



## Precipitation-runoff relationship variation of water resources changes

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### ABSTRACT

*The consistency is the basis for hydrological analysis, and consistency has been destroyed in changing environment. Most of the well-studied cases of abrupt changes pay close attention to precipitation or runoff. This study focuses on detecting changes in annual precipitation and runoff for the five states in Wei River Basin (WRB) of the Northeastern China. Using Mann-Kendall test, double mass curve, we analyze relationship of precipitation and runoff observations to detect WRB that have undergone decreases and abrupt changes in precipitation and runoff for each of the five stations for the time period, 1951 to 2010. To further validate variation of the relationship between precipitation and runoff, the Euclidean distance of copula function and experience copula function indicate the distribution type of precipitation and runoff has changed in the variation before and after. Our results show that in the past 60 years, precipitation and runoff have undergone variation in WRB, and a significant decreasing trend in runoff. The attribution of climate variability and human activities for runoff changes was quantitatively assessed in different station of the WRB. The results of the study will be beneficial to the regional water resources management in Wei River Basin under the changing environment.*

**Keywords:** precipitation and runoff, variation, WRB

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### INTRODUCTION

In the global warming and human activities, space-time distribution scope and scale of precipitation and runoff are changed<sup>[1]</sup>. Especially in the last decades, the importance of abrupt precipitation and runoff changes, and its inclusion in the development of climate change adaptation strategies, has been increasingly emphasized<sup>[2]</sup>. To date, most of the well-studied cases of abrupt change are focused on precipitation or runoff. Richter et al.,<sup>[3]</sup> who combined river ecosystem management objectives, annual runoff series variations were studied, and pointing out that the main driving force affecting river ecosystem is runoff magnitude variations. After a lot of calculation and verification, Perreault<sup>[4]</sup> used a Bayesian approach to detect changes in annual energy inflows of eight hydropower systems in Québec. Using the Mann-Kendall non-parametric test to detect abrupt changes and trends in runoff, Donald<sup>[5]</sup> investigated 18 hydrological variables reflecting various components of hydrological cycles in a network of 248 Canadian catchments selected to reflect natural conditions. Moreover various non-linear analysis methods have been introduced recently, such as entropy<sup>[6]</sup> techniques.

There is a certain relationship of precipitation and runoff in WRB<sup>[7]</sup> that can be reflected in the double mass curve, and the slope of the curve changes to some extent reflects variations in precipitation-runoff relationship. Using Mann-Kendall test, double mass curve and bivariate Copula function, we analyzed variations of precipitation and runoff in WRB, and the joint distribution function of precipitation and runoff, by comparing the differences in different periods of runoff, calculated contribution of climate changes and human activities on runoff changes.

### Study Area

The River Weihe originates in Weiyuan County, Gansu Province, northwest China. It passes through several major cities, including Tianshui, Xianyang and Weinan situated in the southern margins of the Loess Plateau before discharging into the Yellow River at Tongguan. The basin is in the continental monsoon climate zone, with cold, dry winters and hot, wet summers governed by the Mongolian and West Pacific subtropical high pressure systems, respectively.

## EXPERIMENTAL SECTION

### Mann-Kendall test

Mann Kendall test is a statistical test widely used for the analysis of trend in climatologic and in hydrologic time series<sup>[8]</sup>.

Let the time series  $X=\{x_1, x_2, \dots, x_n\}$ , the Mann-Kendall test is computed as follows:

$$D_\tau = \sum_{i=1}^{\tau} R_i \quad (\tau = 1, 2, \dots, n) \quad (1)$$

$$\text{where } R_i = \begin{cases} +1, & x_i > x_j; \\ 0, & x_i \leq x_j. \end{cases} \quad (j = 1, 2, \dots, i)$$

$$UF_\tau = \frac{|D_\tau - E(D_\tau)|}{\sqrt{V(D_\tau)}} \quad (2)$$

Where  $E(D_\tau)$   $V(D_\tau)$  are the mean and variance of  $D_\tau$ . In reverse sequence  $\{x_n, x_{n-1}, \dots, x_1\}$ , repeat the above process, make  $UB_\tau = -UF_\tau$  ( $\tau = n, n-1, \dots, 1$ ), draw graphs  $UF_\tau$  and  $UB_\tau$ , If  $UF_\tau$  or  $UB_\tau$  value is greater than 0, then the sequence on the rise, and vice versa decline; the intersection of the two curves is the abrupt change point.

### The double mass curve

The double mass curve is used to check the consistency of many kinds of hydrometeorological data by plotting the graph. The graph of the cumulative data of two variables is a straight line so long as the relation between the variables is a fixed ratio. Breaks in the double-mass curve of such variables are caused by changes in the relation between the variables<sup>[9]</sup>. In this method proposed nearly a century, scholars have made great contributions to double cumulative curve<sup>[10]</sup>. Double mass curve is a simple method, computed as follows.

