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Petri nets of groups in biogeochemical cycles

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ABSTRACT

Petri nets concepts provide additional tool for the modeling of metabolic networks. Especially the concepts of siphon and trap are used to study the role of trios phosphate isomerase's (TPI) in trypanosome brucei metabolism. In this paper we show that the marked graph *for groups with generating set has subset of places whose input transitions is equal to the output transitions and both of them equal to the set of all transitions of the* marked graph *if the group is (D_{2n} , A) and hence that the dual of this Marked graph can be associated with a Phosphorus cycle.*

Keywords: Marked graph, siphon and traps, Dihedral group, Bio inspired computing, phosphorus cycle.
AMS Mathematical Subject Classification: 68R10, 68Q85

INTRODUCTION

Petri nets are one of the most popular models for the representation and analysis of parallel processes. It has a wide range of application including information systems, operation systems, databases, communication protocols, computer hardware architectures, security systems, manufacturing systems, defence command and control, business processes, banking systems, chemical processes, nuclear waste systems, system biology and telecommunications [4, 5].

There are many sub classes for Petri nets namely marked graph, free choice Petri nets, conflict free Petri nets etc. The study of structural properties and behavioral properties for the marked graph has been made utilizing siphons and traps [6,7]. A nonempty subset of places J is called a siphon if every transition having an output place in J has an input place in J. A nonempty subset of places Q is called a trap if every transition having an input place in Q has an output place in Q.

Phosphorus is an essential nutrient for plants and animals in the form of ions $PO4^{3-}$ and $HPO4^{2-}$. It is a part of DNA- molecules, of molecules that store energy (ATP and ADP) and of fats of cell membranes. Phosphorus is also a building block of certain parts of the human and animal body, such as the bones and teeth. Phosphorus can be found on earth in water, soil and sediments. Phosphorus cannot be found in air in the gaseous state. This is because phosphorus is usually liquid at normal temperatures and pressures. It is mainly cycling through water, soil and sediments. In the atmosphere phosphorus can mainly be found as very small dust particles.[1,2,3]

Phosphorus is a limiting nutrient for aquatic organisms. Phosphorus forms parts of important life sustaining molecules but are very common in the biosphere. Phosphorus does not enter the atmosphere, remaining mostly on land and in rock and soil minerals. Eighty percent of the phosphorus is used to make fertilizers and a type of phosphorus such as dilute phosphoric acid is used in soft drinks [8].

In this paper I construct a marked graph for a given dihedral group D_{2n} with a generating set. I prove that the resulting marked graph associated with this group has subsets of places which are both siphon and trap whose input transitions equal to output transitions and both of them equal the set of all transitions. This leads us to establish that the dual of this constructed marked graph represents a Phosphorus cycle.

PRELIMINARIES

In this section I present some basic definitions relevant to this paper.

Definition: 2.1: A Petri net is triple N = (P, T, F) where P is a finite set of places, T is finite set of transitions such that

(i) $P \cup T \neq \varphi$; $P \cap T = \varphi$

(ii) $F \subseteq (P \times T) \cup (T \times P)$ is set of directed arcs.

For all $p \in P$, ${}^{\bullet}p = \{t \in T / (t, p) \in F\}$ and $p^{\bullet} = \{t \in T / (p, t) \in F\}$ be the input and output sets of p respectively. Similarly for all $t \in T$, ${}^{\bullet}t = \{p \in P / (p, t) \in F\}$ and $t^{\bullet} = \{p \in P / (t, p) \in F\}$ be the input and output sets of t respectively.

Definition2.2:

A Petri net is said to be a marked graph if $|{\bullet}p| = |p{\bullet}| = 1$ for all $p \in P$. A Petri net is said to be conservative if $|{\bullet}t| = |t{\bullet}|$ for all $t \in T$. If the number of tokens in a place p is always less than or equal to a constant k, then such a place is called k-bounded. A Petri net is said to be k-bounded if all its places are k-bounded. If the constant k = 1 then it is called a safe Petri net.

