



## Oscillating gear transmission with hypocycloidal shockwave push rod

Xiaoyi Che, Youxin Luo and Qiyuan Liu

College of Mechanical Engineering, Hunan University of Arts and Science, Changde, P. R. China

### ABSTRACT

Transmission principle is the basis of studying design theory. This work proposed new oscillating gear transmission of hypocycloidal shockwave push rod with arbitrary tooth difference. Tooth profile curve of the shockwave gear is hypocycloidal or its equidistant curve; profiles at both ends of push-rod oscillating tooth are cylindrical curves; internal tooth profile is the envelope curve in cylindrical profile group: shockwave cam keeps fixed ratio of transmission with oscillating tooth carrier, and oscillating teeth move along the radial hole of oscillating tooth carrier with circumferential movement. After describing the basic structure, tooth profile equations for shockwave device and center gear are derived based on speed conversion and envelope principle. Then we propose a design condition that tooth interfere does not occur in center gear, laying the foundation for design of such transmission.

**Key words:** Arbitrary tooth difference, hypocycloidal, tooth profile equation, curvature radius, push-rod oscillating gear transmission

### INTRODUCTION

Two-teeth-difference oscillating teeth transmission has a compact structure, high transmission efficiency, large transmission ratio and carrying capacity. In addition, due to the axis symmetry structure of transmission, static and dynamic self-balance can be achieved in the whole transmission process, avoiding vibration excitation of the machine by theory. It includes cycloidal-cam oscillating gear transmission [1], spatial-cam oscillating gear transmission [2], rolling oscillating gear transmission of cam shockwave [3], push-rod oscillating gear transmission with two-teeth difference [4] and swing oscillating gear transmission with two-teeth difference [5, 6]. Oscillating gear transmission of two-tooth difference was developed from that of one-tooth difference, with improved performance but little option in transmission ratio. With the axis symmetry structure of transmission, multi-teeth-difference oscillating teeth transmission can achieve static and dynamic self-balance in the whole transmission process, thereby avoiding vibration excitation of the machine by theory. Therefore, with more research value, multi-teeth-difference oscillating gear transmission has drawn more attention in recent years [7].

Literature [8] proposed pure rolling oscillating gear transmission with arbitrary tooth difference, and profile of shockwave gear and fixed gear were equidistance lines of cosine curve, thus achieving isokinetic conjugate transmission with arbitrary tooth difference. This transmission was simple in design and optimization, and its oscillating teeth utilized rolling bearing components to achieve pure rolling contact transmission. Literature [9] proposed the meshing curve of double-cosine oscillating gear transmission. Literature [10] studied the three-shockwave roller gear transmission which was actually a special case of cosine shockwave. Literatures [8-10] were oscillating gear transmission based on the structure of cosine shockwave roller. Literature [11] studied the high-order polynomial curve tooth profile for push-rod oscillating gear with arbitrary tooth difference. While in Literature [12], the high-order elliptic curve tooth profile for push rod oscillating gear with arbitrary tooth difference was studied. Tooth profile form has a great influence on oscillating transmission performance, so research on internal gear tooth profile and the performance of new oscillating gear transmission is of great significance. Literatures [13] researched the oscillating gear transmission of hypocycloidal shockwave push rod with arbitrary tooth difference. Literatures [14] researched the shockwave swing oscillating gear transmission—swing oscillating gear transmission of

shockwave isometric polygonal profile. Literatures [15] researched swing oscillating gear Transmission with hypocycloid shockwave.

This work has studied the oscillating gear transmission of hypocycloidal shockwave push rod. After describing the principle of oscillating gear transmission with hypocycloidal shockwave push rod, tooth profile equations for shockwave device and center gear are derived based on speed conversion and envelope principle. Then we propose design condition that tooth interfere does not occur in center gear, laying the foundation for design of such transmission. In the new transmission, moving parts such as the input axis are self-balancing speed variator with arbitrary tooth difference. With easy dismounting and large torque, the mechanical transmission is suitable for heavy duty.

