Available online www.jocpr.com

Journal of Chemical and Pharmaceutical Research, 2014, 6(7):1370-1377

Research Article

ISSN : 0975-7384
CODEN(USA) : JCPRC5

Oscillation of the systems of impulsive hyperbolic partial differential equations

Xiaoqi Ning* and Shujun You

Department of Mathematics, Huaihua University, Huaihua, China

---

ABSTRACT

The systems of impulsive hyperbolic partial differential equations with Robin boundary value condition are investigated. Several new sufficient conditions of oscillation for such systems are established by employing impulsive differential inequalities and integration.

Key words: Impulsive; Delay; The systems of partial differential equations; Oscillation

---

INTRODUCTION

As an important study field of impulsive partial differential equation, the oscillation theory of impulsive partial differential equation has various applications therefore it has aroused great study interests in recent years, and the study about oscillation theory of impulsive partial differential equations also gradually draws people's attention [1-6]. This paper considers equations

$$\begin{cases} \frac{\partial^2}{\partial t^2}[u_i(t,x)+c(t)u_i(t-\tau,x)]+\frac{\partial}{\partial t}[u_i(t,x)+c(t)u_i(t-\tau,x)] \\ \quad =a_i(t)\Delta u_i(t,x)+\sum_{l=1}^{s}a_{il}(t)\Delta u_i(t-\rho_l(t),x) \\ \quad\quad -\sum_{j=1}^{m}p_{ij}(t,x)u_j(t-\sigma(t),x),\ (t,x)\in R_+\times\Omega, \\ \quad\quad t\neq t_k,\ k=1,2,\cdots,\ i\in I_m=\{1,2,\cdots,m\}, \\ u_i(t_k^+,x)-u_i(t_k^-,x)=\alpha_k u_i(t_k,x),\quad k=1,2,\cdots,\ i\in I_m, \\ \frac{\partial u_i(t_k^+,x)}{\partial t}-\frac{\partial u_i(t_k^-,x)}{\partial t}=\beta_k\frac{\partial u_i(t_k,x)}{\partial t},\quad k=1,2,\cdots,\ i\in I_m, \end{cases} \tag{1}$$

with boundary condition

$$\frac{\partial u_i(t,x)}{\partial N}+g_i(t,x)u_i(t,x)=0,\quad x\in\partial\Omega,\ t\neq t_k,\ i\in I_m, \tag{2}$$

where, $R_+=[0,+\infty)$, $G\equiv R_+\times\Omega$, $u_i=u_i(t,x)$, $\Omega\subset R^n$ denotes a bounded domain With piecewise smooth boundary ,

$$\Delta u_i(t,x)=\sum_{r=1}^{n}\frac{\partial^2 u_i(t,x)}{\partial x_r^2},\ (t,x)\in G,\ i\in I_m.$$

We study the oscillation problem of equation (1) with boundary condition (2), obtain sufficient condition for the oscillation of all solutions and extend the previous results by using existence condition of the eventually positive solution, combined with the impulsive differential inequality. In the following, we will use the conditions as follows:

$(H1)$ $0<t_1<t_2<\cdots<t_k<\cdots,\ \lim_{k\to+\infty}t_k=+\infty,\ \lim_{t\to+\infty}(t-\sigma(t))=\lim_{t\to+\infty}(t-\rho_j(t))=+\infty.$

$0\le\alpha_k\le\beta_k,\ k=1,2,\cdots.\ \sigma,\rho_l,a_i,a_{il}\in PC([0,+\infty),[0,+\infty)),\ c(t)\in C^1(R_+,R_+);$

$(H2)$ $p_{ij}(t,x)\in PC(\bar{G},R),\ p_{ii}(t,x)>0,\ p_{ii}(t)=\min_{x\in\bar{G}}p_{ii}(t,x),\quad \bar{p}_{ij}(t)=\sup_{x\in\bar{G}}|p_{ij}(t,x)|,$

