Available online www.jocpr.com

Journal of Chemical and Pharmaceutical Research, 2014, 6(5):1431-1436

Research Article

ISSN : 0975-7384
CODEN(USA) : JCPRC5

Practical Model for Predicting Compressive Strength of Aerated Concrete Based on Its Pore Structure Distribution

Youwu Chen[1], Tao Sun*[2], Kun Tian[1], Shengyin Zhou[1], Ge Yan[2]

[1]National Energy-saving Construction Material Quality Supervision Inspection Center, Wuhan, China
[2]State Key Laboratory of Silicate Materials for Architectures, Wuhan University of Technology, Wuhan, China

---

ABSTRACT

A practical model for predicting compressive strength of aerated concrete based on its pore structure was proposed. The aerated concrete's pore structure is characterized by using a MATLAB-based image analysis method. After modeling study of compressive strength, we propose a model to predict the compressive strength according to the Griffith fracture theory. Then by using complicated multiple regression analysis, we get the model between pore structure and compressive strength

Key words: pore structure; image analysis method; model; multiple regression analysis

---

INTRODUCTION

After many years of development, aerated concrete has became one of the major varieties of new wall materials. However, the compressive strength is a major mechanical property index of the aerated concrete. It has always been a focus of material scientific research of aerated concrete[1-2]. Beside, aerated concrete is a porous concrete. As aerated concrete is characterized by porosity, the pore structure is the primary factor that affects aerated concrete's compressive strength.

Nowadays, research on the model of compressive strength of aerated concrete is still lacking, and the lack of systematic research on aerated concrete's most important characteristic - the pore structure - has hindered the efforts to enhance the control of aerated concrete performance and remedy its common quality problems[3-6]. Therefore, if we can establish a model between pore structure and compression strength to predict compressive strength of aerated concrete, which has important application value and theoretical value for guiding practice.

In light of the failure of current pore structure testing methods and approaches to describe the characteristics of aerated concrete, a Matlab-based image analysis method is proposed for describing the characteristics of aerated concrete's pore structure. This method can be used to accurately measure and characterize the porosity, size and distribution of macro and micro pores in aerated concrete. Based on this, the paper introduces the principle of Griffith, and establishes a model between pore structure and compression strength to study the effect of pore and pore distribution on compression strength of aerated concrete in quantitative.

EXPERIMENTAL SECTION

2.1 Measurement of bulk density and compressive strength

The 100×100×100 mm^3 aerated concrete cubes were used to determine the bulk density according to Chinese standard GB/T 11970-1997. Compressive strength test is performed on 100×100×100 mm^3 aerated concrete cubes according to Chinese standard GB/T 11971-1997.

2.2 Image analysis method on pore structure
Image analysis method on pore structure is a method that to obtain the morphology of the pore structure using by optics and digital image processing technology. In this paper, a MATLAB-based image analysis method is used to characterize the pore structure of aerated concrete pore. The flow chart of image analysis method is shown in Fig 1.

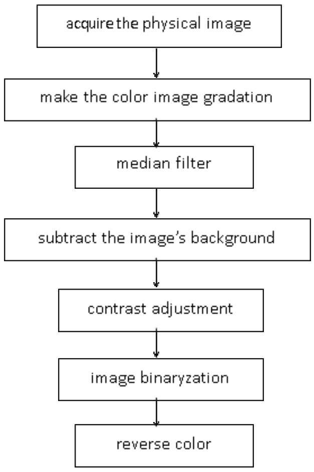


Fig. 1: The flow chart of image analysis method

Finally, we can get pore structure parameters by using MATLAB, such as porosity, pore area directly from the processed photos processed by image processing software, and pore size distribution indirectly from the data of hole area.

2.3 Modeling study of compressive strength
The porosity, bulk density and other factors of aerated concrete can affect its compression strength, among which the pore structure and pore walls of aerated concrete influence strongly. Bulk density is also an important factor in affecting its compression strength: the compressive strength decreased with the reduced of density of aerated concrete[7]. There exhibits a linear correlation between bulk density and compressive strength.