### TRANSMISSION PRINCIPLE AND STRUCTURE

With the same rotation center, shockwave gear, internal gear and oscillating tooth group can all be regarded as fixed member, input member or output member, thus achieving different transmission effects. The uniform rotation of input axis will drive shockwave to slew uniformly. Restricted by shockwave gear, internal gear and oscillating tooth group, oscillating gear will also slew with constant speed ratio. Shockwave periodically drives oscillating gear, so that a continuous, relative movement with fixed transmission ratio can be maintained among shockwave gear, oscillating teeth gear and internal gear. Fig.1 shows the principle of oscillating tooth transmission based on hypocycloidal shockwave push rod. Due to the driving torque, shockwave cam maintain counterclockwise rotation and drive push-rod oscillating teeth (No. 4, 5, 6, 9 and 10) to move along the radial holes of tooth carrier in the working area. Meshing force between oscillating teeth and the working segment of internal gear will rotate the oscillating tooth carrier, achieving speed conversion and power output. Meanwhile, under the repulsion of oscillating tooth carrier, oscillating teeth (No. 1, 2, 3, 7 and 8) at non-working area will rotate along internal gear and return to the working position in order.

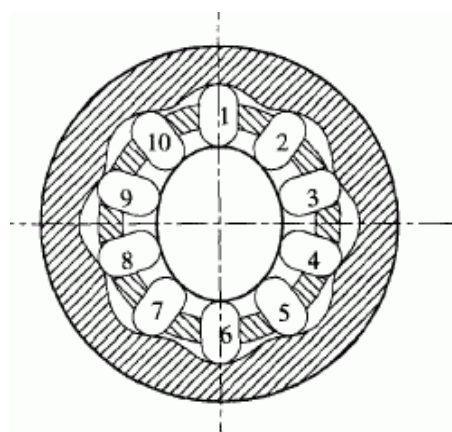


Fig. 1: Oscillating tooth transmission diagram of hypocycloidal shockwave push rod

### TRANSMISSION RATIO CALCULATION

In oscillating gear transmission with hypocycloidal shockwave push rod, fixed shockwave gear can be regarded as conversion mechanism without planetary gear. Then an angular velocity is added to the shockwave oscillating gear system, which is equal to the angular velocity of shockwave gear but with contrary direction. According to transmission ratio equation of fixed-axis gear,

$$i_{GK}^H = \frac{\omega_G^H}{\omega_K^H} = \frac{\omega_G - \omega_H}{\omega_K - \omega_H} = \frac{Z_K}{Z_G} \quad (1)$$

Then we can be obtained from Equation (1):

$$\omega_G = \omega_K i_{GK}^H + \omega_H (1 - i_{GK}^H) \quad (2)$$

where H is the gear shockwave; G the oscillating gear; K the internal gear; i the transmission ratio. The superscript letters indicate the corresponding fixed member, while subscript letters indicate the relative state of left member to the right member. For example,  $i_{GK}^H$  is the specific value between the relative angular velocity of oscillating gear G (compared with shockwave gear H) and that of internal gear K. Superscript letter of  $\omega$  indicates corresponding

fixed member, and subscript letter is the corresponding member;  $Z_H$  is the shockwave number of shockwave gear;  $Z_G$  the number of pin rollers in oscillating gear;  $Z_K$  the wave number of internal gear. Tab.1 shows transmission ratio of different installing forms.