Let the observed variable values  $X_i$  and  $Y_j$ , the cumulative amount calculated:

$$X'_i = \sum_{j=1}^i X_j, \quad Y'_i = \sum_{j=1}^i Y_j. \quad X_i \text{ represents precipitation, and } Y_j \text{ is the runoff. The slope of the curve changes are abrupt changes points.}$$

### Copulas functions

Copulas functions are useful tools for the construction of multivariate models<sup>[11]</sup>. The joint cumulative distribution function<sup>[12]</sup>  $H(x, y)$  of any pair  $(X, Y)$  of random variables may be written in the form.

$$H(x, y) = C[F(x), G(y)], \quad x, y \in R \quad (3)$$

$F(x)$  and  $G(y)$  are marginal distributions, and  $C: [0, 1]^2 \rightarrow [0, 1]$ .

The Copula theory can be extended to multi-dimensional joint distribution (Nelsen, 2006). Previous studies of the joint distribution required making some assumptions on the random variables, but copula function can avoid the limitations of assumptions.

Bivariate T-copula function is described equation (4).

$$C^t(u, v; \rho, k) = \int_{-\infty}^{t_k^{-1}(u)} \int_{-\infty}^{t_k^{-1}(v)} \frac{1}{2\pi\sqrt{1-\rho^2}} \left[ 1 + \frac{s^2 - 2\rho st + t^2}{k(1-\rho^2)} \right]^{-(k+2)/2} ds dt \quad (4)$$

Where  $\rho$  is the linear correlation coefficient,  $k$  is the degree of freedom,  $t^{-1}$  is the inverse function of the distribution of T.

Archimedean Copula function contains more types and the Frank-Copula relatively wide scope of application.

Bivariate Frank-Copula function is described as equation (5).

$$C^{Fr}(u, v; \theta) = -\frac{1}{\theta} \ln \left[ 1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{(e^{-\theta} - 1)} \right], \theta \in (-\infty, \infty) \setminus \{0\} \tag{5}$$

Let  $(x_i, y_i) (i = 1, 2, \dots, n)$  be samples taken from the general  $(X, Y)$ .  $F_n(x)$  and  $G_n(y)$  are empirical distribution functions  $X$  and  $Y$  respectively.

$$\hat{C}_n(u, v) = \frac{1}{n} \sum_{i=1}^n I_{[F_n(x_i) \leq u]} I_{[G_n(y_i) \leq v]}, u, v \in [0, 1] \tag{6}$$

$I_{[a]}$  is indicator function, when  $F_n(x_i) \leq u$ ,  $I_{[F_n(x_i) \leq u]} = 1$  else  $I_{[F_n(x_i) \leq u]} = 0$

The Euclidean distance (ED) of E-Copula and bivariate copula is defined as equation (7)

$$d = \sqrt{\sum_{i=1}^n \left| C(u_i, v_i) - \hat{C}_n(u_i, v_i) \right|^2} \tag{7}$$

Euclidean distance reflects goodness of fit, the smaller the value, the model is more appropriate.

### RESULTS AND DISCUSSION

#### Trends and abrupt changes of precipitation and runoff

As can be seen from Fig 1, runoff significantly declined in the WRB, precipitation tends to decrease, but not significant (decreasing trend significant in XY).