$Q(t)=\min_{1\le i\le m}\{p_{ii}(t)-\sum_{j=1,j\ne i}^{m}\bar{p}_{ji}(t)\}\ge 0,\ i\in I_m,\ j\in I_m.$ here PC denotes piecewise smooth continuous function with the following properties: only discontinuous on $t=t_k,\ k=1,2,\cdots$ which are discontinuity point of the first kind and left continuous on $t=t_k,\ k=1,2,\cdots$;

$(H3)$ the N in (2) is the unit vector normal to $\partial\Omega$,pointing out of Ω , $g_i(t,x)$ is continuous nonnegative function on $[0,+\infty)\times\partial\Omega$;

$(H4)$ $u_i(t,x)$ is the solution satisfied the boundary conditions of equation (1), and $\frac{\partial u_i(t,x)}{\partial t}$ is the partial derivative of $u_i(t,x)$ both are piecewise continuous function with discontinuity point of the first kind on $t=t_k,\ k=1,2,\cdots$. Suppose that they satisfy the following equation at impulsion:

$$u_i(t_k^-,x)=u_i(t_k,x),\ u_i(t_k^+,x)=(1+a_k)u_i(t_k,x),\ k=1,2,\cdots,i\in I_m; \tag{3}$$

$$\frac{\partial u_i(t_k^-,x)}{\partial t}=\frac{\partial u_i(t_k,x)}{\partial t},\ \frac{\partial u_i(t_k^+,x)}{\partial t}=(1+\beta_k)\frac{\partial u_i(t_k,x)}{\partial t},\ k=1,2,\cdots,i\in I_m. \tag{4}$$

1. THE MAIN RESULT

Theorem 1. Suppose condition $(H1)-(H4)$ satisfied, if second-order impulsive differential inequalities

$$\begin{cases} w''(t)+w'(t)+Q(t)w(t-\sigma(t))[1-c(t-\sigma(t))]\le 0,\\ \quad t\ge t_0, t\ne t_k, k=1,2,\cdots,\\ w(t_k^+)=(1+\alpha_k)w(t_k),\quad k=1,2,\cdots,\\ w'(t_k^+)\le(1+\beta_k)w'(t_k),\quad k=1,2,\cdots, \end{cases} \tag{5}$$

doesn't have eventually positive solutions, then every non zero solution of problems (1)-(2) is oscillation in domain G.

Proof. Suppose systems (1)-(2) have a nonzero and non-oscillation solution

$$u(t,x)=(u_1(t,x),u_2(t,x),\cdots,u_m(t,x))^{\mathrm{T}},$$

let us assume when $t\ge t_0\ge 0$,we have $|u_i(t,x)|>0, i=1,2,\cdots,m$. Set

$$\delta_i=\mathrm{sgn}u_i(t,x),\ z_i(t,x)=\delta_i u_i(t,x),$$

Then $z_i(t,x)>0, (t,x)\in[t_0,+\infty)\times\Omega.$ From condition $(H1)$ we can see there is $T_1>t_0$, when $t\ge T_1$ we have $z_i(t,x)>0, z_i(t-\rho_l(t),x)>0, z_i(t-\sigma(t),x)>0, \ (t,x)\in[T_1,+\infty)\times\Omega, i=1,2,\cdots,m, l=1,2,\cdots,s.$

When $t\ne t_k$ 时, we integrate both sides of equation (1) on Ω

$$\begin{aligned}&\frac{d^2}{dt^2}\left(\int_\Omega u_i(t,x)dx+c(t)\int_\Omega u_i(t-\tau,x)dx\right)+\frac{d}{dt}\left(\int_\Omega u_i(t,x)dx+c(t)\int_\Omega u_i(t-\tau,x)dx\right)\\&=a_i(t)\int_\Omega \Delta u_i(t,x)dx+\sum_{l=1}^{s}a_{il}(t)\int_\Omega \Delta u_i(t-\rho_l(t),x)dx+ \qquad (6)\\&-\sum_{j=1}^{m}\int_\Omega p_{ij}(t,x)u_j(t-\sigma(t),x)dx,\ \ t\ge T_1, i\in I_m, k=1,2,\cdots,\end{aligned}$$