**TAB.1 Transmission ratio in different forms of installing**

Transmission program	Transmission ratio	Rotation direction of master-slave member	Number of oscillating gear roller
<b>Internal gear fixed</b> $\omega_K = 0$	$i_{HG}^k = \frac{Z_G}{Z_G - Z_K}$	$Z_G > Z_K$ , syntropy $Z_G < Z_K$ , reverse	$Z_G = Z_K + Z_H$ $Z_G = Z_K - Z_H$
	$i_{GH}^k = \frac{Z_G - Z_K}{Z_G}$	$Z_G > Z_K$ , syntropy $Z_G < Z_K$ , reverse	$Z_G = Z_K + Z_H$ $Z_G = Z_K - Z_H$
<b>Oscillating gear fixed</b> $\omega_G = 0$	$i_{HK}^G = \frac{Z_K}{Z_K - Z_G}$	$Z_G > Z_K$ , reverse $Z_G < Z_K$ , syntropy	$Z_G = Z_K + Z_H$ $Z_G = Z_K - Z_H$
	$i_{KH}^G = \frac{Z_K - Z_G}{Z_K}$	$Z_G > Z_K$ , reverse $Z_G < Z_K$ , syntropy	$Z_G = Z_K + Z_H$ $Z_G = Z_K - Z_H$
<b>Shockwave gear fixed</b> $\omega_H = 0$	$i_{GK}^H = \frac{Z_K}{Z_G}$	$Z_G > Z_K$ , syntropy $Z_G < Z_K$ , syntropy	$Z_G = Z_K + Z_H$ $Z_G = Z_K - Z_H$
	$i_{KG}^H = \frac{Z_G}{Z_K}$	$Z_G > Z_K$ , syntropy $Z_G < Z_K$ , syntropy	$Z_G = Z_K + Z_H$ $Z_G = Z_K - Z_H$

#### TOOTH PROFILE EQUATION AND CURVATURE RADIUS

In hypocycloidal oscillating gear transmission with shockwave push rod, the profile of shockwave cam is the equidistant curve of hypocycloidal shockwave curve; profiles at two end of push-rod oscillating teeth are cylindrical curves; internal tooth profile is the envelope curve in cylindrical profile group: shockwave cam keeps fixed ratio of transmission with oscillating tooth carrier, and oscillating tooth moves along the radial hole of oscillating tooth carrier with circumferential movement.

Fig.2 shows the profile generation principle of hypocycloidal oscillating gear transmission with shockwave push rod. In the figure, shockwave gear has two hypocycloidal curve waves.  $xOy$  is regarded as the body-fixed coordinate system of internal gear, coordinate origin  $O$  as the geometric center of internal gear.  $x'Oy'$  and  $x''Oy''$  are fixed coordinate systems of shockwave cam and oscillating tooth carrier, respectively. Three coordinate systems coincide at the initial position of transmission. The upper, lower end centers of push-rod oscillating teeth locate on axis  $Ox''$ . During the transmission, the track of lower end center of push-rod oscillating teeth is the theoretical profile of shockwave cam (curve 3) in  $xOy$ . Regarding oscillating teeth radius  $r_1$  as offset distance, the inner equidistant line is the working profile of shockwave cam (curve 4). In addition, the track of upper end center of push-rod oscillating teeth (curve 2) is the theoretical tooth profile of internal gear in  $xOy$ . Working tooth profile of the internal gear is outer equidistance line (curve 1) of its theoretical tooth profile regarding oscillating teeth radius  $r_2$  as the offset distance.  $r_1$  is not necessarily equal to  $r_2$ .

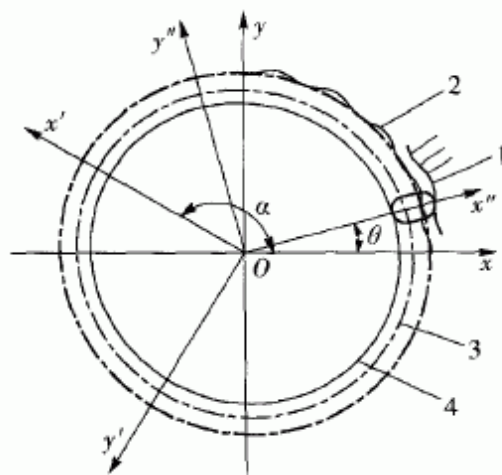


Fig. 2: Forming principle diagram of inner gear profile

(1. Actual internal gear profile 2. Theoretical internal gear profile curve 3. Theoretical profile curve of shockwave gear 4. Actual gear profile of shockwave gear)