Definition 2.3: A non empty subset Z of places in a marked graph is said to be both siphon and trap if ${}^{\bullet}Z = Z^{\bullet}$. That is, every transition having an input place in Z has an output place in Z and vice versa.

Definition 2.4: A Dihedral Group is a group whose elements correspond to a closed set of rotations and reflections in the plane. The Dihedral Group with 2*n* elements is usually written as D_{2n}. It is generated by a single rotation *r* with order *n*, and a reflection *f* with order 2. i.e.,$D_{2n} = \{r, f / r^{2n} = e, f^2 = e, frf = r^{-1}\}$. The 2n elements of D_{2n} can be written as e, r, $r^2, \ldots, r^{n-1}$, f, rf, $r^2f, \ldots, r^{n-1}f$.

Examples 2.5

(a) Dihedral group D_4 of order 8 = $\{a, b / a^4 = e, b^2 = e, bab = a^{-1}\}$. The elements of D_4 are 1, a, a^2, a^3, b, ab, a^2b, a^3b.

(b) Dihedral group D_6 of order 12 = $\{a, b / a^6 = e, b^2 = e, bab = a^{-1}\}$. The elements of D_6 are 1, a, a^2, a^3, a^4, a^5, b, ab, a^2b, a^3b, a^4b, a^5b.

RESULTS

3.1 Construction of petri net

In this section I construct a marked graph for a given group with a generating set and prove that it has some structural properties.

Theorem 3.1.1: Let D_{2n} be a Dihedral group with a generating set A. Then there exists a marked graph for every group D_{2n} with a generating set A.

Proof: Let $(D_{2n}, A) = \{a, b / a^{2n} = e, b^2 = e, bab = a^{-1}\}$ be a group with generating set A= {a, b}. Let us take the elements of D_{2n} as the transitions of the marked graph. Clearly $A \subseteq D_{2n.}$ Now let us introduce places as follows. For every $t_i \in D_{2n}$ and $s_k \in A$ such that $ti{*}s_k = t_j$, make a place p such that. ${}^{\cdot}p = t_i$ and $p^{\cdot} = t_j$. Also deposits tokens in a place p if p is the input of $s_{ki} + s_{kj}$, for every $s_{ki}, s_{kj} \in A$. Since the generating set A has 2 elements, we have each transition with exactly 2 inputs and 2 outputs and each place with exactly one input and one output. Thus I have constructed a marked graph with initial marking.

Examples 3.1.2

The constructed marked graph for the Dihedral group D_4 of order 8 = $\{a, b / a^4 = e, b^2 = e, bab = a^{-1}\}$ is shown in Figure 1. The elements of D_4 are $\{1, a, a^2, a^3, b, ab, a^2b, a_3b\}$.

In the constructed marked graph of (D_4, A) there exists a subset of places Z = $\{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, \}$ such that $Z^{\cdot} = \{t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, \}$and ${}^{\cdot}Z = \{t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, \}$. Hence Z is both Siphon and trap.

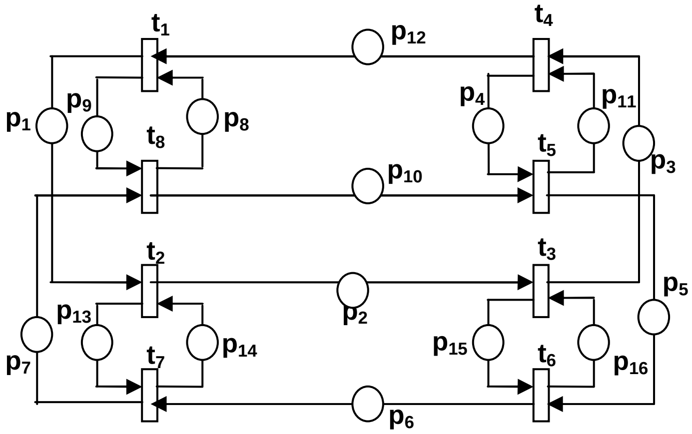

Figure 1: Constructed marked graph for the group (D_4, A)

