



## On the study of majority and minority shareholders' symbiosis theory-based dynamical behaviors and countermeasures

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### ABSTRACT

*In light of the features of listed company's majority and minority shareholders' behaviors and relations, this dissertation takes the symbiosis theory in Ecology as the foundation and demonstrates the feasibility of symbiosis between majority and minority shareholders in listed companies on the basis of non-linear dynamical system. By analyzing majority and minority shareholders' income contribution parameters, it lists out status interval of symbiosis between majority and minority shareholders, analyzes their income and behavioral characteristics under different statuses and offers corresponding support as countermeasures in supervision.*

**Key words:** Shareholders' behavior; Dynamical system; Parameter bifurcation; Symbiosis theory; Decision-making support.

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### INTRODUCTION

With the evolution of the nature and the human society, various "populations" have come into being, including an entirety assembled by different biological individuals or human beings or objects of the same kind. In order to adapt themselves to the environment, these different "populations" conduct a variety of "behaviours" continuously. For example, ants migrate to the higher to avoid the upcoming heavy rains; pigeons stick to democracy while they are flying; human beings make further operation plans and decisions for their companies through general meetings of shareholders. It is actually because of the population behaviour that corresponding "relations" come into being among populations, for example, the preying and being preyed relations between sharks and edible fish, the competitive relations among species on the field, and the symbiosis relations between flowering plants and pollinating insects. For a long time, by studying the variety of populations, the relations among them and their behaviours, scholars have been making significant contributions to exploring this mysterious and beautiful world for us.

There are two different types of "populations" in listed companies. One is majority shareholders represented by controlling shareholders and the other is medium and minority shareholders in the subordinate status. Under the economic environment as a whole, these two populations will conduct different behaviours to realize the goal of making profit, for example, majority shareholders' "tunnelling" and "propping", medium and minority shareholders' stock holding or selling. Thus, different relations come into being-cooperate to create a win-win situation or encroach each other's interest. Studying the behaviours and relations between these two groups can facilitate to classify listed company's shareholder's behavioural characteristics, seek for the optimal symbiotic solutions and provide policy references positively.

It was pointed out by American scholars Berle and Means [1] in 1930s that as shareholding structure became scattered, the weight of ownership and right of operation led to interest conflicts between owners and operators and there's a big possibility that operators would take advantage of all accessible convenient conditions to impair owners'

interest. Based on study, Shleifer and Vishny [2] believed that the most basic agency issue of the majority of state-owned companies is the agency between exterior investors and interior controlling shareholders. La Porta [3] found out through study that the stock right of listed companies in many countries is not highly scattered and the agency issue is mainly focused on the agency between majority shareholders, minority shareholders and creditors. Johnson et al [4] proposed the concept of controlling shareholders' "Tunnelling", which stands for various legal or illegal behaviours that the company's controlling shareholders transfer the assets and profits to their accounts, leading to the violations to the minority shareholders' interests. Although "tunnelling" exists, Shleifer and Vishny [5] insisted that the wealth brought by increasing stock prices made the controlling shareholders and minority shareholders' interests converge. According to Friedman [6], controlling shareholders encroach on companies' profits by tunnelling, they may provide support with their private resources when the company is in trouble, from which, minority shareholders can benefit, this is called "propping". Riyanto and Toolsema [7] show that tunnelling alone cannot justify the pyramidal structure unless outside investors are myopic, since rational outside investors anticipate tunnelling and adjust their willingness-to-pay for the firm's shares accordingly. With propping, however, they may be willing to be expropriated in exchange for implicit insurance against bankruptcy. By employing connected transaction data from China, Peng et al [8] find that tunnelling or propping depends on different financial situations of the firms. The word "Ecology" came out in the second half of the 19th century. Its definition was put forward by a German scholar named Haeckel [9] in 1866 at the earliest. As he defined, Ecology refers to the whole science concerning living things and their relations with the outside world. The well-known mycologist founder and German doctor De Bary [10] came up with the concept of symbiosis in the biological circle in the broad sense at the earliest in 1879. It was clearly indicated that "symbiosis means different living things live closely together." In 1976, Robert M. May [11] based on the application of differential equation in Biology and put forward the mathematical model of group symbiosis, which offers basis to the study of group symbiosis mathematically. Yuan Chunqing [12] introduced the symbiosis concept in Biology and relevant theories into the study of small economics and thus opened up the era of the study and application of symbiosis theory in Economics. Yang Songling and Liu Tingli [13] introduced the biometric concept of symbiosis into research in this regard for the first time, which constituted a new breakthrough in the research in this field. Yang Songling and Liu Tingli [14] thoroughly expounded the compatibility of Biology theory and the study of shareholder's relations in listed companies and suggested the framework of studying symbiosis theory-based majority and minority shareholders' relations.