### SHOCKWAVE GEAR PROFILE EQUATION

Hypocycloidal curve equation of shockwave gear [16,17] is:

$$\begin{cases} x_1 = (R - r_0) \cos \theta + nr_0 \cos\left[\left(\frac{R}{r_0} - 1\right)\theta\right] \\ y_1 = (R - r_0) \sin \theta - nr_0 \sin\left[\left(\frac{R}{r_0} - 1\right)\theta\right] \end{cases} \quad (3)$$

where  $R$  is the radius of fixed circle;  $r_0$  the radius of movable circle;  $\theta \in [0, 2\pi]$ ;  $Z_H$  the wave number of shockwave gear;  $R = Z_H r_0$ ;  $n \in [0, 0.27]$ .

Shockwave gear profile (curve 3) is the inner equidistant offset of theoretical gear profile, so the actual shockwave gear profile equation [18] can be obtained through mechanical principles.

$$\begin{cases} x_2 = x_1 - \frac{r_1 \frac{dy_1}{d\theta}}{\sqrt{\left(\frac{dx_1}{d\theta}\right)^2 + \left(\frac{dy_1}{d\theta}\right)^2}} \\ y_2 = y_1 + \frac{r_1 \frac{dx_1}{d\theta}}{\sqrt{\left(\frac{dx_1}{d\theta}\right)^2 + \left(\frac{dy_1}{d\theta}\right)^2}} \end{cases} \quad (4)$$

where  $\frac{dx_1}{d\theta} = -\sin \theta (R - r_0) - nr_0 \sin[R/r_0 - 1)\theta](R/r_0 - 1)$

and  $\frac{dy_1}{d\theta} = \cos \theta (R - r_0) - nr_0 \cos[R/r_0 - 1)\theta](R/r_0 - 1)$

### INTERNAL GEAR PROFILE EQUATION

At any moment, the body-fixed rectangular coordinates of shockwave gear  $x'Oy'$  and oscillating teeth carrier  $x''Oy''$  have turned the angle of  $\alpha$  and  $\theta$  (relative to the fixed rectangular coordinate system), respectively. Meanwhile, shockwave gear has the same direction of rotation with transmission ring, so  $\alpha$  and  $\theta$  should meet

$i_{HG}^K = \frac{\alpha}{\theta} = \frac{Z_G}{Z_H}$ . From Figure 3, the origin  $O$ , as well as the upper, lower end center of push-rod oscillating teeth  $O_1'$  and  $O'$  have turned the angle of  $\alpha - \theta$  relative to axis  $Ox'$ . Theoretical gear profile of internal gear is the track of  $O_1'$  in coordinate system  $xOy$ .

$$\begin{cases} x_{30} = (R - r_0) \cos[(i-1)\theta] + nr_0 \cos\left[\left(\frac{R}{r_0} - 1\right)(i-1)\theta\right] \\ y_{30} = (R - r_0) \sin[(i-1)\theta] - nr_0 \sin\left[\left(\frac{R}{r_0} - 1\right)(i-1)\theta\right] \end{cases} \quad (5)$$

Theoretical gear profile of internal gear is the track of  $O_1'$  in coordinate system  $xOy$ , and the theoretical profile equation of internal gear in  $xOy$  is:

$$\begin{cases} x_3 = (s + L) \cos \theta \\ x_3 = (s + L) \sin \theta \end{cases} \quad (6)$$

where  $s = \sqrt{x_{30}^2 + y_{30}^2}$ . When  $L = 0$ , the push-rod oscillating gear transmission becomes roller oscillating gear transmission.