then

$$\begin{aligned}&\frac{d^2}{dt^2}\left(\int_\Omega z_i(t,x)dx+c(t)\int_\Omega z_i(t-\tau,x)dx\right)+\frac{d}{dt}\left(\int_\Omega z_i(t,x)dx+c(t)\int_\Omega z_i(t-\tau,x)dx\right)\\&=a_i(t)\int_\Omega \Delta z_i(t,x)dx+\sum_{l=1}^{s}a_{il}(t)\int_\Omega \Delta z_i(t-\rho_l(t),x)dx+ \qquad (7)\\&-\sum_{j=1}^{m}\frac{\delta_i}{\delta_j}\int_\Omega p_{ij}(t,x)z_j(t-\sigma(t),x)dx,\ \ t\ge T_1, i\in I_m, k=1,2,\cdots.\end{aligned}$$

It can be derived from Green Theorem and the boundary condition (2) that

$$\begin{aligned}&\int_\Omega \Delta z_i(t,x)dx=\int_{\partial\Omega}\frac{\partial z_i(t,x)}{\partial N}ds=-\int_{\partial\Omega}g_i(t,x)z_i(t,x)ds\le 0,\\&\qquad t\ge T_1, i\in I_m, k=1,2,\cdots, \qquad (8)\end{aligned}$$

$$\begin{aligned}\int_\Omega \Delta z_i(t-\rho_l(t),x)dx&=\int_{\partial\Omega}\frac{\partial z_i(t-\rho_l(t),x)}{\partial N}ds\\&=-\int_{\partial\Omega}g_i(t-\rho_l(t),x)z_i(t-\rho_l(t),x)ds \qquad (9)\\&\le 0, \qquad t\ge T_1, i\in I_m, k=1,2,\cdots,\end{aligned}$$

here ds denotes the area element on $\partial\Omega$. Combining (7),(8),(9) and condition $(H2)$, we can work out the form

$$\begin{aligned}&\frac{d^2}{dt^2}\left(\int_\Omega z_i(t,x)dx+c(t)\int_\Omega z_i(t-\tau,x)dx\right)+\frac{d}{dt}\left(\int_\Omega z_i(t,x)dx+c(t)\int_\Omega z_i(t-\tau,x)dx\right)\\&\le -p_{ii}(t)\int_\Omega z_i(t-\sigma(t),x)dx+\sum_{j=1,j\ne i}^{m}\overline{p}_{ij}(t)\int_\Omega z_j(t-\sigma(t),x)dx, \qquad (10)\\&\qquad t\ge T_1, i\in I_m, k=1,2,\cdots.\end{aligned}$$

Set

$$v_i(t)=\int_\Omega z_i(t,x)dx,\quad t\ge T_1, i\in I_m,$$

By form (10), we have

$$\begin{aligned}&[v_i(t)+c(t)v_i(t-\tau)]''+[v_i(t)+c(t)v_i(t-\tau)]'+\\&+[p_{ii}(t)v_i(t-\sigma(t))-\sum_{j=1,j\ne i}^{m}\overline{p}_{ij}(t)v_j(t-\sigma(t))]\le 0, \qquad (11)\\&\qquad t\ge T_1, t\ne t_k, k=1,2,\cdots.\end{aligned}$$