Although regulatory bodies, organizations and scholars in different countries have made much efforts and attempts, and have achieved considerable progress and development in the theory and system of corporate governance, and listed companies and shareholders tend to show increasingly standard behaviours, it is undeniable that the injurious acts of such majority shareholders as "tunnelling" still exist, and the relations among shareholders still show many inharmonious factors.

On the basis of symbiosis theory in Biology and non-linear dynamical system, this dissertation demonstrates the feasibility of harmonious symbiosis between majority and minority shareholders in listed companies and listed out the status interval of symbiosis between majority and minority shareholders by analyzing their income contribution parameters. Also, in light of different statuses, it analyzes the income and behavioural characteristics of majority and minority shareholders and offers corresponding support as countermeasures in supervision.

## MODEL BUILDING

### QUADRATIC SUM MODEL

Set  $\mathbf{x}(t) = (x_1(t), x_2(t))^T$  as the total of two species at the time  $t$  and assume  $x_1(t)$  and  $x_2(t)$  are continuous function about  $t$ . Assume the two species' contribution coefficient on their quantity is the function of  $x_1(t)$  and  $x_2(t)$ , namely:

$$\mathbf{h} = (h_1, h_2)^T = \begin{pmatrix} a_0 & a_1 & a_2 \\ b_0 & b_1 & b_2 \end{pmatrix} \begin{pmatrix} 1 \\ x_1(t) \\ x_2(t) \end{pmatrix} \quad (1)$$

In which,  $a_i, b_i$  ( $i = 0, 1, 2$ ) are constants. Thus, the quadratic sum model is achieved:

$$\frac{d\mathbf{x}(t)}{dt} = \text{diag}(x_1(t), x_2(t))\mathbf{h} \quad (2)$$

## SYMBIOSIS SYSTEM OF MAJORITY SHAREHOLDERS AND MEDIUM AND SHAREHOLDERS IN LISTED COMPANIES

In the symbiosis relations between majority shareholders and medium and minority shareholders in listed companies, assume there were no medium or minority shareholders, the company's benefit is  $x_1(t)$ , accordingly, when there were only medium and minority shareholders, the company's profit is  $x_2(t)$ .

Describe the symbiosis system of majority shareholders and medium and minority shareholders in listed companies as differential equation group:

$$\frac{dx(t)}{dt} = \text{diag}(x_1(t), x_2(t))\mathbf{h} \quad (3)$$

In which,  $\mathbf{x}(t) = (x_1(t), x_2(t))^T$ ,  $\mathbf{h} = (h_1, h_2)^T$ ,  $h_1 = r_1(1 + \frac{\sigma_1 x_2}{K_2} - \frac{x_1}{K_1})$ ,  $h_2 = r_2(-1 + \frac{\sigma_2 x_1}{K_1} - \frac{x_2}{K_2})$ .  $r_i > 0$ ,

$i = 1, 2$ , which are the mean growth rate of company benefit when there were only majority shareholders or medium and minority shareholders respectively.  $\sigma_i > 0$ ,  $i = 1, 2$ , which represent each majority shareholder or medium and minority shareholder's contributions to each other's profit respectively.

### MODEL ANALYSIS

#### BIFURCATION OF THE EQUILIBRIUM POINT

Set the right-hand item of differential equation (3) as zero.

$$\begin{cases} r_1 x_1 (1 + \frac{\sigma_1 x_2}{K_2} - \frac{x_1}{K_1}) = 0 \\ r_2 x_2 (-1 + \frac{\sigma_2 x_1}{K_1} - \frac{x_2}{K_2}) = 0 \end{cases} \quad (4)$$

Find the solution to (4), and locate four equilibrium points existing in the original equation group  $(0, 0)$ 、 $(K_1, 0)$ 、 $(0, -K_2)$  and  $(\frac{K_1(-1+\sigma_1)}{\sigma_1\sigma_2-1}, -\frac{K_2(-1+\sigma_2)}{\sigma_1\sigma_2-1})$ .

**Theorem 1 (Bifurcation of the equilibrium point)** If the coefficient  $\sigma_1, \sigma_2$  of equation (3) meet the following conditions:

(1) When  $\sigma_1 = 1$  and  $\sigma_2 \neq 1$ , (or  $\sigma_2 = 1$  and  $\sigma_1 \neq 1$ ), the equation has three equilibrium points (namely,  $(0, 0)$ 、 $(K_1, 0)$  and  $(0, -K_2)$ );

(2) When  $\sigma_1\sigma_2 \neq 1$  and  $\sigma_1, \sigma_2 \neq 1$ , the original equation has four equilibrium points (namely,  $(0, 0)$ 、 $(K_1, 0)$ 、 $(0, -K_2)$  and  $(\frac{K_1(-1+\sigma_1)}{\sigma_1\sigma_2-1}, -\frac{K_2(-1+\sigma_2)}{\sigma_1\sigma_2-1})$ ).