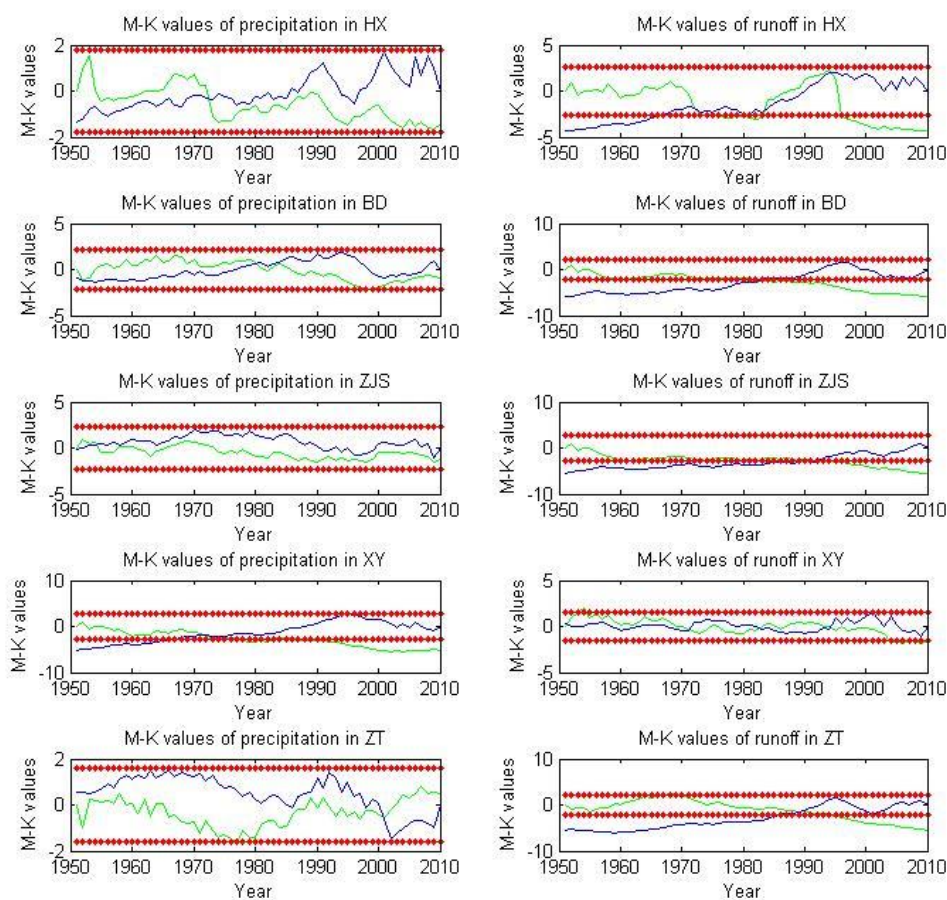


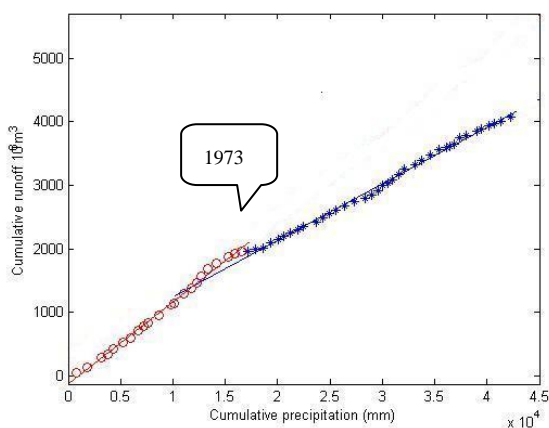
Fig 1 M-K test values for precipitation and runoff at 95% confidence level (the horizontal line) using data from 5 stations (1951-2010)

There were abrupt changes in precipitation at HX in 1972, in 1982 at BD, in 1972 at XY, in 2002 at ZT. There were abrupt changes in runoff at HX in 1973, 1982 and 1994. Abrupt changes were also detected in 1984 at Beidao, 1972 and 1982 at XY, 1994 at ZJS and 1990 at ZT. In the past 60 years, there were no obvious abrupt changes in precipitation of ZJS.

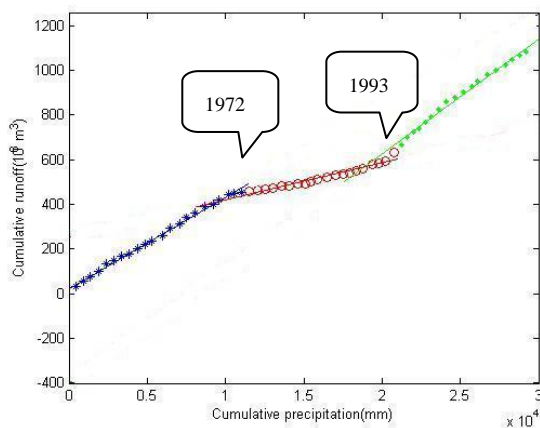
**The abrupt changes of Precipitation-runoff relation**

When the double-mass curve is used to study trends or possible changes in precipitation-runoff relations or when it is used with precipitation to check the consistency the cumulative measured runoff should be plotted against the cumulative computed runoff, taken from a precipitation-runoff relation. Fig 2 showed that, in the WRB, the relations was change in 1973 at HX, in 1972 and 1993 at BD, in 1993 at ZJS and XY, in 1989 at ZT.

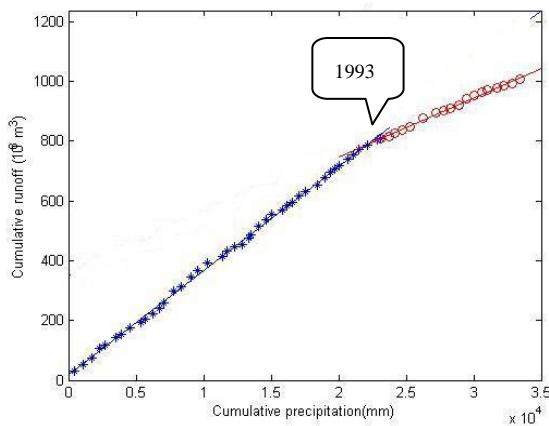
Before the abrupt change precipitation and runoff has a strong correlation, and correlation becomes weak after the change at HX, ZJS, ZT. There are two variants at BD; its correlation descending again becomes large. There is slightly larger correlation at XY.