3.2 Phosphorus Cycle

The phosphorus cycle is the biogeochemical cycle that describes the movement of phosphorus through the lithosphere, hydrosphere, and biosphere. Phosphorus moves slowly from deposits on land and in sediments, to living organisms, and than much more slowly back into the soil and water sediment. Phosphorus is most commonly found in rock formations and ocean sediments as phosphate salts. Phosphate salts that are released from rocks through weathering usually dissolve in soil water and will be absorbed by plants. Because the quantities of phosphorus in soil are generally small, it is often the limiting factor for plant growth. That is why humans often apply phosphate fertilizers on farmland. Phosphates are also limiting factors for plant-growth in marine ecosystems, because they are not very water-soluble. Animals absorb phosphates by eating plants or plant-eating animals. Phosphorus cycles through plants and animals much faster than it does through rocks and sediments. When animals and plants die, phosphates will return to the soils or oceans again during decay. After that, phosphorus will end up in sediments or rock formations again, remaining there for millions of years. Eventually, phosphorus is released again through weathering and the cycle starts over [1, 2, 3].

a) Biological Function

The primary biological importance of phosphates is as a component of nucleotides, which serve as energy storage within cells (ATP) or when linked together, form the nucleic acids DNA and RNA. The double helix of our DNA is only possible because of the phosphate ester bridge that binds the helix. Besides making bio-molecules phosphorus is also found in bones, whose strength is derived from calcium phosphate in enamel of mammalian teeth; exoskeleton of insects and phospholipids (found in a biological membranes) It also functions as buffering agent in maintaining acid base homeostasis in the human body.

b) Phosphorus Minerals

The availability of phosphorus in ecosystem is restricted by the rate of release of this element during weathering. The release of phosphorus from apatite dissolution is key control on ecosystem productivity. The primary mineral with significant phosphorus content, apatite [$Ca_5(PO_4)_3OH$] undergoes carbonation weathering releasing phosphorus contained different forms. This process decreases the total phosphorus in the system due to losses in runoff.

Little of this released phosphorus is taken by biota (organic form) whereas; large proportion reacts with other soil minerals leading to precipitation in unavailable forms. Available phosphorus is found in a biogeochemical cycle in the upper soil profile, while phosphorus found at lower depths is primarily involved in geochemical reactions with secondary minerals. Plant growth depends on the rapid root uptake of phosphorus released from dead matter in the biochemical cycle. Phosphorus is limited in supply for plant growth.

Low-molecular-weight (LMW) organic acids are found in soils. They originate from the activities of various microorganisms in soils or may be exuded from the roots of living plants. Several of those organic acids are capable of forming stable oregano-metal complexes with various metal ions found in soil solutions. As a result, these processes may lead to the release of inorganic phosphorus associated with aluminum, iron, and calcium in soil minerals. The production and release of oxalic acid by my corrival fungi explain their importance in maintaining and supplying phosphorus to plant.

c) Human Inputs to the Phosphorus Cycle
Human influences on the phosphate cycle come mainly from the introduction and use of commercial synthetic fertilizers. The phosphate obtained through mining of certain deposits of calcium phosphate is called apatite. Huge quantities of sulfuric acid are used in the conversion of the phosphate rock into a fertilizer product called "super phosphate". Plants may not be able to utilize all of the phosphate fertilizer applied, as a consequence, much of it is lost form the land through the water run-off. The phosphate in the water is eventually precipitated as sediments at the bottom of the body of water. In certain lakes and ponds this may be re-dissolved and recycled as a problem nutrient.