#### BIFURCATION OF INCOME PARAMETERS

It is not difficult to find out from Theorem 1 that equilibrium points  $(0, 0)$ 、 $(K_1, 0)$  and  $(0, -K_2)$  represent the final income of at least one party, either majority shareholder or medium and minority shareholder, is zero. This can be hardly explained as the long-term balance between two parties; under condition (2) of Theorem 1, equilibrium points exist in equation group (7)  $(\frac{K_1(-1+\sigma_1)}{\sigma_1\sigma_2-1}, -\frac{K_2(-1+\sigma_2)}{\sigma_1\sigma_2-1})$ . This point is of great importance to the discussion and analysis in this dissertation. Therefore, condition (2) in Theorem 1 needs to be attached more importance to, and a plane region graph of parameters  $\sigma_1$  and  $\sigma_2$  should be drafted under the conditions (See Fig. 2.1, the first quadrant of plane  $\sigma_1 - \sigma_2$  is segmented into six areas by Theorem 1, See Table 1 the parameter conditions of each area.

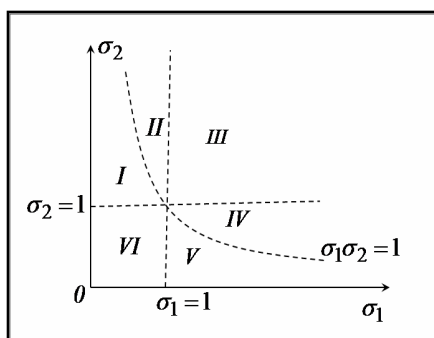


Fig. 2.1: The first quadrant of plane  $\sigma_1 - \sigma_2$  is segmented into six areas by Theorem 1.

In order to describe more conveniently, mark the condition of Area  $i$  in Table 1 as  $i$ . Set  $l_1$  and  $l_2$  as straight lines  $1 + \frac{\sigma_1 x_2}{K_2} - \frac{x_1}{K_1} = 0$  and  $-1 + \frac{\sigma_2 x_1}{K_1} - \frac{x_2}{K_2} = 0$ . Under different conditions of parameters  $\sigma_1$  and  $\sigma_2$ , the part where  $x_1, x_2 > 0$  in plane  $x_1 - x_2$  is divided into several different areas by straight lines  $l_1$  and  $l_2$ . Next, the different conditions in Table 1 will be expounded separately.

In condition I, the equilibrium  $(\frac{K_1(-1+\sigma_1)}{\sigma_1\sigma_2-1}, -\frac{K_2(-1+\sigma_2)}{\sigma_1\sigma_2-1})$  is on the first quadrant of  $x_1 - x_2$ . Straight line  $l_1$  and  $l_2$  segment the part where  $x_1, x_2 > 0$  in  $x_1 - x_2$  into four areas as shown in Fig 2.2 (a). Take parameters  $\sigma_1 = \frac{1}{2}$  and  $\sigma_2 = \frac{3}{2}$ , and draw the phase diagram of the original equation group, as shown in Fig 2.2 (b). Each solution to equation group (7),  $(x_1(t), x_2(t))^T$ , when  $t$  tends to infinity, it is approaching equilibrium point  $(\frac{K_1(-1+\sigma_1)}{\sigma_1\sigma_2-1}, -\frac{K_2(-1+\sigma_2)}{\sigma_1\sigma_2-1})$ .

Table. 1: Parameter Conditions of Ares in Fig. 2.1.

Area	Parameter Conditions	Area	Parameter Conditions
I	$0 < \sigma_1 < \sigma_1\sigma_2 < 1 < \sigma_2$	IV	$0 < \sigma_2 < 1 < \sigma_1\sigma_2 < \sigma_1$
II	$0 < \sigma_1 < 1 < \sigma_1\sigma_2 < \sigma_2$	V	$0 < \sigma_2 < \sigma_1\sigma_2 < 1 < \sigma_1$
III	$1 < \sigma_1, \sigma_2$	VI	$0 < \sigma_1, \sigma_2 < 1$

Under condition IV, equilibrium point  $(\frac{K_1(-1+\sigma_1)}{\sigma_1\sigma_2-1}, -\frac{K_2(-1+\sigma_2)}{\sigma_1\sigma_2-1})$  is also in the first quadrant of  $x_1 - x_2$ .