2.4 Analysis of modeling between porosity and compressive strength
All this time, scholars from various countries do a lot of theoretical and experimental research on relationship between porosity and compression strength of aerated concrete, Finally they achieved many accomplishments in the theoretical model, among which the commonly used and valuable were Balshin equation(1), Ryshkewitch equation(2) and Schiller equation(3).

$$\text{Balshin equation：} \sigma = \sigma_0 (1-P)^{\mathrm{n}} \tag{1}$$

$$\text{Ryshkewitch equation：} \sigma = \sigma_0 \mathrm{e}^{-BP} \tag{2}$$

$$\text{Schiller equation：} \sigma = D \ln \frac{\sigma_0}{P} \tag{3}$$

Where σ is the compressive strength when the porosity is P, σ_0 is the compressive strength when the porosity is 0, where P is the porosity of aerated concrete, where n, B, D is the empirical constants.

Hansen use the simplest physical model to explain this problem. He assumed that the porous material can be decomposed into individual cube unit, and each unit has a round hole, the radius of the circular hole is r, then the volume of the circular hole (porosity) is shown in equation(4),

$$P = \frac{4}{3}\pi r^3 \tag{4}$$

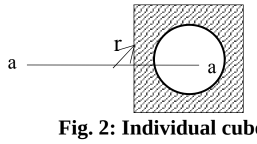


Fig. 2: Individual cube unit

The exist of the round holes in the unit weaken area of porous materials bearing pressure, and the weakest place is the section a-a in the fig 8, the area of the circular holes is A = πr^2, so the reduced area due to the presence of porosity is shown in equation(5),

$$\mathrm{A} = \pi r^2 = \left(\frac{3}{4\pi}\right)^{\frac{2}{3}} P^{\frac{2}{3}} = 1.22P^{\frac{2}{3}} \tag{5}$$

While the effective area under bearing pressure is shown in equation(6),

$$1 - A = 1 - 1.22P^{\frac{2}{3}} \tag{6}$$

So, to the porous material, its compressive strength is shown in equation(7),

$$\sigma = \sigma_0\left(1 - 1.22P^{\frac{2}{3}}\right) \tag{7}$$

In addition, there are many other similar equations, we are no longer listed. While Balshin equation, Ryshkewitch equation, Schiller equation is derived from experimental data using mathematical statistical regression, it is a statistical empirical formula. It only reflects the objective fact that the compressive strength increases with the porosity of the specimens decreasing. However, Hansen assumes too much simple during the derivation, so it has a great difference with the actual situation, such as material stress concentration caused by the presence of holes, breaking effect caused by the presence of cracks, the compressive strength changes caused by hole shape. Under low porosity, Hansen equation is quite different from actual results. The all above indicate that there is a correlation between porosity and compressive strength, but it is not the only the pore structure parameters affect compressive strength.

DEVELOPMENT OF PORE STRUCTURE AND COMPRESSIVE STRENGTH

Recent studies show that not only impact on the compressive strength of porosity, pore size and shape also have an impact on the compressive strength, and many multi-parameter concrete strength theories including different levels of component structure was gradually established.

In 1980, On the Seventh International Conference of Cement, J.Jamber published an article that talk about the relationship between porosity, pore size distribution of cement hydration products and the compressive strength. It is the first time that proposed the pore size distribution, which is a factor of compressive strength. According to J.Jamber's the curve of strength - average pore size, the result obtained is shown in Tab 1.

Tab. 1: The relationship of strength - average pore size

average pore size/$\times 10^{-10}$m	strength/MPa
100	>140.0
250	40.0
1000	<10.0
5000~10000	<5.0

Professor Yunyuan Huang suggests that the relationship between compressive strength and pore structure can be expressed as equation (8) after study,

$$\sigma = \left[K_1 K_2^{(K_3 K_4)}\right] \cdot \left[K_2 K_3^{(S-1)}(1-P)^{K_3 S + K_4}\right] \cdot \left[\frac{1-P}{1+2P}\right] \tag{8}$$

Where S is the relative specific surface area of the pores, K_1~ K_4 is material constants determined by experiment.

