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On interval arithmetic method of connection number a+bi

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ABSTRACT

In this paper, the similarity between connection number and interval number is established. Then four arithmetic methods on connection number including addition, subtraction, multiplication and division are constructed via the corresponding operations on interval number. At last, some operating laws are discussed.

Key words: Set Pair Analysis, Connection number, Arithmetic, Interval number

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INTRODUCTION

Set Pair Analysis, SPA for short, a new system analysis technology proposed in 1989, proposed a new type of digital form-connection number, which describes the comprehensive features of complex phenomenon mixed with uncertainty in the nature world [1-3]. Currently, so many significant results of SPA have received in hydrological analysis, network planning, service evaluation and other areas [2-8]. However, in many applications, the imperfect of basic methodology restricts the further development. This issue has aroused the concern of scholars; especially the connection number computational problems have become one of the hot. Various methods scattered among the literatures [9, 10].

Huang and Zhao have proposed a kind of Arithmetic for connection number a+bi in 2000 [9]. In multiplication, the product of a1+b1i and a2+b2i is still a connection number a+bi, and a=a1a2+b1b2，b=a1b2+a2 b1. As a special case the product of b1i and b2i is b1b2, namely, two uncertain quantities become determined after multiplying. The amount of uncertainty disappeared in the process of computing. Liu and Zhao gave complex algorithms of connection number [10]. The connection number was rewritten as $\mu = r(\cos\theta + i\sin\theta), r = \sqrt{a^2+b^2}$. The product of i and itself was treated as i2 and fussed the value on -1 (complex arithmetic method).

So, connection number had many operation methods, but there is not a standardized theoretical system. Among the operations, uncertainty quantities are often treated as certain quantity, which under normal circumstances is not established. Therefore, the operations of connection number still need further discussion and research.

Uncertainty is the basic characteristics of connection number and should not disappear according to the different computing methods, but should always run through the operation [11]. As the coexistence body of certainty and uncertainty, connection number's operations should be different from the ordinary number. In fact, if all values of i are traversed, a+bi coincided with an interval centered of a. Therefore, the authors introduce the algorithms of interval number, and construct a new kind of four algorithms on connection number.

BASIC CONCEPTS

Set Pair Analysis

Set Pair Analysis studies two sets (systems) from the three aspects, identity, difference and contrary. The core idea is that any system is constituted by certain and uncertain information. In this system, certainty and uncertainty are interlinked and influenced each other, and they can be transformed into each other under certain conditions. The certainty and uncertainty are described with the IDC connection degree $\mu = a + bi + cj$.

Definition 2.1 Suppose A and B be two given sets, and H. (A, B) denote a set pair made up with the two sets. Under some specific background W , set pair H have N features, in which S features are mutual of A and B , P features are opposite of A and B , F features are neither mutual nor opposite of A and B. We define the ratio as follows:

$\frac{S}{N}$ is the identity degree of A and B; $\frac{F}{N}$ is the discrepancy degree of A and B; $\frac{P}{N}$ is the contrary degree of A and B.

Then the connection degree of set pair $H = (A, B)$ can be recorded as: $\mu = \frac{S}{N} + \frac{F}{N} i + \frac{P}{N} j$.

For short, $\frac{S}{N}$ is written as a, $\frac{F}{N}$ is written as b and $\frac{P}{N}$ is written as c. Then, the connection degree can be recorded as $\mu = a + bi + cj$, where i is the coefficient of the discrepancy degree, an value between -1 and 1; the j is the coefficient of the contrary degree, and is specified as -1. Obviously, $0 \le a, b, c \le 1$, and a b c 1.

Many times, $a + bi + cj$ is written as $a + bi$, $a + cj$ or $bi + cj$ for short, and the most one of important types is $a + bi$, which is studied in this paper.

Interval number and its operations

Definition 2.2 (Atanu Sengupta and Tapan Kumar Pal [11]) in the real number filed R, $X^I = [\underline{x}, \overline{x}] = \{x \mid \underline{x} \le x \le \overline{x}\}$ is called close interval number, where $\underline{x}, \overline{x} \in R$.

