and the other is spare component. S_4 represents the working component for repairing or entering the warehouse storage, while spare component works.

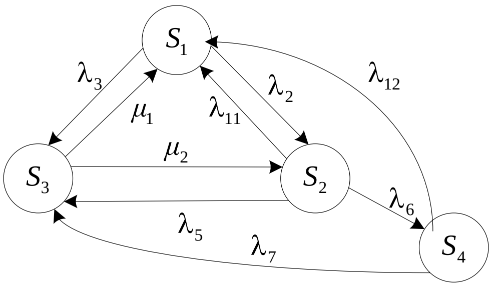


Figure 5: The state transition diagram with redundant component

The state transition equation is

$$\mathrm{d}\boldsymbol{x}(t)/\mathrm{d}t = \boldsymbol{A}_{33}\boldsymbol{x}(t) + \boldsymbol{\mu}_1 \tag{18}$$

where

$$\boldsymbol{A}_{33} = \begin{bmatrix} -d_{11}^{'} & a_1 & a_2 \\ b & -d_{22}^{'} & -\mu_2 \\ 0 & \lambda_6 & -d_{33} \end{bmatrix}, \boldsymbol{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_4(t) \end{bmatrix}, \boldsymbol{\mu}_1 = \begin{bmatrix} \mu_1 \\ \mu_2 \\ 0 \end{bmatrix}. \quad \begin{aligned} d_{11}^{'} = \lambda_2 + \lambda_3 + \mu_1, a_1 = \lambda_{11} - \mu_1, a_2 = \lambda_{12} - \mu_1 \\ b = \lambda_2 - \mu_2, d_{22}^{'} = \lambda_{11} + \lambda_5 + \lambda_6 + \mu_2 \\ d_{33} = \lambda_{12} + \lambda_7 \end{aligned}$$

As $t\to\infty$, the steady state value of the system is

$$\left.\begin{aligned} x_1(\infty) &= \frac{(d_{22}^{'}d_{33} + \lambda_6\mu_2)\mu_1 + (a_1d_{33} + a_2\lambda_6)\mu_2}{-d_{11}^{'}d_{22}^{'}d_{33} + a_2b_1\lambda_6 - d_{11}^{'}\lambda_6\mu_2 + d_{33}a_1b_1} \\ x_2(\infty) &= \frac{b_1d_{33}\mu_1 + d_{11}^{'}d_{33}\mu_2}{-d_{11}^{'}d_{22}^{'}d_{33} + a_2b_1\lambda_6 - d_{11}^{'}\lambda_6\mu_2 + d_{33}a_1b_1} \\ x_4(\infty) &= \frac{-b_1\lambda_6\mu_1 + d_{11}^{'}\lambda_6\mu_2}{-d_{11}^{'}d_{22}^{'}d_{33} + a_2b_1\lambda_6 - d_{11}^{'}\lambda_6\mu_2 + d_{33}a_1b_1} \\ x_3(\infty) &= 1 - x_1(\infty) - x_2(\infty) - x_4(\infty) \end{aligned}\right\} \tag{19}$$

When being at steady state, during time T, let the time being in state S_1, S_2, S_3, and S_4 be $T_1^{''}$, $T_2^{''}$, $T_3^{''}$, and $T_4^{''}$, respectively. Then

$$T_1^{''} = Tx_1(\infty), T_2^{''} = Tx_2(\infty), T_3^{''} = Tx_3(\infty), T_4^{''} = Tx_4(\infty) \tag{20}$$

Let C be component cost, and C_{m3} be the component's desired storage cost per unit time, and C_{f3} be the component's desired preventive maintenance cost per unit time, and $N^{''}$ be the component's expected number of failures during time T, thus

$$N^{''} = \int_0^{T_1^{''}} v_1 dt + \int_0^{T_2^{''}} v_2 dt + \int_0^{T_4^{''}} \lambda_7 dt = T_1^{''}v_1 + T_2^{''}v_2 + T_4^{''}\lambda_7 \tag{21}$$

If a component failure occurs, it will be replaced with the new. So average maintenance cost rate during time T can be written as

$$f_3 = \frac{C_{m3}T_1^{''} + CN^{''} + C_{f3}T_3^{''}}{T} = C_{m3}x_1(\infty) + C\left(x_1(\infty)v_1 + x_2(\infty)v_2 + \lambda_7 x_4(\infty)\right) + C_{f3}x_3(\infty) \tag{22}$$

5. EXAMPLE

The transition rates per month of the parameters of the production devices of an enterprise are given as follows: λ_1= 1.5625, λ_2= 1.875, v_1= 0.0020, v_2= 0.0020, μ_1=1.2500, μ_2=0.6250, u_1= 1.2500, u_2= 0.6250, λ_5=0.4685, λ_6= 1.2500, λ_7= 0.4685, λ_{11}=0.7813, λ_{12}= 0.7813[14].

Let C be the component cost. In the first model, let C_{m1}=0.01C. According to parameters given above, we have $x_1(\infty)$=0.4542, and $x_2(\infty)$=0.5448, and so average maintenance cost rate f_1 is 0.006542C during time T.

In the second model, let C_{m2}=0.01C，C_{r2}=0.01C, C_{f2}=0.01C. According to parameters given above, we have $x_1(\infty)$=0.3113, and $x_2(\infty)$=0.3603, and $x_3(\infty)$=0.3284, so average maintenance cost rate f_2 is 0.006398C during time T.

In the third model, let C_{m3}=0.01C, and C_{f3}=0.01C. According to parameters given above, we have $x_1(\infty)$=0.2318, and $x_2(\infty)$=0.2405, and $x_3(\infty)$=0.2663, $x_4(\infty)$=0.2614, so average maintenance cost rate f_3 is 0.126421C during time T.

From the above calculation, we have $f_2<f_1<f_3$. Clearly, the average maintenance cost rate of preventive maintenance scheme is the lowest, and the highest for redundant maintenance. The main reason for this result lies in that the model 1 and the model 3 adopt corrective maintenance, while model 2 adopts preventive maintenance. Therefore, it is easy to draw conclusion that preventive maintenance model is the best model for some repairable components.

CONCLUSION

This paper firstly analyzes transient process and stability of state equation of repairable components. Secondly, three different maintenance models are introduced. At last the average maintenance cost rates of all models are calculated. The above study can provide the theoretical basis for system operation and maintenance, helping people make scientific, reasonable production plans and maintenance works. It is necessary for enterprise to build optimal model that can help its manager make the decision to obtain maximum profit.

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