Straight line  $l_1$  and  $l_2$  segment the part where  $x_1, x_2 > 0$  in  $x_1 - x_2$  into four areas as shown in Fig 2.3 (a). Take parameters  $\sigma_1 = \frac{5}{2}$  and  $\sigma_2 = \frac{1}{2}$ , draw the phase diagram of the original equation group, as shown in Fig 2.3 (b).

The stability of the equilibrium point  $(\frac{K_1(-1+\sigma_1)}{\sigma_1\sigma_2-1}, -\frac{K_2(-1+\sigma_2)}{\sigma_1\sigma_2-1})$  will be discussed in details below.

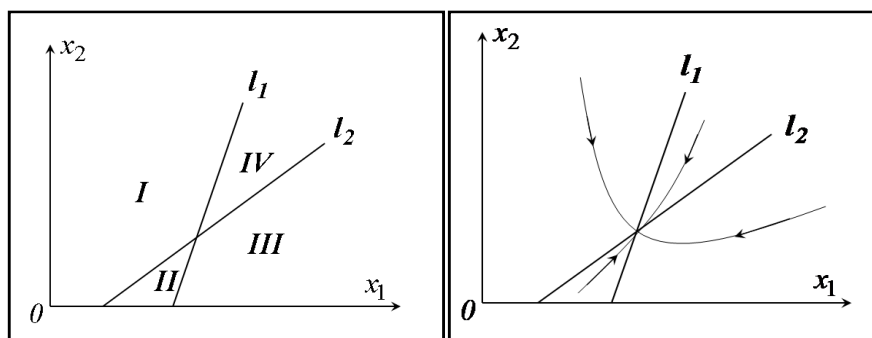


Fig 2.2(a). Under condition I,  $l_1$  and  $l_2$  divides the part where  $x_1, x_2 > 0$ .

Fig 2.2(b). The phase of the original equation group when  $\sigma_1 = \frac{1}{2}$  and  $\sigma_2 = \frac{3}{2}$ .

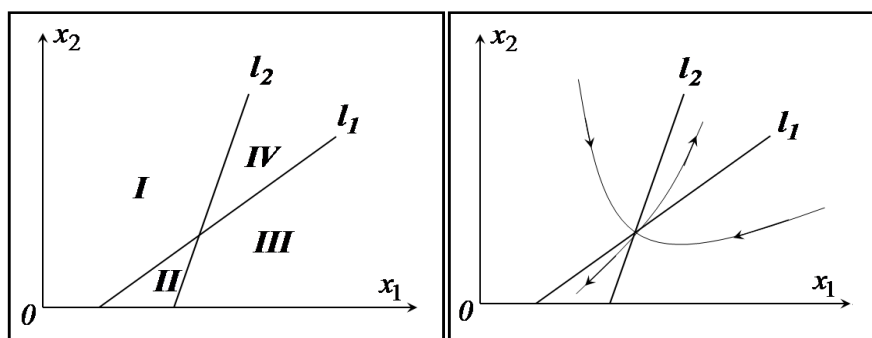


Fig 2.3(a). Under condition IV,  $l_1$  and  $l_2$  divide the part where  $x_1, x_2 > 0$ .

Fig 2.3(b). The phase of the original equation group when  $\sigma_1 = \frac{5}{2}$  and  $\sigma_2 = \frac{1}{2}$ .

Under condition II, equilibrium point  $(\frac{K_1(-1+\sigma_1)}{\sigma_1\sigma_2-1}, -\frac{K_2(-1+\sigma_2)}{\sigma_1\sigma_2-1})$  is in the fourth quadrant of  $x_1 - x_2$ , which means both parties' income is negative when income is balanced. This can also hardly be explained as the long-term balance between two parties. Straight line  $l_1$  and  $l_2$  segment the part where  $x_1, x_2 > 0$  in  $x_1 - x_2$  into three areas as shown in Fig 2.4 (a). Take parameters  $\sigma_1 = \frac{1}{2}$  and  $\sigma_2 = \frac{5}{2}$ , draw the phase diagram of the original equation group, as shown in Fig 2.4 (b).