Definition 2.3 (QY Wang and RH Lv [12]) suppose $X^I = [\underline{x}, \overline{x}]$ and $Y^I = [\underline{y}, \overline{y}]$ are two close interval numbers, and then

$$X^I + Y^I = [\underline{x}, \overline{x}] + [\underline{y}, \overline{y}] = [\underline{x} + \underline{y}, \overline{x} + \overline{y}],$$

$$X^I - Y^I = [\underline{x}, \overline{x}] + [\underline{y}, \overline{y}] = [\underline{x} - \overline{y}, \overline{x} - \underline{y}],$$

$$X^I * Y^I = [\underline{x}, \overline{x}] * [\underline{y}, \overline{y}] = [\min\{\underline{x} * \underline{y}, \underline{x} * \overline{y}, \overline{x} * \underline{y}, \overline{x} * \overline{y}\}, \max\{\underline{x} * \underline{y}, \underline{x} * \overline{y}, \overline{x} * \underline{y}, \overline{x} * \overline{y}\}],$$

$$X^I / Y^I = [\underline{x}, \overline{x}] / [\underline{y}, \overline{y}] = [\underline{x}, \overline{x}] / [\frac{1}{\underline{y}}, \frac{1}{\overline{y}}] \ \ if\ 0 \notin [\underline{y}, \overline{y}].$$

Translation between connection number and interval number

Theorem 2.1. Suppose a connection number $a + bi$. According to the Definition 2.2, $i \in [-1,1]$, so $a + bi \xrightarrow{i \in [-1,1]} [a - b, a + b]$.

Theorem 2.2. According to the Theorem 1, let $[\underline{x}, \overline{x}] = a + bi$ and then the following results are obvious.

$a - b = \underline{x},\ a + b = \overline{x} \quad \Rightarrow \quad a = \frac{\overline{x} + \underline{x}}{2}, b = \frac{\overline{x} - \underline{x}}{2}$; So, $[\underline{x}, \overline{x}] = \frac{\overline{x} + \underline{x}}{2} + \frac{\overline{x} - \underline{x}}{2} i$.

INTERVAL ARITHMETIC

Assume:

$\mu_1 = a_1 + b_1 i$, $\mu_2 = a_2 + b_2 i$, $\mu_3 = a_3 + b_3 i$, and $a_1 b_1 \neq 0, a_2 b_2 \neq 0, a_3 b_3 \neq 0$.

Then according to the Theorem 2.1, we can get

$\mu_1 = [a_1 - b_1, a_1 + b_1]$, $\mu_2 = [a_2 - b_2, a_2 + b_2]$, $\mu_3 = [a_3 - b_3, a_3 + b_3]$.

Let $A_1 = a_1 - b_1$, $B_1 = a_1 + b_1$, $A_2 = a_2 - b_2$, $B_2 = a_2 + b_2$, $A_3 = a_3 - b_3$, $A_3 = a_3 + b_3$.

So $\mu_1 = [A_1, B_1]$, $\mu_2 = [A_2, B_2]$, $\mu_3 = [A_3, B_3]$.

Definition 3.1 (Addition)

$\mu_1 + \mu_2 = [a_1 - b_1 + a_2 - b_2, a_1 + b_1 + a_2 + b_2] = [(a_1 + a_2) - (b_1 + b_2), (a_1 + a_2) + (b_1 + b_2)] = (a_1 + a_2) + (b_1 + b_2)i$.

Definition 3.2 (Subtraction)

$\mu_1 - \mu_2 = [a_1 - b_1 - a_2 - b_2, a_1 + b_1 - a_2 + b_2] = [(a_1 - a_2) - (b_1 + b_2), (a_1 - a_2) + (b_1 + b_2)] = (a_1 - a_2) + (b_1 + b_2)i$.

For example, if $\mu_1 = 0.5 + 0.5i$, $\mu_2 = 0.7 + 0.3i$, Then $\mu_1 - \mu_2 = -0.2 + 0.8i$. This result can be also written as $\mu_1 - \mu_2 = -0.2 + 0.8i = 0.8i + 0.2j$.

Definition 3.3 (Multiplication)

$$\begin{aligned}\mu_1 * \mu_2 &= [\min\{(a_1 - b_1)*(a_2 + b_2),(a_2 - b_2)*(a_1 + b_1),(a_1 - b_1)*(a_2 - b_2),(a_1 + b_1)*(a_2 + b_2)\},\\ &\quad \max\{(a_1 - b_1)*(a_2 + b_2),(a_2 - b_2)*(a_1 + b_1),(a_1 - b_1)*(a_2 - b_2),(a_1 + b_1)*(a_2 + b_2)\}]\\ &= [\min\{A_1A_2, A_1B_2, B_1A_2, B_1B_2\}, \max\{A_1A_2, A_1B_2, B_1A_2, B_1B_2\}]\\ &= \frac{\max\{A_1A_2, A_1B_2, B_1A_2, B_1B_2\} + \min\{A_1A_2, A_1B_2, B_1A_2, B_1B_2\}}{2}\\ &\quad + \frac{\max\{A_1A_2, A_1B_2, B_1A_2, B_1B_2\} - \min\{A_1A_2, A_1B_2, B_1A_2, B_1B_2\}}{2} i\end{aligned}$$.