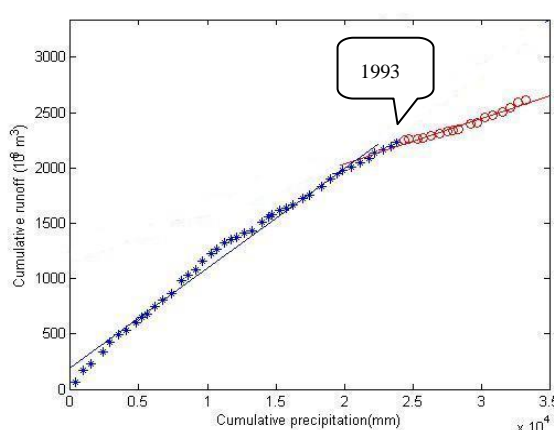
**Figure2-1 Double mass curve of HX**



**Figure2-2 Double mass curve of BD**



**Figure2-3 Double mass curve of ZJS**



**Figure2-4 Double mass curve of XY**

In the Euclidean distance criterion, before the variation of precipitation-runoff relationship, T-Copula fits better than Frank-Copula (Frank-Copula has been better than T-Copula at XY), but Frank-Copula better after the variation. For most stations, the probability density function more obeys a certain copula function type before variation; however, even to obey another function after variation, as shown in table 1.

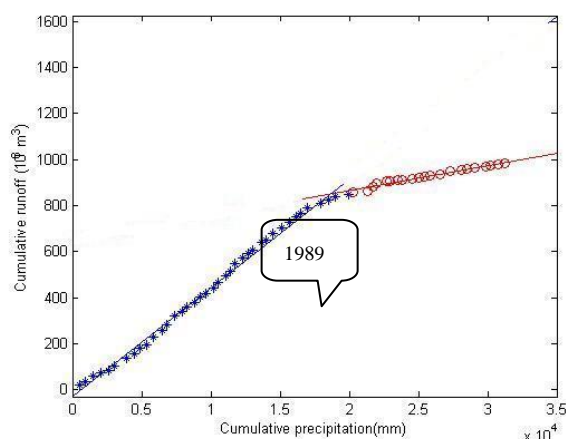


Figure2-5 Double mass curve of ZT

Fig 2 Double mass curve of each station (the turning point is variation corresponding to the year)

Table1 The parameters of copula function

Station	Time	Type	Parameters	ED
HX	1951-1973	T	0.5693	0.1004
		Frank	5.0797	0.1363
	1974-2010	T	0.2916	0.1211
		Frank	2.8874	0.0991
BD	1951-1972	T	0.6595	0.1029
		Frank	6.0410	0.1282
	1973-1993	T	0.2926	0.1103
		Frank	2.0181	0.1012
ZJS	1994-2010	T	0.6061	0.1322
		Frank	5.3163	0.0987
	1951-1993	T	0.4876	0.1003
		Frank	3.0044	0.1218
XY	1994-2010	T	-0.3816	0.1077
		Frank	-1.9297	0.0839
	1951-1993	T	0.5369	0.0928
		Frank	3.5459	0.1126
ZT	1994-2010	T	0.6856	0.0867
		Frank	5.9913	0.1192
	1951-1989	T	0.5767	0.0821
		Frank	2.2156	0.1083
1990-2010	T	0.0562	0.1109	
	Frank	2.387	0.0889	

## CONCLUSION

During the period of records, WRB had been becoming drier. Meantime, the local human activities had become more and more extensive. Observed annual precipitation and stream flow were detected as decreasing trends in 5 hydrological stations located in different parts of the WRB. Correlations of precipitation and runoff become smaller with increased human activities.

These findings also indicate that changes were more frequent and complex in the mainstream and downstream reaches than in tributaries and upstream reaches, respectively, presumably because the mainstream is affected by changes in all of its tributaries and all points upstream. ENSO events, variations in West Pacific subtropical high pressure systems and various identified human activities appear to have contributed to the changes in runoff. However, sunspot activity patterns, other climatic factors and other human activities may also have contributed. Thus, further research is required to elucidate the causal factors comprehensively.

The results could be useful for water resources planners and managers to understand the changing process of

hydrological cycle and driving factors for runoff change and could also be a reference for water resources planning and management in the HRB.

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