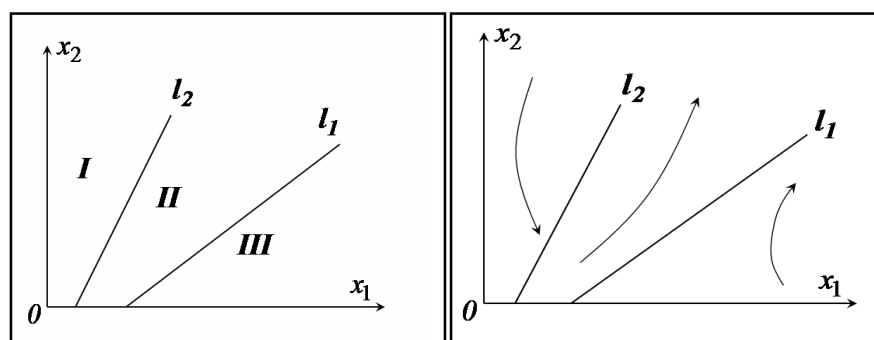


Fig 2.4(a). Under condition II,  $l_1$  and  $l_2$  divide the part where  $x_1, x_2 > 0$ .

Fig 2.4(b). The phase of the original equation group when  $\sigma_1 = \frac{1}{2}$  and  $\sigma_2 = \frac{5}{2}$ .

Under condition *V* , equilibrium point  $(\frac{K_1(-1+\sigma_1)}{\sigma_1\sigma_2-1}, -\frac{K_2(-1+\sigma_2)}{\sigma_1\sigma_2-1})$  is also in the fourth quadrant of  $x_1 - x_2$ , which can also hardly be explained as the long-term balance between two parties. Straight line  $l_1$  and  $l_2$  segment the part where  $x_1, x_2 > 0$  in  $x_1 - x_2$  into three areas as shown in Fig 2.5 (a). Take parameters  $\sigma_1 = \frac{3}{2}$  and  $\sigma_2 = \frac{1}{2}$ , draw the phase diagram of the original equation group, as shown in Fig 2.5 (b).

Under condition *III* and *VI* , coefficient  $\sigma_1$  and  $\sigma_2$  can represent as a linear relation  $\sigma_2 = \frac{r_1}{r_2}\sigma_1 + (1 - \frac{r_1}{r_2})$ . Set  $l_3$ ,  $l_4$  and  $l_5$  to represent  $\sigma_2 = \frac{r_1}{r_2}\sigma_1 + (1 - \frac{r_1}{r_2})$  under  $r_1 > r_2$ ,  $r_1 = r_2$  and  $r_1 < r_2$  respectively, as shown in Fig 2.6 (a). Apparently,  $l_3$ ,  $l_4$  and  $l_5$  pass through (1, 1). Take parameters  $\sigma_1 = \sigma_2 = \frac{1}{2}$ , draw the phase diagram of the original equation group, as shown in Fig 2.6(b).

Based on the above analysis, it can be concluded:

**Theorem 2** If the value range of coefficient  $\sigma_1$  and  $\sigma_2$  in differential equation (7) is within area *I*、*II*、*IV* and *V* in Fig 2.1, no cycle exists in the phase diagram of the original equation group.

**Theorem 3** If the value range of coefficient  $\sigma_1$  and  $\sigma_2$  in differential equation (7) is within area *III* and *VI* , the cycle shown in Fig 2.6 (b) exists in the original equation group.

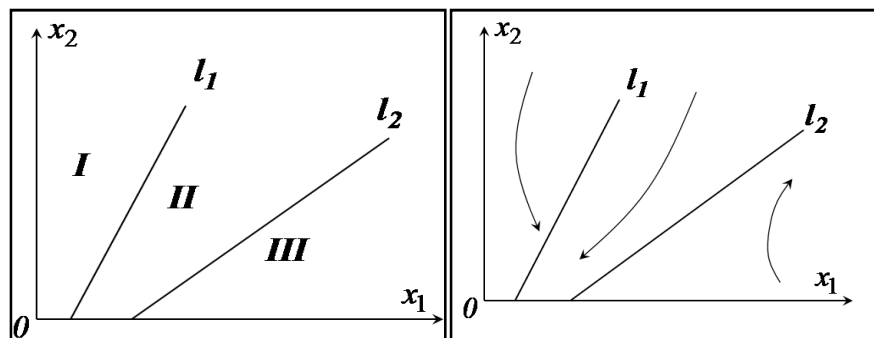


Fig 2.5(a). Under condition *V* ,  $l_1$  and  $l_2$  divide the part where  $x_1, x_2 > 0$ .

Fig 2.5(b). The phase of the original equation group when  $\sigma_1 = \frac{3}{2}$  and  $\sigma_2 = \frac{1}{2}$ .