Set

$$V(t)=\sum_{i=1}^{m}v_i(t),\quad t\ge T_1,$$

Combining this form and form (11), we have

$$[V(t)+c(t)V(t-\tau)]''+[V(t)+c(t)V(t-\tau)]'+ \\ +\sum_{i=1}^{m}[p_{ii}(t)v_i(t-\sigma(t))-\sum_{j=1,\,j\ne i}^{m}\overline{p}_{ij}(t)v_j(t-\sigma(t))]\le 0, \\ t\ge T_1,\ k=1,2,\cdots \tag{12}$$

and hence

$$[V(t)+c(t)V(t-\tau)]''+[V(t)+c(t)V(t-\tau)]'+ \\ +\Big\{\Big[p_{11}(t)v_1(t-\sigma(t))-\sum_{j=1,\,j\ne 1}^{m}\overline{p}_{1j}(t)v_j(t-\sigma(t))\Big]+ \\ +\Big[p_{22}(t)v_2(t-\sigma(t))-\sum_{j=1,\,j\ne 2}^{m}\overline{p}_{2j}v_j(t-\sigma(t))\Big]+ \\ +\cdots+\Big[p_{mm}(t)v_m(t-\sigma(t))-\sum_{j=1,\,j\ne m}^{m}\overline{p}_{mj}(t)v_j(t-\sigma(t))\Big]\Big\}\le 0, \\ t\ge T_1,\ k=1,2,\cdots.$$

furthermore we have

$$[V(t)+c(t)V(t-\tau)]''+[V(t)+c(t)V(t-\tau)]'+ \\ +\Big\{\Big[p_{11}(t)-\sum_{j=1,\,j\ne 1}^{m}\overline{p}_{j1}(t)\Big]v_1(t-\sigma(t))+\Big[p_{22}(t)-\sum_{j=1,\,j\ne 2}^{m}\overline{p}_{j2}(t)\Big]v_2(t-\sigma(t))+ \\ +\cdots+\Big[p_{mm}(t)-\sum_{j=1,\,j\ne m}^{m}\overline{p}_{jm}(t)\Big]v_m(t-\sigma(t))\Big\}\le 0,\quad t\ge T_1,\ k=1,2,\cdots,$$

then

$$[V(t)+c(t)V(t-\tau)]''+[V(t)+c(t)V(t-\tau)]'+ \\ +\min_{1\le i\le m}\Big\{p_{ii}(t)-\sum_{j=1,\,j\ne i}^{m}\overline{p}_{ji}(t)\Big\}\sum_{i=1}^{m}v_i(t-\sigma(t))\le 0,\ t\ge T_1,\ k=1,2,\cdots,$$

Consequently, we have

$$[V(t)+c(t)V(t-\tau)]''+[V(t)+c(t)V(t-\tau)]'+Q(t)V(t-\sigma(t))\le 0, \\ t\ge T_1,\ k=1,2,\cdots. \tag{13}$$

Set

$$w(t)=V(t)+c(t)V(t-\tau)\ge V(t)>0,\quad t\ge T_1,$$

then form (13) can be rewrite as

$$w''(t)+w'(t)+Q(t)V(t-\sigma(t))\le 0,\quad t\ge T_1,\ k=1,2,\cdots, \tag{14}$$

Thus, we have

$$[e^t w'(t)]'+e^t Q(t)V(t-\sigma(t))\le 0,\quad t\ge T_1,\ k=1,2,\cdots.$$

From the above discussion , we can see that

$$w(t)>0,\ V(t-\sigma(t))>0,\ [e^t w'(t)]'\le 0,\quad t\ne t_k,\ k=1,2,\cdots,$$

So $e^t w'(t)$ is a monotone decreasing function in $(t_k,t_{k+1}]$, $w'(t)$ is also a monotone decreasing function in $(t_k,t_{k+1}]$, it means $w''(t)\le 0,\quad t\ne t_k, k=1,2,\cdots.$

On the other hand, when $t=t_k$, by using (3),(4) in $(H4)$ we have

$$u_i(t_k^+,x)=(1+\alpha_k)u_i(t_k,x),\quad \frac{\partial u_i(t_k^+,x)}{\partial t}=(1+\beta_k)\frac{\partial u_i(t_k,x)}{\partial t},$$