In 1985, to analyze the impact of different size porosity on compressive strength, Odler and RÖβler[8-10] proposed a completely different form of compressive strength- porosity relationship. By using complex linear regression analysis method, they established equation(9),(10).

$$25℃\text{：} \quad \sigma = 121 - 0.1P_{<10} - 3.7P_{10-100} - 3.0P_{>100} \tag{9}$$

$$All\,temperature\text{：}\ \sigma = 100 - 0.08P_{<10} - 1.8P_{10-100} - 1.9P_{>100} \tag{10}$$

Where $P_{<10}$ is the porosity below 10×10^{-10}m, $P_{<10}$ is the porosity between 10×10^{-10}m and 100×10^{-10}m, $P_{>100}$ is the porosity more than 100×10^{-10}m.

In 1987, Atzeni[11] amended equation(9), the porosity is further divided, namely pore radius greater than 106nm; pore radius of 106 ~ 53nm; pore radius of 53 ~ 10.6nm and pore radius of less than 1.06nm, and finally got equation(11)，

$$\sigma = \sigma_0 - \mathrm{a}P_{>106} - bP_{106-53} - cP_{53-10.6} - dP_{<10.6} \tag{11}$$

As can be seen from the above equation, all of them considered the effect on the strength of aerate concrete with different pore sizes, achieving a shift from qualitative analysis to quantitative calculation on the relationship between pore sizes and compressive strength. However, due to the lack of rigorous theoretical derivation, its applicability is not very widely, so we introduce Griffith theory, and apply it to the study between pore structure and compressive strength, making the model closer to the actual situation.

PRACTICAL MODEL FOR PREDICTING COMPRESSIVE STRENGTH

4.1 Proposed prediction model

According to Griffith fracture theory, when the stress reaches a certain level, the cracks began to expand and lead to fracture, stress is called the critical stress at this time, critical stress is calculated as follows:

$$\sigma = \sqrt{\frac{2E\gamma}{\pi C}} \tag{12}$$

Where σ is breaking strength when the length of crack is 2C, Where C is a half of the length of crack, Where E is elastic modulus of the material, Where γ is fracture surface energy per unit area.

When calculating the elastic modulus of aerated concrete, we assume its compression process to be the parallel model, and the elastic modulus of aerated concrete is shown in equation(13),

$$E = E_0 \cdot V_0 \tag{13}$$

i.e,

$$E = E_0 \cdot (1 - P) \tag{14}$$

Where E_0 is the elastic modulus of aerated concrete matrix, Where V_0 is the volume fraction of the aerated concrete matrix.

According to Brandt, who suggested the relationship between fracture surface energy and porosity of porous cementitious composites is shown in equation(15),

$$\gamma = \gamma_0 (1 - P) \tag{15}$$

Where γ_0 is the fracture surface energy of aerated concrete matrix.

The original crack of Griffith fracture theory is approximated as porosity, then the functioned formula between half-value width of crack C and the pore radius r is shown in equation(16),

$$C = \mathrm{k} \cdot \mathrm{r} \tag{16}$$

Where k is a constant that rectify aerated concrete pore shape. The equation(12), equation(14), equation(15), equation(16) is integrated, we get equation(17),

$$\sigma = \sqrt{\frac{2E_0 \cdot \gamma_0}{\mathrm{k} \cdot \pi}} \cdot \frac{1 - P}{\sqrt{\mathrm{r}}} \tag{17}$$

The equation(17) derived from Griffith fracture theory, and Griffith fracture theory assumes that the material is elastically deformed. But in fact, aerated concrete is not completely brittle materials, which still have a certain degree of plastic deformation after the force. Consider this, we get the modified equation(18),

$$\sigma = \sqrt{\frac{2E_0 \cdot (\gamma_0 + \gamma_p)}{k \cdot \pi}} \cdot \frac{1 - P}{\sqrt{r}} \tag{18}$$

Where γ_p is the plastic work that expand unit area crack.