Animal wastes or manure may also be applied to the land as fertilizer. If misapplied on frozen ground during the winter, much of it may lose as run-off during the spring thaw. In certain area very large feed lots of animals, may result in excessive run-off of phosphate and nitrate into streams. Other human sources of phosphate are in the out flows from municipal sewage treatment plants. Without an expensive tertiary treatment, the phosphate in sewage is not removed during various treatment operations. Again an extra amount of phosphate enters the water.

d) Crop Uptake
One goal with field crop management is to optimize crop uptake of available phosphorus .A typical corn silage crop will remove about 4.3 lbs of P_2O_5 per ton of silage (35% dry matter). Soil testing of available phosphorus can help avoid application of fertilizer phosphorus that is not needed for optimum production. Applying fertilizer beyond crop needs is a waste of time and money, and can be harmful to the environment. Crop uptake is the goal of applying P fertilizer or manure to the soil. If soil tests P levels are already optimum, P additions through fertilizer or manure should not exceed crop removal.

e) Weathering and Precipitation
Soils naturally contain phosphorus rich minerals, which are weathered over long periods of time and slowly made available to plants. Phosphorus can become unavailable through precipitation, which happens if plant available inorganic phosphorus reacts with dissolved iron, aluminum, manganese (in acid soils), or calcium (in alkaline soils) to form phosphate minerals.

f) Mineralization and Immobilization
Mineralization is the microbial conversion of organic phosphorus to H2PO4- or HPO42-forms of plant available phosphorus known as orthophosphates. Immobilization occurs when these plant available phosphorus forms are consumed by microbes, turning the phosphorus into organic phosphorus forms that are not available to plants. The microbial phosphorus will become available over time as the microbes die. Maintaining soil organic matter levels is important in phosphorus management. Mineralization of organic matter results in the slow release of phosphorus to the soil solution during the growing season, making it available for plant up take. This process reduces the need for fertilizer applications and the risk of runoff and leaching that may result from additional phosphorus. Soil temperatures between 65 and 105°F favor phosphorus mineralization.

g) Adsorption and Desorption
Adsorption is the chemical binding of plant available P to soil particles, which makes it unavailable to plants. Desorption is the release of adsorbed P from its bound state into the soil solution. Adsorption (or “fixing” as it is sometimes called) occurs quickly whereas desorption is usually a slow process. Adsorption differs from precipitation: adsorption is reversible chemical binding of P to soil particles while precipitation involves a more permanent change in the chemical properties of the P as it is removed from the soil solution. Soils that have higher iron and/or aluminum contents have the potential to adsorb more P than other soils. Phosphorus is in its most plant available form when the pH is between 6 and 7. At higher pH, P can precipitate with Ca. At lower pH, P tends to be sorbed to Fe and Al compounds in the soil. Every soil has a maximum amount of P that it can adsorb. Phosphorus losses to the environment through runoff and/or leaching increase with P saturation level. Precise fertilizer placement can decrease P adsorption effects by minimizing P contact with soil and concentrating P into a smaller area.

h) Runoff
Runoff is a major cause of P loss from farms Water carries away particulate (soil-bound) P in eroded sediment, as well as dissolved P from applied manure and fertilizers. Erosion control practices decrease P losses by slowing water flow over the soil surface and increasing in filtration.

i) Leaching

Leaching is the removal of dissolved P from soil by vertical water movement. Leaching is a concern in relatively high P soils (near or at P saturation), especially where preferential flow or direct connections with tile drains exist.