Hence

$$\text{sgn}u_i(t_k^+,x)=\text{sgn}u_i(t_k,x),\quad \text{sgn}\left(\frac{\partial u_i(t_k^+,x)}{\partial t}\right)=\text{sgn}\left(\frac{\partial u_i(t_k,x)}{\partial t}\right),$$

then

$$\begin{aligned}V(t_k^+)-V(t_k^-)&=\sum_{i=1}^{m}[v_i(t_k^+)-v_i(t_k^-)]\\&=\sum_{i=1}^{m}\int_\Omega (z_i(t_k^+,x)-z_i(t_k^-,x))dx\\&=\sum_{i=1}^{m}\int_\Omega\left[\text{sgn}(u_i(t_k^+,x))u_i(t_k^+,x)-\text{sgn}(u_i(t_k^-,x))u_i(t_k^-,x)\right]dx\\&=\sum_{i=1}^{m}\int_\Omega \text{sgn}(u_i(t_k,x))[u_i(t_k^+,x)-u_i(t_k^-,x)]dx\\&=\sum_{i=1}^{m}\int_\Omega \text{sgn}(u_i(t_k,x))\alpha_k u_i(t_k,x)dx\\&=\alpha_k V(t_k),\end{aligned}$$

furthermore ,we have

$$\begin{aligned}w(t_k^+)-w(t_k^-)&=V(t_k^+)+c(t_k^+)V(t_k^+-\tau)-[V(t_k^-)+c(t_k^-)V(t_k^--\tau)]\\&=V(t_k^+)-V(t_k^-)+c(t_k)[V(t_k^+-\tau)-V(t_k^--\tau)]\\&=\alpha_k[V(t_k)+c(t_k)V(t_k-\tau)]\\&=\alpha_k\omega(t_k),\end{aligned}$$

Thus, we have $w(t_k^+)-w(t_k^-)=\alpha_k w(t_k)$. Note that $w(t_k^-)=w(t_k)$, we can also have

$$w(t_k^+)=(1+\alpha_k)w(t_k). \tag{15}$$

Similarly, we can prove

$$w'(t_k^+)-w'(t_k^-)\le\beta_k w'(t_k).$$

note that

$$\begin{aligned}V'(t_k^+)-V'(t_k^-)&=\sum_{i=1}^{m}[v_i'(t_k^+)-v_i'(t_k^-)]\\&=\sum_{i=1}^{m}\int_\Omega\left[\frac{\partial z_i(t_k^+,x)}{\partial t}-\frac{\partial z_i(t_k^-,x)}{\partial t}\right]dx\\&=\sum_{i=1}^{m}\int_\Omega\left[\text{sgn}(\frac{\partial u_i(t_k^+,x)}{\partial t})\frac{\partial u_i(t_k^+,x)}{\partial t}-\text{sgn}(\frac{\partial u_i(t_k^-,x)}{\partial t})\frac{\partial u_i(t_k^-,x)}{\partial t}\right]dx\\&=\sum_{i=1}^{m}\int_\Omega \text{sgn}(\frac{\partial u_i(t_k,x)}{\partial t})\left[\frac{\partial u_i(t_k^+,x)}{\partial t}-\frac{\partial u_i(t_k^-,x)}{\partial t}\right]dx\\&=\sum_{i=1}^{m}\int_\Omega \text{sgn}(\frac{\partial u_i(t_k,x)}{\partial t})\beta_k\frac{\partial u_i(t_k,x)}{\partial t}dx\\&=\beta_k\sum_{i=1}^{m}v_i'(t_k)=\beta_k V'(t_k),\end{aligned}$$