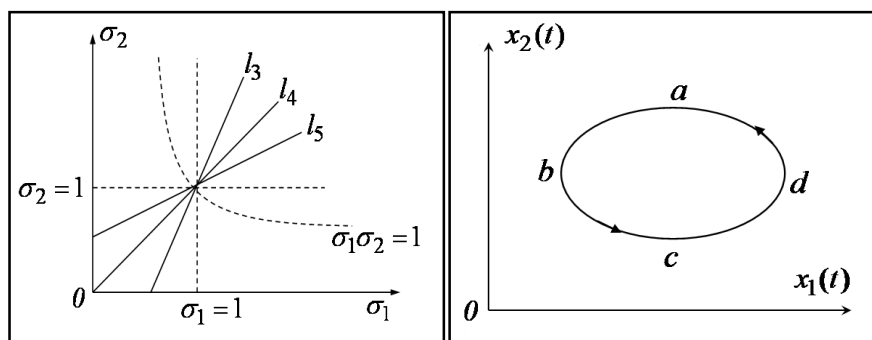


Fig 2.6(a). Straight line  $l_1$ ,  $l_2$  and  $l_3$ 's position in  $\sigma_1 - \sigma_2$ .

Fig 2.6(b). The phase diagram of the original equation group when  $\sigma_1 = \sigma_2 = \frac{1}{2}$ .

## STABILITY ANALYSIS

Firstly, we consider the stability of equilibrium point  $(\frac{K_1(-1+\sigma_1)}{\sigma_1\sigma_2-1}, -\frac{K_2(-1+\sigma_2)}{\sigma_1\sigma_2-1})$ . It is found out by analyzing the Figures and Table in 3.2 that when parameters  $\sigma_1$  and  $\sigma_2$  meet conditions *I* and *V*, each solution to equation group (3) is  $(x_1(t), x_2(t))^T$ , when  $t$  tends to infinity, it approaches to the equilibrium point, which means the point is balanced; when parameters  $\sigma_1$  and  $\sigma_2$  meet condition *II* and *IV*, the stability of this equilibrium point needs to be discussed in groups. Take parameter condition *IV* as an example, each solution to equation group (3) is  $(x_1(t), x_2(t))^T$ , within area *I* and *III* in Fig 2.3 (a), when  $t$  tends to infinity, it can approach to the equilibrium point, and within area *II* and *IV* in Fig 2.3 (a), when  $t$  tends to infinity, it cannot approach to the equilibrium point. The discussion of parameter condition *II* is similar.

Secondly, cycle existing in the original phase diagram is also taken into consideration in this dissertation. As a matter of fact, biological cycle is generally stable, which means after a period of time, it is doomed that it will be free from the unavoidable impact or influence of the real world. When certain disturbance makes one point  $p$  in the diagram to self-cycle  $\Gamma_1$  and deviate to the nearby point  $q$  at the time  $t_1$ , and then such disturbance stops immediately. Afterwards, the diagram gradually returns to the cycle  $\Gamma_1$  along the route of  $q$ , just as shown in Fig 2.7. As to the system studied in this dissertation, when parameters  $\sigma_1$  and  $\sigma_2$  meet conditions *III* and *VI*, it means the phase diagram of the corresponding original equation group begins to cycle. If such parameter condition is met, changing the value of parameters  $\sigma_1$  and  $\sigma_2$  can create more cycles in the phase diagram of the original equation group, as shown in Fig 2.8. Disturbance from the outside world can make one point deviate from its self-cycle to other cycles and retain the point at the new cycle, unless another disturbance sends the point back to the original cycle. Otherwise, it is almost impossible for the point to gradually return to the original cycle along certain route. Therefore, the cycle studied in this dissertation is not ecologically stable.

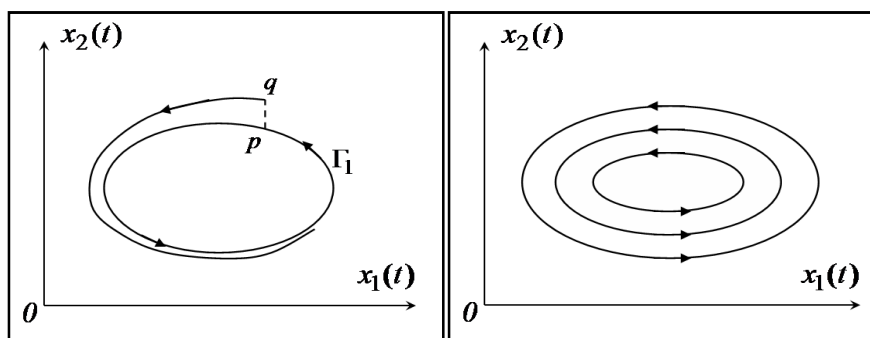


Fig 2.7. Ecological cycle.