Let $m = \min\{A_1A_2, A_1B_2, B_1A_2, B_1B_2\}, M = \max\{A_1A_2, A_1B_2, B_1A_2, B_1B_2\}$, then $\mu_1 * \mu_2 = [m, M] = \frac{M+m}{2} + \frac{(M-m)}{2} i$.

Definition 3.4 (Division) ($a_2 \neq b_2$) $\mu_1 / \mu_2 = \mu_1 * (1/\mu_2) = [\min\{\frac{A_1}{A_2}, \frac{A_1}{B_2}, \frac{B_1}{A_2}, \frac{B_1}{B_2}\}, \max\{\frac{A_1}{A_2}, \frac{A_1}{B_2}, \frac{B_1}{A_2}, \frac{B_1}{B_2}\}]$.

Let $m' = \min\{\frac{A_1}{A_2}, \frac{A_1}{B_2}, \frac{B_1}{A_2}, \frac{B_1}{B_2}\}, M' = \max\{\frac{A_1}{A_2}, \frac{A_1}{B_2}, \frac{B_1}{A_2}, \frac{B_1}{B_2}\}$, then $\mu_1 / \mu_2 = [m', M'] = \frac{M'+m'}{2} + \frac{M'-m'}{2} i$.

OPERATING LAWS

Theorem 4.1. The Addition operation of connection number meets the following properties.

1) $\mu_1 + \mu_2 = \mu_2 + \mu_1$.
2) $\mu_1 + (\mu_2 + \mu_3) = (\mu_1 + \mu_2) + \mu_3$.

Theorem 4.2. The Subtraction operation of connection number meets the following properties.

1) $-\mu_1 = -a_1 + b_1 i$, especially, if $a_1 = 0$, then $-\mu_1 = b_1 i = \mu_1$.
2) $\mu_1 - \mu_1 = 2b_1 i$.
3) $\mu_1 - (\mu_2 + \mu_3) = \mu_1 - \mu_2 - \mu_3 = \mu_1 - \mu_3 - \mu_2$.

Theorem 4.3. The Multiplication operation of connection number meets the following properties.

1) If $a_1 = a_2 = 0$, then $\mu_1 * \mu_2 = b_1 b_2 i$.

2) $\mu_1 * \mu_2 = \mu_2 * \mu_1$.

3) $\mu_1 * (\mu_2 + \mu_3) = \mu_1 * \mu_2 + \mu_1 * \mu_3$.

Proof. According to the definition, 1) is obvious.

2) $\mu_1 * \mu_2 = [\min\{A_1A_2, A_1B_2, B_1A_2, B_1B_2\}, \max\{A_1A_2, A_1B_2, B_1A_2, B_1B_2\}] = \mu_2 * \mu_1$.

3) $\mu_1 * \mu_2 = [\min\{A_1A_2, A_1B_2, B_1A_2, B_1B_2\}, \max\{A_1A_2, A_1B_2, B_1A_2, B_1B_2\}]$.

$$\begin{aligned}\mu_1 * (\mu_2 + \mu_3) &= \mu_1 * [(a_2 + a_3) + (b_2 + b_3)i] \\ &= (a_1 + b_1 i) * [(a_2 + a_3) + (b_2 + b_3)i] \\ &= [\min\{A_1(A_2 + A_3), A_1(B_2 + B_3), B_1(A_2 + A_3), B_1(B_2 + B_3)\}, \\ &\quad \max\{A_1(A_2 + A_3), A_1(B_2 + B_3), B_1(B_1 + A_3), B_1(B_2 + B_3)\}]\end{aligned}$$.

$\mu_1 * \mu_3 = [\min\{A_1A_3, A_1B_3, B_1A_3, B_1B_3\}, \max\{A_1A_3, A_1B_3, B_1A_3, B_1B_3\}]$.

$$\begin{aligned}\mu_1 * \mu_2 + \mu_1 * \mu_3 &= [\min\{A_1A_2, A_1B_2, B_1A_2, B_1B_2\}, \max\{A_1A_2, A_1B_2, B_1A_2, B_1B_2\}] \\ &\quad + [\min\{A_1A_3, A_1B_3, B_1A_3, B_1B_3\}, \max\{A_1A_3, A_1B_3, B_1A_3, B_1B_3\}] \\ &= [\min\{A_1A_2, A_1B_2, B_1A_2, B_1B_2\} + \min\{A_1A_3, A_1B_3, B_1A_3, B_1B_3\}, \\ &\quad \max\{A_1A_2, A_1B_2, B_1A_2, B_1B_2\} + \max\{A_1A_3, A_1B_3, B_1A_3, B_1B_3\}]\end{aligned}$$.

So $\mu_1 * (\mu_2 + \mu_3) = \mu_1 * \mu_2 + \mu_1 * \mu_3$.