Actual profile of internal gear (curve 1) is the outer equidistant curve with a theoretical profile offset distance of  $r_2$ , thus obtaining the actual gear profile of internal gear [15].

$$\begin{cases} x_4 = x_3 + \frac{r_2 \frac{dy_3}{d\theta}}{\sqrt{\left(\frac{dx_3}{d\theta}\right)^2 + \left(\frac{dy_3}{d\theta}\right)^2}} \\ y_4 = y_3 - \frac{r_2 \frac{dx_3}{d\theta}}{\sqrt{\left(\frac{dx_3}{d\theta}\right)^2 + \left(\frac{dy_3}{d\theta}\right)^2}} \end{cases} \quad (7)$$

#### CURVATURE RADIUS OF INNER GEAR PROFILE

Curvature of the center point of tooth profile curve indicates the curving degree of the point. As an important parameter for the carrying capacity and lubrication of oscillating gear transmission, it describes the geometric characteristics of tooth profile. Then we can obtain the curvature of center profile curve through differential geometry.

$$Kr = \frac{\frac{dx_3}{d\theta} \frac{d^2 y_3}{d^2 \theta} - \frac{d^2 x_3}{d^2 \theta} \frac{dy_3}{d\theta}}{\left(\left(\frac{dx_3}{d\theta}\right)^2 + \left(\frac{dy_3}{d\theta}\right)^2\right)^{\frac{3}{2}}}$$

Therefore, the curvature radius of center profile curve is:

$$\rho = \frac{1}{Kr} = \frac{\left(\left(\frac{dx_3}{d\theta}\right)^2 + \left(\frac{dy_3}{d\theta}\right)^2\right)^{\frac{3}{2}}}{\frac{dx_3}{d\theta} \frac{d^2y_3}{d^2\theta} - \frac{d^2x_3}{d^2\theta} \frac{dy_3}{d\theta}} \quad (8)$$

The actual curvature radius of center gear is  $\rho_w = |\rho| \pm r_2$ . Curvature radius of concave segment around the tooth root is positive (+), while curvature radius of convex segment is negative (-).

### CALCULATION EXAMPLE

Fig. 3 shows the obtained tooth profile curve when the parameter  $r_0 = 40\text{mm}$ ,  $n = 0.2$ ,  $R = 120\text{mm}$ ,  $L = 30\text{mm}$ ,  $i = 5$  and  $r = 10\text{mm}$  are given. The curvature radius around the addendum of theoretical tooth profile is  $8.8321\text{mm}$ ; curvature radius around the addendum convex segment of actual inner gear profile is  $-1.1679\text{mm}$ . According to the figure, there is overlap and crossing at the addendum.

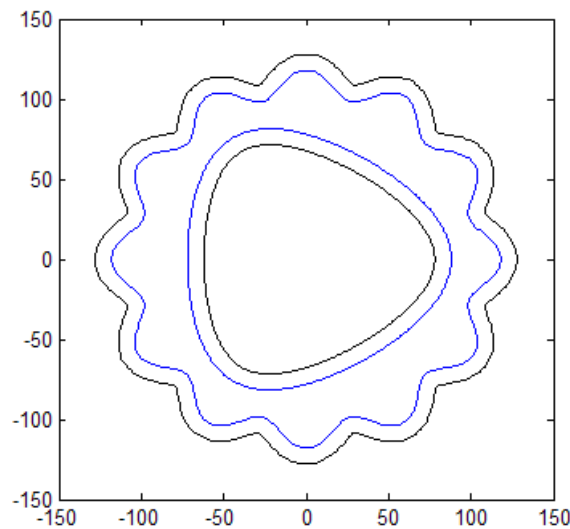


Fig. 3: Theoretical and actual profile of hypocycloidal shockwave and internal gear (L=30mm)

Fig.4 shows the obtained tooth profile curve when  $L = 50\text{mm}$  and the other parameters as above are given. In Figure 3 and 4, curves from the inside to the outside are actual profile of hypocycloidal shockwave gear, theoretical tooth profile of hypocycloidal shockwave gear, theoretical tooth profile of inner gear, actual tooth profile of internal gear, respectively.