then

$$\begin{aligned} w'(t_k^+) - w'(t_k^-) &= [V(t_k^+) + c(t_k^+)V(t_k^+ - \tau)]' - [V(t_k^-) + c(t_k^-)V(t_k^- - \tau)]' \\ &= V'(t_k^+) + [c(t_k^+)V(t_k^+ - \tau)]' - V'(t_k^-) - [c(t_k^-)V(t_k^- - \tau)]' \\ &= V'(t_k^+) - V'(t_k^-) + c'(t_k^+)V(t_k^+ - \tau) + c(t_k^+)V'(t_k^+ - \tau) \\ &\quad - c'(t_k^-)V(t_k^- - \tau) - c(t_k^-)V'(t_k^- - \tau) \\ &= [V'(t_k^+) - V'(t_k^-)] + c'(t_k)[V(t_k^+ - \tau) - V(t_k^- - \tau)] \\ &\quad + c(t_k)[V'(t_k^+ - \tau) - V'(t_k^- - \tau)] \\ &= \beta_k V'(t_k) + c'(t_k)\alpha_k V(t_k - \tau) + c(t_k)\beta_k V'(t_k - \tau) \\ &\le \beta_k [V'(t_k) + c'(t_k)V(t_k - \tau) + c(t_k)V'(t_k - \tau)] \\ &= \beta_k [V(t_k) + c(t_k)V(t_k - \tau)]' \\ &= \beta_k w'(t_k), \end{aligned}$$

thus, we have

$$w'(t_k^+) - w'(t_k^-) \le \beta_k w'(t_k).$$

furthermore , because $w'(t_k^-) = w'(t_k),$ we have

$$w'(t_k^+) \le (1+\beta_k) w'(t_k). \tag{16}$$

By using (15),(16),we can work out

$$w'(t) \ge 0, \quad t \ge T_1, \, t \ne t_k.$$

In fact, if the above form is wrong then $\exists T_2 \ge T_1,$ such that $w'(T_2) < 0,$ let us assume $w'(T_2) = -c, (c > 0).$ By

$$w''(t) \le 0, \quad t \ne t_k, \, k = 1,2,\cdots, \quad w'(t_k^+) \le (1+\beta_k) w'(t_k), \quad t = t_k, \, k = 1,2,\cdots,$$

together with lemma 5.1.3 in [6], we have

$$w'(t) \le w'(T_2) \prod_{T_2 < t_k < t} (1+\beta_k) = -c \prod_{T_2 < t_k < t} (1+\beta_k), \quad t \ge T_2.$$

Note that $w(t_k^+) = (1+\alpha_k) w(t_k),$ then we have

$$\begin{aligned} w(t) &\le w(T_2) \prod_{T_2 < t_k < t} (1+\alpha_k) - c\int_{T_2}^{t} \prod_{s < t_k < t} (1+\alpha_k) \prod_{T_2 < t_k < s} (1+\beta_k) ds \\ &= \prod_{T_2 < t_k < t} (1+\alpha_k) \left[w(T_2) - c\int_{T_2}^{t} \prod_{T_2 < t_k < s} \frac{1+\beta_k}{1+\alpha_k} ds \right], \end{aligned}$$

note $0 < \alpha_k \le \beta_k$ in $(H1)$, therefor we have $w(t) \le 0, \quad t \ge T_2,$ according the above form , this contradicts to $w(t) > 0, \quad t \ge T_1$. By $w(t) = V(t) + c(t)V(t-\tau), \quad t \ge T_1, t \ne t_k, k = 1,2,\cdots$. together with form (14) we have

$$w''(t) + w'(t) + Q(t)[w(t-\sigma(t)) - c(t-\sigma(t))V(t-\sigma(t)-\tau)] \le 0, \qquad t \ge T_1, t \ne t_k, k = 1,2,\cdots. \tag{17}$$

Since

$$w(t) \ge V(t), \quad w'(t) \ge 0, \quad t \ne t_k, \, k = 1,2,\cdots,$$

the form (17) can be rewrite as

$$w''(t)+w'(t)+Q(t)w(t-\sigma(t))[1-c(t-\sigma(t))]\le 0,\, t\ge T_1, t\ne t_k, k=1,2,\cdots. \quad (18)$$

therefor combining form (18) and form (15),(16) we can see that $w(t)$ is a eventually positive solution of differential inequality (5),this contradicts to condition of theorem 1,and we complete the proof of theorem1.