For example, $\mu_1 = 0.3 + 0.5i$, $\mu_2 = 0.7 + 0.3i$, then

$\mu_1 = 0.3 + 0.5i = [-0.2, 0.8] = [A_1, B_1]$, $\mu_2 = 0.7 + 0.3i = [0.4, 1] = [A_2, B_2]$.

$$\mu_1 * \mu_2 = [\min\{-0.8, -0.2, 0.32, 0.8\}, \min\{-0.8, -0.2, 0.32, 0.8\}] = [-0.8, 0.8] = \frac{-0.8 + 0.8}{2} + \frac{0.8 - (-0.8)}{2} i = 0.8i \cdot$$

Theorem 4.4. The Division operation of connection number meets the following properties.

1) $1/\mu_1$ called the reciprocal of μ_1, and

$$1/\mu_1 = [\min\{\frac{1}{B_1}, \frac{1}{B_2}\}, \max\{\frac{1}{B_1}, \frac{1}{B_2}\}] = \frac{\max\{\frac{1}{B_1}, \frac{1}{B_2}\} + \min\{\frac{1}{B_1}, \frac{1}{B_2}\}}{2} + \frac{\max\{\frac{1}{B_1}, \frac{1}{B_2}\} - \min\{\frac{1}{B_1}, \frac{1}{B_2}\}}{2} i \ .$$

2)

$$\begin{aligned}\mu_1 / 1 &= [\min\{A_1, B_1\}, \max\{A_1, B_1\}] \\ &= \frac{\max\{A_1, B_1\} + \min\{A_1, B_1\}}{2} + \frac{\max\{A_1, B_1\} - \min\{A_1, B_1\}}{2} i = \frac{a_1 + b_1 + a_1 - b_1}{2} + \frac{a_1 + b_1 - a_1 + b_1}{2} i = a_1 + b_1 i = \mu_1\end{aligned}$$.

3) $\mu_1 / \mu_2 / \mu_3 = \mu_1 / \mu_3 / \mu_2$.

CONCLUSION

This paper has established the interval arithmetic methods of connection number a+bi. In fact, connection number a+bi is an interval number in some sense. The connection number and interval number can translate from one to the other one. For example, 0.3+0.7i can be written as [0.3-0.7, 0.3+0.7], that is, [-0.4, 1]. Accordingly, interval number [0.1, 0.5] can be translated to [0.3-0.2, 0.3+0.2], and then 0.3+0.2i.

What's more, the four arithmetic of a+bi were discussed, and the main results:

$\mu_1 + \mu_2 = (a_1 + a_2) + (b_1 + b_2)i$.

$$\mu_1 - \mu_2 = (a_1 - a_2) + (b_1 + b_2)i\ .$$

$$\mu_1 * \mu_2 = \frac{M+m}{2} + \frac{(M-m)}{2}i\ .$$

$$\mu_1 / \mu_2 = \frac{M'+m'}{2} + \frac{M'-m'}{2}i\ .$$

Some operating laws were given and proved. Future works should deal with the following points: The more operating methods may be discussed, such as exponentiation, trigonometric and so on. Some theorems were considered in some special situation, so the general model of a+bi should be studied deeply.

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REFERENCES

[1] Keqin Zhao. *CAAI Transactions on Intelligent Systems*, **2007**, 2(5), 20-35.
[2] Chunying Zhang; Fengchun Liu; Baoxiang Liu. *Computer Engineering and Design*, **2007**, 28(20), 4871-4873.
[3] Decai Huang; Jiangqian Chen. *Compute Science*, **2012**, 39(1), 210-214.
[4] Huimei Wang; Ming Xian; Guoyu Wang. *Compute Science*, **2012**, 39(4), 53-56.
[5] Chao Xu; Shuxin Du; Yi Luo. *Bulletin of Science and Technology*, **2013**, 29(3), 208-212.
[6] Wensheng Wang; Juliang Jin; Jing Ding. *Science in china Press*, **2009**, 39(9): 1529-1534.
[7] Decai Huang; Keqin Zhao; Yaozhong Lu. *Journal of Zhejiang University of Technology*, **2000**, 28(3), 190-194.
[8] Xiumei Liu; Keqin Zhao. *Mathematics in Practice and theory*, **2008**, 38(23), 57–64.
[9] Atanu Sengupta;Tapan Kumar Pal. *European Journal of Operational Research*, **2000**, 127, 28-43.
[10] Qingyin Wang; Ruihua Lv. *Mathematics in practice and theory*, **2005**, 35(6), 216-222.
[11] Zhang B.; Zhang S.; Lu G.. *Journal of Chemical and Pharmaceutical Research*, **2013**, 5(9), 256-262.