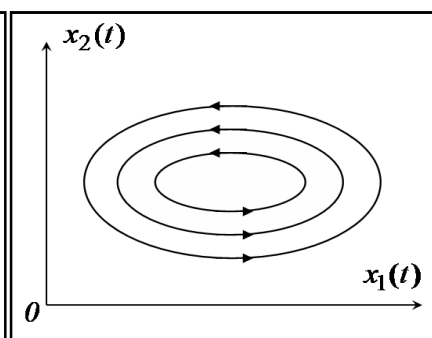


Fig 2.8. Cycles in the phase of the original equation group.

## THE SIGNIFICANCE OF INCOME PARAMETERS AND SUPERVISION COUNTERMEASURES

This paper considers the income parameters in the symbiosis system of majority and minority shareholders' income in listed companies under different conditions, illustrates the phase diagram of the original equation group and analyzes the equilibrium point and the stability of cycle in the phase diagram. It is hoped that through the detailed discussion of majority and minority shareholders' income changing trend, more reasonable strategies of realizing the symbiosis between majority and minority shareholders can be offered, as shown in Table 2 to Table 6.

In accordance with the different statuses analyzed above, four different supervision countermeasures are suggested here. They are:

Strategy A: reinforce supervision. In case majority shareholders encroach the interest of minority shareholders and lead to a rise of their income and a drop of the latter's income, supervision should be reinforced, encroach behaviours should be investigated, so as to protect the interest of medium and small investors;

Strategy B: foster more energetically. In case majority and minority shareholders' income drop in the same time, offer the listed companies some support through policies and encourage majority shareholders' "propping" behaviour, with an aim to help enterprises walk back to the normal operation.

Strategy C: conduct precaution-oriented supervision. Although the income of majority and minority shareholders is rising, the continuously accumulated wealth will definitely trigger majority shareholder's desire to encroach. Therefore, any early trends must be caught early, supervision must be conducted in time in a targeted way, so as to avoid "tunnelling";

Strategy D: give priority to precaution in fostering. When majority shareholders start implementing "propping" for the interest of their enterprises, minority shareholders will benefit from it, and that's when medium and small investors should be encouraged to enter the market to create better investment environment and avoid bigger risks caused by the broken capital chain.

**Table 2: Under condition I ,  $\frac{dx_1}{dt}$  and  $\frac{dx_2}{dt}$  's symbol, changing trend and supervision countermeasures in four areas of Fig 2.2(a).**

Area	$\frac{dx_1}{dt}$ and $\frac{dx_2}{dt}$ Symbol	Income Changing Trend	Supervision Countermeasures
I	$\frac{dx_1}{dt} > 0, \frac{dx_2}{dt} < 0$	Majority shareholders' income rises while medium-minority shareholders' income drops	Strategy A
II	$\frac{dx_1}{dt} > 0, \frac{dx_2}{dt} > 0$	Majority and minority shareholders' income rise	Strategy C
III	$\frac{dx_1}{dt} < 0, \frac{dx_2}{dt} > 0$	Majority shareholders' income drops while medium-minority shareholders' income rises	Strategy D
IV	$\frac{dx_1}{dt} < 0, \frac{dx_2}{dt} < 0$	Majority and minority shareholders' income drop	Strategy B

**Table 3: Under condition IV ,  $\frac{dx_1}{dt}$  and  $\frac{dx_2}{dt}$  's symbol, changing trend and supervision countermeasures in four areas of Fig 2.3 (a).**

Area	$\frac{dx_1}{dt}$ and $\frac{dx_2}{dt}$ Symbol	Income Changing Trend	Supervision Countermeasures
I	$\frac{dx_1}{dt} > 0, \frac{dx_2}{dt} < 0$	Majority shareholders' income rises while medium-minority shareholders' income drops	Strategy A
II	$\frac{dx_1}{dt} < 0, \frac{dx_2}{dt} < 0$	Majority and minority shareholders' income drop	Strategy B
III	$\frac{dx_1}{dt} < 0, \frac{dx_2}{dt} > 0$	Majority shareholders' income drops while medium-minority shareholders' income rises	Strategy D
IV	$\frac{dx_1}{dt} > 0, \frac{dx_2}{dt} > 0$	Majority and minority shareholders' income rise	Strategy C

Parameters  $\sigma_1$  and  $\sigma_2$  meet condition I and II, which mean medium and minority shareholders contribute less to majority shareholder's income while majority shareholders contribute more to minority shareholder's income. In condition IV and V, which mean the contrary.