In this experiment, the porosity of aerated concrete is divided into less than 0.1mm, 0.1~1mm, 1~2mm, greater than 2mm. For different porosity of aerated concrete, $\sqrt{\frac{2E_0 \cdot (\gamma_0 + \gamma_p)}{k \cdot \pi}}$ can be seen as corresponding different parameters. Besides，the total porosity also have an effect on the compressive strength. Combined data of pore structure derived from image analysis method, we get the model between pore structure and compressive strength as follows

$$\sigma = a_1 \frac{1 - P_{<0.1}}{\sqrt{r_{<0.1}}} + a_2 \frac{1 - P_{0.1\sim1}}{\sqrt{r_{0.1\sim1}}} + a_3 \frac{1 - P_{1\sim2}}{\sqrt{r_{1\sim2}}} + a_4 \frac{1 - P_{>2}}{\sqrt{r_{>2}}} + bP + c \tag{19}$$

Where a_i (i = 1, 2, 3, 4, 5),b,c is the unknown parameter of prediction model.

As we all known, $\sqrt{\frac{2E_0 \cdot (\gamma_0 + \gamma_p)}{k \cdot \pi}}$ can't be calculated by using the parameter in the equation. Therefore, by using complicated multiple regression analysis, we get the model between pore structure and compressive strength.

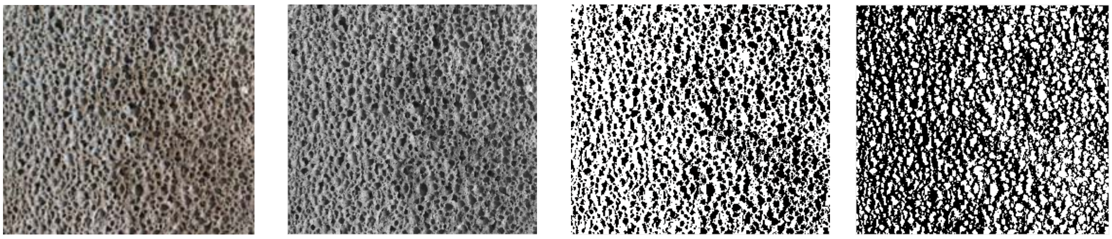
4.2 Verification of proposed model

The proposed model should be used in real life to predict the compressive strength of the aerated concrete. Therefore, we select some 100×100×100 mm^3 aerated concrete cubes to verify the availability of the proposed model.

RESULTS AND DISCUSSION

5.1 Image analysis

The result of image processing by using the image analysis method is shown as follows.



Physical image Grey-scale image Binarization Reverse color

Fig. 3:The result of image processing

According to the image analysis method, we get the data of pore structure, which is shown in the Tab 2. The total porosity obtained by the image analysis method is 20~30%. When it comes to different aperture hole, Pore diameter range always located in 0.1~1mm.

Tab. 2: Pore structure distribution and the compressive strength of aerated concrete

Number	Porosity/%				Total porosity	Compressive strength
	$P_{<0.1}$	$P_{0.1\sim1}$	$P_{1\sim2}$	$P_{>2}$	P/%	σ/MPa
S1	3.06	17.87	3.39	0.05	24.83	3.4
S2	3.29	25.88	2.38	0.94	32.49	2.1
S3	5.35	17.87	2.54	3.72	29.48	1.6
S4	2.87	15.36	0.64	0	18.88	4.1
S5	3.41	17.83	2.13	0.21	23.58	2.7
S6	2.74	19.87	2.18	0.15	24.93	2.8
S7	2.97	16.81	1.89	0.79	22.45	3.3
S8	4.82	18.75	2.1	0.4	26.07	2.9
S9	2.94	18.26	3.57	0	24.77	2.3
S10	2.36	21.81	3.75	0.43	28.37	1.6
S11	3.24	17.59	1.67	0.54	23.05	3.7
S12	3.1	16.5	1.66	0	21.26	3.7
S13	4.99	18.55	0.87	0.67	25.07	3.3
S14	4.9	18.19	1.76	0.63	25.47	2.9

5.2 Effect of pore structure on compressive strength
Pore structure distribution and the compressive strength of aerated concrete is shown in Tab 2.