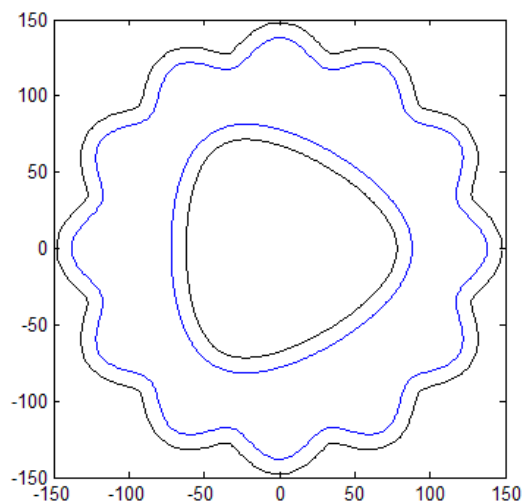
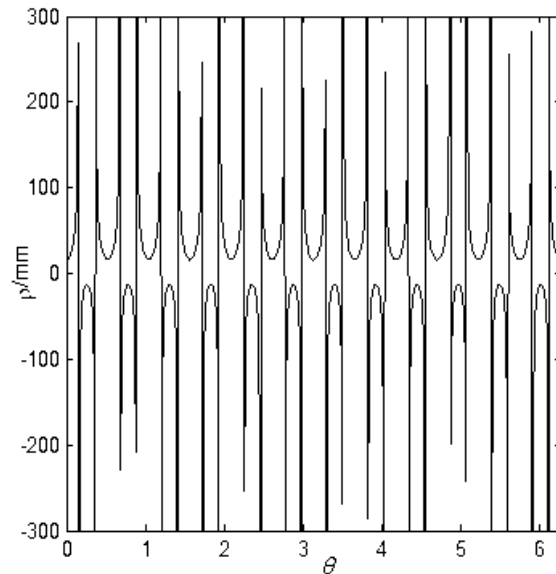


Fig. 4: Theoretical and actual profile of hypocycloidal shockwave and internal gear (L=50mm)

Fig.5 shows the curvature radius of inner gear theoretical profile. The curvature radius around the addendum of theoretical tooth profile is 12.8534mm, while the curvature radius of the convex segment around the addendum of actual internal gear profile is 2.8534mm.



**Fig. 5: Curvature radius of inner gear theoretical profile**

The MATLAB program of calculation is:

```

syms R r0 n L i seta r;
x1=(R-r0)*cos(seta)+n*r0*cos((R/r0-1)*seta); y1=(R-r0)*sin(seta)-n*r0*sin((R/r0-1)*seta);
dxseta=diff(x1,seta);
dyseta=diff(y1,seta);
x2=x1-r*dyseta/(dxseta^2+dyseta^2)^(1/2); y2=y1+r*dxseta/(dxseta^2+dyseta^2)^(1/2);
alfa=i*seta;
x30=subs(x1,seta,alfa-seta);
y30=subs(y1,seta,alfa-seta);
s2=(x30^2+y30^2)^(1/2);
x3=(s2+L)*cos(seta);
y3=(s2+L)*sin(seta);
dxseta20=diff(x3,seta);
dyseta20=diff(y3,seta);
x4=x3+r*dyseta20/(dxseta20^2+dyseta20^2)^(1/2);
y4=y3-r*dxseta20/(dxseta20^2+dyseta20^2)^(1/2); dxseta21=diff(x3,seta,2);
dyseta21=diff(y3,seta,2);
rho=(dxseta20.^2+ dyseta20.^2).^(3/2)./(dxseta20.*dyseta21-dxseta21.*dyseta20);
%%%%%%%%%%%%%%
r0=40;n=0.2;R=3*r0;L=30;i=5;r=10; seta=0:0.01:2*pi;
rho=subs(rho);
rhomin=min(abs(rho));
disp(['%%%%%%%%%%'])
disp(['r0=',num2str(r0),'n=',num2str(n),'R=',num2str(R)])
disp(['L=',num2str(L),'i=',num2str(i),'r=',num2str(r),'rhomin=',num2str(rhomin)])
disp(['^rhomin-r=',num2str(rhomin-r)]);
disp(['%%%%%%%%%%',date,'%%%%%%%%%%'])
x1=subs(x1);
y1=subs(y1);
x2=subs(x2);
y2=subs(y2);
plot(x1,y1,'b');
hold on;plot(x2,y2,'k');
x3=subs(x3);