Theorem 2. Suppose condition $(H1)-(H4)$ satisfied, further more suppose

(i) $0\le\sigma(t)\le\sigma,\, 0\le\rho_j(t)\le\rho,\ j=1,2,\cdots,m;$

(ii) if $\sum_{k=1}^{\infty}\alpha_k<+\infty, \sum_{k=1}^{\infty}\beta_k<+\infty,$ and for each are sufficiently large T , we have

$$\liminf_{t\to+\infty}\frac{\int_T^t\prod_{\eta<t_k<t}(1+\alpha_k)\int_T^\eta\prod_{s<t_k<\eta}(1+\beta_k)\{-Q(s)[1-c(s-\sigma(s))]\}dsd\eta}{\int_T^t\prod_{T<t_k<\eta}(1+\beta_k)\prod_{\eta<t_k<t}(1+\alpha_k)d\eta}=-\infty,$$

Then every nonzero solution of problems (1)-(2) is oscillation in domain G

Proof. By theorem 1 we only need to prove the nonexistence of the eventually positive solution of the second order impulsive differential inequality (5). Suppose $w(t)$ is a eventually positive solution of the impulsive differential inequality (5), then there exists a real number $T\ge\max\{\sigma,\rho\}$, and $w(T-\sigma)\ge 0,$ in form (5), we notice that $w'(t)\ge 0$ and $w(t-\sigma(t))\ge 0, t\ge T, t\ne t_k, k=1,2,\cdots,$ hence we have

$$w''(t)+w(T-\sigma)Q(t)[1-c(t-\sigma(t))]\le 0. \quad (19)$$

combining (19) and $w'(t_k^+)\le(1+\beta_k)w'(t_k),$ in (5), we can use lemma 5.1.3 in [6] to get

$$\begin{aligned}w'(t)\le\ & w'(T)\prod_{T<t_k<t}(1+\beta_k)+\\ &+\int_T^t\prod_{s<t_k<t}(1+\beta_k)\Big\{-w(T-\sigma)Q(s)[1-c(s-\sigma(s))]\Big\}ds,\end{aligned} \quad (20)$$

and together with $w(t_k^+)=(1+\alpha_k)w(t_k)$ in (5) we have

$$\begin{aligned}w(t)\le\ & w(T)\prod_{T<t_k<t}(1+\alpha_k)+\int_T^t\prod_{\eta<t_k<t}(1+\alpha_k)\Big\{w'(T)\prod_{T<t_k<\eta}(1+\beta_k)+\\ &+\int_T^\eta\prod_{s<t_k<\eta}(1+\beta_k)\Big[-w(T-\sigma)Q(s)[1-c(s-\sigma(s))]\Big]ds\Big\}d\eta\\ =\ & w(T)\prod_{T<t_k<t}(1+\alpha_k)+w'(T)\int_T^t\prod_{\eta<t_k<t}(1+\alpha_k)\prod_{T<t_k<\eta}(1+\beta_k)d\eta+\\ &+w(T-\sigma)\int_T^t\prod_{\eta<t_k<t}(1+\alpha_k)\int_T^\eta\prod_{s<t_k<\eta}(1+\beta_k)\Big[-Q(s)[1-c(s-\sigma(s))]\Big]dsd\eta\end{aligned} \quad (21)$$