We apply the equation(19) to the data of Tab 2, then use the multiple regression analysis, we get equation(20) as follow,

$$\sigma = -123.215\frac{1-P_{<0.1}}{\sqrt{r_{<0.1}}} - 291.48\frac{1-P_{0.1\sim1}}{\sqrt{r_{0.1\sim1}}} - 420.996\frac{1-P_{1\sim2}}{\sqrt{r_{1\sim2}}} - 530.633\frac{1-P_{>2}}{\sqrt{r_{>2}}} - 401.335P + 1507.874 \quad (20)$$

The related coefficient (R) of equation(20) is 0.9661, which show the model have high reliability.

Besides, in order to verify the availability of the equation(20), we select some 100×100×100 mm^3 aerated concrete cubes. Calculate the compressive strength by using the model above, the calculated result contrast with the measured compressive strength is shown in Tab 3.

Tab. 3: Calculated result contrast with the measured compressive strength

Number	Calculated compressive strength/MPa	Measured compressive strength/MPa	Error/%
A1	2.67	2.6	2.6
A2	2.90	2.6	11.5
A3	1.84	2.0	8
A4	2.25	2.5	10
A5	3.38	3.2	5.6

CONCLUSION

Based on the Griffith fracture theory and a MATLAB-based image analysis method, a practical model for predicting compressive strength of aerated concrete based on its pore structure was proposed. According to the model we got and experimental verification, the following conclusions can be drawn:

By using the MATLAB-based image analysis method, we can easily get the pore structure distribution. The characterization of aerated concrete pores indicate that the total porosity is 20~30% and the pore diameter range always located in 0.1~1mm.

The equation(20) have high reliability as we introduce the Griffith fracture theory, besides as we consider the porosity less than 0.1mm, 0.1~1mm, 1~2mm, greater than 2mm and the total porosity, The related coefficient (R) of equation(20) can reach 0.9661.

Acknowledgments
The authors wish to thank the Special Fund for AQSIQ Scientific Research in the Public Welfare, under which the present work was possible.

REFERENCES

[1] Ropelewski L, Neufeld RD. *Journal of Energy Engineering-Asce*, v.125, n.2, pp.59-75, **1999**.
[2] Laukaitis A, Fiks B. *Applied Acoustics*, v.67, n.3, pp.284-296, **2006**.
[3] Kumar R, Bhattacharjee B. *Cement and Concrete Research,* v.33, n.1, pp.155-164, **2003**.
[4] Shi CJ. *Cement and Concrete Research,* v.26, n.12, pp.1789-1799, **1996**.
[5] Takahashi T, Yamamoto M, Ioku K, et al. *Advances in Cement Research,* v.9, n.33, pp.25-30, **1997**.
[6] O'Farrell M, Wild S, Sabir BB. *Cement and Concrete Composites,* v.23, n.1, pp.81-91, **2001**.
[7] I.Older.et. *Cement and Concrete Research,* v.16, n.1, pp. 87-96,**1986**.
[8] I.Older. *Materials and Structures,* v.24, n.2, pp.143-157, **1991**.
[9] L.Konnecy. Relationship between strength, permeability and microstructural characteristics of blended mortars. M.Sc.Thesis,The University of Calgary,Canada,**1987**.
[10] Odler 、RÖβler. *Cement and Concrete Research,* v.15, n.2, pp. 401-410,**1985**.
[11] C.Atzeni et. Effect of pore size distribution on strength of hardened cement pastes, proceedings of the First International RILEM Congress on Pore Structure and Material Properties, Paris, pp.195-202, **1987**.