**Table 4: Under condition II,  $\frac{dx_1}{dt}$  and  $\frac{dx_2}{dt}$ 's symbol, changing trend and supervision countermeasures in four areas of Fig 2.4 (a).**

Area	$\frac{dx_1}{dt}$ and $\frac{dx_2}{dt}$ Symbol	Income Changing Trend	Supervision Countermeasures
I	$\frac{dx_1}{dt} > 0, \frac{dx_2}{dt} < 0$	Majority shareholders' income rises while medium-minority shareholders' income drops	Strategy A
II	$\frac{dx_1}{dt} > 0, \frac{dx_2}{dt} > 0$	Majority and minority shareholders' income rise	Strategy C
III	$\frac{dx_1}{dt} < 0, \frac{dx_2}{dt} > 0$	Majority shareholders' income drops while medium-minority shareholders' income rises	Strategy D

**Table 5: Under condition V,  $\frac{dx_1}{dt}$  and  $\frac{dx_2}{dt}$ 's symbol, changing trend and supervision countermeasures in four areas of Fig 2.5 (a).**

Area	$\frac{dx_1}{dt}$ and $\frac{dx_2}{dt}$ Symbol	Income Changing Trend	Supervision Countermeasures
I	$\frac{dx_1}{dt} > 0, \frac{dx_2}{dt} < 0$	Majority shareholders' income rises while medium-minority shareholders' income drops	Strategy A
II	$\frac{dx_1}{dt} < 0, \frac{dx_2}{dt} < 0$	Majority and minority shareholders' income drop	Strategy B
III	$\frac{dx_1}{dt} < 0, \frac{dx_2}{dt} > 0$	Majority shareholders' income drops while medium-minority shareholders' income rises	Strategy D

Parameters  $\sigma_1$  and  $\sigma_2$  meet condition I and V mean when majority and medium and minority shareholders' income reach a balanced state,  $\sigma_1$  should be smaller while  $\sigma_2$  should be larger for each medium and minority shareholder. In condition II and IV, which mean the contrary.

When parameters  $\sigma_1$  and  $\sigma_2$  meet condition III and VI, the cycle shown in Fig 2.8 (b) will show up in the phase diagram of the original equation group.

**Table 6: Under III and VI,  $\frac{dx_1}{dt}$  and  $\frac{dx_2}{dt}$ 's changing trend and supervision countermeasures in the four arcs of Fig 2.6 (b).**

Arcs	Income Changing Trend	Supervision Countermeasures
cd	Majority's income rises but the rate decreases; medium and minority shareholders' income rises and the rate also rises	Strategy C
da	Majority's income drops and the rate increases; medium and minority shareholders' income rises but the rate decreases	Strategy D
ab	Majority's income drops but the rate decreases; medium and minority shareholders' income drops and the rate also rises	Strategy B
bc	Majority's income rises and the rate also rises; medium and minority shareholders' income drops and the rate also decreases	Strategy A

## CONCLUSION

On the basis of previous studies, this paper further starts from the angle of ecology and applies non-linear dynamics to describe and analyse shareholders' relations and behaviours. Firstly, this paper discussed bifurcations of the equilibrium point and the stability of equilibrium point  $(\frac{K_1(-1+\sigma_1)}{\sigma_1\sigma_2-1}, -\frac{K_2(-1+\sigma_2)}{\sigma_1\sigma_2-1})$  in the symbiosis system of majority and minority shareholders' income in listed companies. Secondly, the positive income of majority and minority shareholders and the periodic changes as time goes by under certain parameter conditions are explored, namely, the closed orbit in the phase diagram of majority and minority shareholders' income symbiosis system under

such conditions and the stability issue are discussed. Through the above study, the following conclusions are drawn:

- (1) The symbiosis between majority and minority shareholders is feasible, which means they can reach a mutually beneficial and win-win symbiotic status;
- (2) By virtue of a series means, for example proper supervision, regulatory constraints and external audit, reasonable external environment is created, which can further influence majority and minority shareholders' behaviours and reach a dynamic stability;
- (3) Under certain conditions, majority shareholders' "tunnelling" and "propping" show up alternatively. Proper guidance should be given to the supervision Layla, so that the listed companies' operation can be as stable as possible;
- (4) After majority shareholders implement "tunnelling" for a period of time, their income will probably drop sometimes, and it means "tunnelling" is a short-term behaviour that should be effectively constrained to avoid minority shareholders' interest loss and market environment deterioration because of majority shareholders' encroaching behaviour;
- (5) Majority shareholders' "propping" behaviour will be conducive to the whole income, so it should be encouraged by policies. In the meanwhile, further loss caused by market environment deterioration should be avoided;
- (6) The behavioural characteristics of the shareholder group under different statuses are analyzed to offer basis for the decision-making in supervision management.

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