for $t>T$, there exists real integer m such that $t_m<t\le t_{m+1},$ and so

$$\begin{aligned}&\int_T^t \prod_{T<t_k<\eta}(1+\beta_k)\prod_{\eta<t_k<t}(1+\alpha_k)d\eta\\ &\leq \int_T^{t_1}\prod_{k=1}^{m}(1+\alpha_k)d\eta+\sum_{j=1}^{m-1}\int_{t_j}^{t_{j+1}}\prod_{k=1}^{j}(1+\beta_k)\prod_{k=j+1}^{m}(1+\alpha_k)d\eta+\int_{t_m}^{t_{m+1}}\prod_{k=1}^{m}(1+\beta_k)d\eta\\ &=\prod_{k=1}^{m}(1+\alpha_k)(t_1-T)+\sum_{j=1}^{m-1}\left[\prod_{k=1}^{j}(1+\beta_k)\prod_{k=j+1}^{m}(1+\alpha_k)(t_{j+1}-t_j)\right]\\ &\quad+\prod_{k=1}^{m}(1+\beta_k)(t_{m+1}-t_m)\\ &<+\infty.\end{aligned} \tag{22}$$

For $t > T$, the inequality (21) with both sides divided by the left side of (22), we obtain

$$\frac{\omega(t)}{\int_T^t \prod_{T<t_k<\eta}(1+\beta_k)\prod_{\eta<t_k<t}(1+\alpha_k)d\eta} \leq \frac{\prod_{T<t_k<t}(1+\alpha_k)\omega(T)}{\int_T^t \prod_{T<t_k<\eta}(1+\beta_k)\prod_{\eta<t_k<t}(1+\alpha_k)d\eta}+\omega'(T)+$$
$$+\frac{\omega(T-\sigma)\int_T^t \prod_{\eta<t_k<t}(1+\alpha_k)\int_T^\eta \prod_{s<t_k<\eta}(1+\beta_k)\{-Q(s)[1-c(s-\sigma(s))]\}dsd\eta}{\int_T^t \prod_{T<t_k<\eta}(1+\beta_k)\prod_{\eta<t_k<t}(1+\alpha_k)d\eta},$$

Notes

$$\int_T^t \prod_{T<t_k<\eta}(1+\beta_k)\prod_{\eta<t_k<t}(1+\alpha_k)d\eta \to +\infty, \quad t\to+\infty,$$

We get

$$\liminf_{t\to+\infty}\frac{w(t)}{\int_T^t \prod_{T<t_k<\eta}(1+\beta_k)\prod_{\eta<t_k<t}(1+\alpha_k)d\eta}=-\infty. \tag{23}$$

According to condition (ii) in theorem 2, on the other hand by $w(t)>0, \quad t\geq T,$ we have

$$\liminf_{t\to+\infty}\frac{w(t)}{\int_T^t \prod_{T<t_k<\eta}(1+\beta_k)\prod_{\eta<t_k<t}(1+\alpha_k)d\eta}\geq 0,$$

This form contradicts to (23), hence impulsive differential equation (5) doesn't have eventually positive solution, and we complete the proof of theorem 2.

REFERENCES

[1] L.P.Luo, Z.G.Ouyang. Oscillation of systems of a class of impulsive delay neutral parabolic equations. *Acta Analysis Functionalis Applicata*, 8(3): 257-264, **2006**.
[2] Q.Zhao,W.A.Liu. Oscillation of solution of a class of impulsive delay neutral hyperbolic equations. *Journal of Mathematics*, 26(5): 563-568, **2006**.
[3] Zhang Y T, Luo Q. On the forced oscillation of solutions for systems of impulsive neutral parabolic differential equations with several delays. *J. of Math*(PRC), 26(3): 272-276, **2006**.
[4] L.P.Luo,Z.G.Ouyang . Oscillation of the system of impulsive neutral felay parabolic partial differential equations. *Acta Mathematicae Applicatae Sinica*, 30(5): 822-830, **2007**.
[5] L.P.Luo. Oscillation of impulsive delay neutral hyperbolic equations with nonlinear diffusion coefficient. *Progress in Nature Science*, 18(3): 341-344, **2008**.
[6] X.L.Fu, B.Q.Yan, Y.S.Liu. An Introduction to the Impulsive Differential Systems. Beijing：Science Press, **2005**.