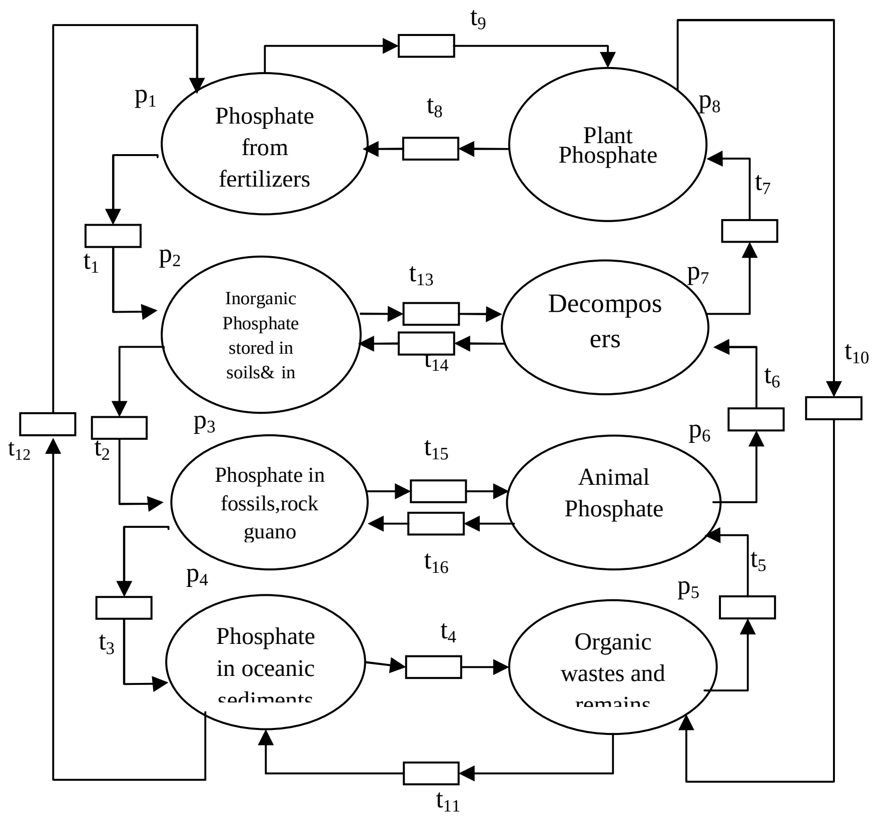

Fig.2: Phosporus cycle

3.3 Description of Phosphorus Cycle in terms of Petri Net

The description of the Phosphorus cycle in the form of Petri net as shown in Fig. 2. Here the places p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8 represent respectively Phosphate from fertilizers, Animal phosphate, Phosphate in fossils and rock guano ,Plant phosphate, Organic wastes and remains, Inorganic phosphate stored in soils and dissolved in rivers, Decomposers and Phosphate in oceanic sediments. The set of transitions t_1, t_2, ..., t_{16} are given in the following description.

The Phosphate from fertilizers is converted to inorganic phosphate stored in soils and dissolved in rivers (p_1) by means of an enabled chemical binding action adsorption (t_1) . This inorganic phosphate (p_2) is then transformed as phosphate (p_2) in fossils, rocks and guano (p_3) due to run off (t_2). The phosphate in fossils and rocks (p_3) are released through weathering (t_3) to phosphate in oceanic sediments (p_4), which will be converted to organic wastes and remains (p_5) by means of immobilization (t_4). This organic wastes and remains will be taken by animals (animal intake) (t_5) and hence the phosphate becomes animal phosphate (p_6). Due to death and excretion (t_6) of animals, the phosphorus in animals becomes decomposers (p_7)which then becomes plant phosphate (p_8) due to fertilization (t_7). The plant phosphate (p_8) can be converted to fertilizers (p_1) as natural manure (t_8). This transformation of phosporus in various forms constitutes one cycle in the marked graph. The other cycles are described as follows. The phosphorus in fertilizers (p_1) goes to plants (p_8) due to assimilation by plant cells (t_9). Due to decomposition (t_{10}) the plant phosphate (p_8) is transformed to phosphorus in organic wates and remains (p_5). The phosphorus in organic waste and remains is then becomes phosphate in oceanic sediments (p_4) due to leaching (t_{11}). Due to manufacturing process (t_{12}) the phosphate in oceanic sediments can be converted to phosphate in fertilizers (p_1). The inorganic phosphate stored in soil and water (p_2) becomes phosphorus in decomposers (p_7) by means of conception (t_{13}). The phosphorus in decomposers (p_7) becomes inorganic phosphate in soils and waters (p_2)

due to precipitation (t_{14}). The phosphate in fossils and rocks (p_3) becomes animal phosphate (p_6) by conception (t_{15}). The animal phosphate (p_6) in animal bones can be converted as phosphate in fossils and rocks(p_3) due to death and decay(t_{16}).

CONCLUSION

I constructed a marked graph for a given group with a generating set. It is proved that the constructed marked graph has subsets of places which are both siphon and trap such that the input transitions equal the output transitions and both of them equal the set of all transitions of the constructed marked graph. As an application, I have proved that the dual of the constructed marked graph has an association with the Phosphorus cycle.

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