```

```

y3=subs(y3);
x4=subs(x4);
y4=subs(y4);
hold on;plot(x3,y3,'b');
hold on;plot(x4,y4,'k');
rho=subs(rho);hold off;
figure ;plot(seta,rho,'k'); hold off;
%%%%%%%%%%

```

## CONCLUSION

In oscillating gear transmission of hypocycloidal shockwave push rod with arbitrary tooth difference, tooth profile curve of shockwave gear is hypocycloidal curve or its equidistant curve; profiles at both ends of push-rod oscillating tooth are cylindrical curves; internal tooth profile is the envelope curve in cylindrical profile group: shockwave cam keeps fixed transmission ratio with oscillating tooth carrier, and oscillating tooth moves along the radial hole of oscillating tooth carrier with circumferential movement. It can achieve isokinetic conjugate transmission of arbitrary tooth difference, with simple design method and optimization. Through calculation in Matlab platform and Solidworks parametric modeling, gear profile equation can create the model for oscillating gear transmission with hypocycloidal shockwave push rod. Then Solidworks motion simulation shows no interference, theoretically verifying the above deduction. As a new practical gear transmission with application prospects, the transmission is superior to other gear transmissions in the transmission efficiency, lubrication and manufacturing processes.

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## REFERENCES

- [1] Chen Bingkui, Li Chaoyang. *Mechanical transmission*, **2003**, 27 (3), 14-17.
- [2] Chen Bingkui, Mou Huan, Li Chaoyang, etc.. *Journal of Mechanical Engineering*, **2004**, 40 (1), 29-32.
- [3] Li Jianfeng, He Aiyang, Dong Xinrui, Zhou Liyan. *Journal of Mechanical Engineering*, **2011**, 47 (1), 24-30.
- [4] Li Jianfeng, Cheng Xing, He Aiyang, Su Jian. *Beijing University of Technology*, **2012**, 38 (7), 967-972.
- [5] Liang Shangming, Zhang Jie, Zhou Rongliang, Luo Pan, Yao Jin. *Coal mining machinery*, **2012**, 33 (1), 40-42.
- [6] Qiao Hui, Li Jianfeng, Chen Xing, Su Jian. *Mechanical design and manufacturing*, **2012**, 11, 40-42.
- [7] Qu jifang. Gear transmission theory, Mechanical Industry Press, Beijing, **1993**.
- [8] Huang Jinzhi, Cheng shigan, Chen Xianxiang. *Machine Design and Research*, **2008**, 24 (4), 44-46.
- [9] Huang Jinzhi, Chen Xianxiang, Cheng shigan. *Mechanical Transmission*, **2008**, 32 (4), 6-7.
- [10] Huang Nan, Tao Dongcai, Li Ming, etc.. *Mechanical Transmission*, **2013**, 37 (3), 5-7.
- [11] Yi Yali, Liu Pengpeng, An Zijun, Qu Jifang. *Mechanical Transmission*, **2014**, 38 (4), 1-4.
- [12] Luo Youxin. *Wulfenia Journal*, **2013**, 20(9), 169-181
- [13] Qiyuan Liu, Youxin Luo, Xiaoyi Che. *Computer Modelling & New Technologies*, **2015**, 19(2), xx-xx.
- [14] Youxin Luo, Qiyuan Liu, Xiaoyi Che. *Computer Modelling & New Technologies*, **2015**, 19(2), xx-xx.
- [15] Zheming He, Youxin Luo, Qiyuan Liu. *Electronic Journal of Geotechnical Engineering*, **2014**, 19(z3), 10625-10634.
- [16] Xiling Liu, Xianghong Liu. *Changsha Railway Institute*, **1998**, 3, 75-79
- [17] Sun Ke, Pu Rui. *Mechanical Engineers*, **2014**, 2, 14-15
- [18] Sun Heng, Chen Zuomo, Ge Wenjie. *Mechanical principles Higher Education Press*, Beijing, **2006**.