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Numerical simulation of one-dimensional flood with WENO scheme

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ABSTRACT

This paper is concerned with a mathematical model for simulating hydrodynamics of 1D flood flows with the WENO(Weighted Essentially Non-oscillatory Scheme) scheme. The time discretization uses the Runge-Kutta TVD(Total Variation Diminishing) scheme. By using the model, the flow property of dam-break was calculated, and the flow velocity and water depth were obtained. The calculated results show that the WENO scheme has higher accuracy and better stability, and has the ability to automatically capture shock waves, and may suppress the oscillations of numerical solution. This model can effectively simulate the hydrodynamics of 1D river flow.

Keywords: WENO scheme; simulation; flood

INTRODUCTION

The flood wave caused by a dam failure can result in the loss of human lives and has a severe economic impact. Therefore, significant efforts have been carried out over the years to produce methods for determination of the extent and timing of the flood wave. Most of these methods are based on the solution of the Shallow Water equations. There are a number of accurate and efficient methods available to solve the Shallow Water equations [1-4].

It is an important basis for validating the numerical method whether the scheme can capture the dam-break bore waves accurately or not. This gives rise to an increasing interest in solving such a problem. From 1980 to 2000 several finite-difference schemes that handle discontinuities effectively were used to compute open-channel flows, such as the approximate Riemann solver [3,4]. Based on the above research results, the goal of the current work is to develop a mathematical model capable of dealing with hydraulic discontinuities such as steep fronts, hydraulic jump and drop, etc. The water governing equations has been solved by the WENO scheme and the Finite Volume Method on unstructured grid.

MATHEMATICAL MODEL

Flow Governing equations

The one-dimensional equations in conservative form in Cartesian coordinates for unsteady gradually varied flow in open channels are[5].

$$\frac{\partial q}{\partial t}+\frac{\partial f(q)}{\partial x}=b(q) \tag{1}$$

in which $q=\begin{bmatrix} h \\ hu \end{bmatrix}$, $f(q)=\begin{bmatrix} hu \\ hu^2+gh^2/2 \end{bmatrix}$, $b(q)=\begin{bmatrix} 0 \\ gh(S_{ax}-S_{fx}) \end{bmatrix}$, and q is the conservation physical quantity; $f(q)$ is the flux vector in x direction; $b(q)$ is the source term vector; h is flow depth; u is depth-averaged velocity in the x-direction; g is the gravity acceleration; S_{ox} is the channel bottom slope in the x-direction and is defined as $S_{ox} = -\partial Z_0 / \partial x$, where Z_0 is the bottom elevation; and S_{fx} is the friction slope in the x-direction, computed using the steady state friction formula $S_{fx} = (n^2 u\sqrt{u^2+v^2}) / h^{4/3}$, in which n is the Manning's roughness coefficient.

Discretization of the flow equation

The integral discretization of Eq. (1) Leads to:

$$q_i^{n+1} = q_i^n - \lambda\left(f(\bar{q})_{i+1/2}^n - f(\bar{q})_{i-1/2}^n \right) + \Delta t b_i \tag{2}$$

in which q^n and q^{n+1} are the conservation variables for n and n+1 time steps; $\lambda = \Delta t / \Delta x$; Δt is time step; Δx is space step; superscript n indicates the time layer; subscript i denotes space node; $\bar{f}_{i+1/2}^n$ and $\bar{f}_{i-1/2}^n$ are the numerical flux in equation (1).

The key problem of solution of Eq. (2) is to determine the normal flux of $f(\bar{q})$. The values of q or $\bar{q}$ may be discontinuous on the contacting surface of the control volumes, which is a FVM discontinuous problem, and brings difficulties to the solution of $f(\bar{q})$, and is written as $f(\bar{q}) = f(q_L, q_R)$. The computation of the normal flux $f(\bar{q})$ has two important aspects: one is the computation of $f(q_L, q_R)$, the other is the computation of q_L and q_R (usually is called reconstruction).

Solution of normal fluxes using FDS(Flux Difference Splitting)

The computation of transformation normal fluxes on crossing unit boundary uses Roe's average FDS method [6]:

$$f(\bar{q}) = \frac{1}{2}\left\{ f\left(q_{i+1/2}^L\right) + f\left(q_{i+1/2}^R\right) - |\tilde{a}|\left(q_{i+1/2}^R - q_{i+1/2}^L\right) \right\} \tag{3}$$

where the Roe's speed is $\tilde{a} = \dfrac{f\left(q_{i+1/2}^R\right) - f\left(q_{i+1/2}^L\right)}{q_{i+1/2}^R - q_{i+1/2}^L}$,and $q_{i+1/2}^L$ and $q_{i+1/2}^R$ are the values of q at the left and right sides of the point $x_{i+1/2}$, respectively; thus the normal flux of the discretization equation (3) may be obtained. The values of $q_{i+1/2}^L$ and $q_{i+1/2}^R$ are obtained by weighting and arraying the stencils with the WENO scheme.

Restructuring of WENO scheme

Now the application of the WENO scheme is explained by the computation of $q_{i+1/2}^L$.

The idea used in the WENO schemes is: Given the stencil size k, supposing the k candidate stencils $S_r(i) = \{x_{i-r}, \cdots, x_{i-r+k-1}\}$, $r = 0, \cdots, k-1$, the k kinds of different restructuring ways may be obtained:

$$q_{i+1/2}^{(r)} = \sum_{j=0}^{k-1} c_{rj}\,\bar{q}_{i-r+j}, \quad r = 0, \cdots, k-1, \tag{4}$$

Three possible interpolation stencils are:

$S_0 = \{\bar{q}_{i-2}, \bar{q}_{i-1}, \bar{q}_i\}$, $S_1 = \{\bar{q}_{i-1}, \bar{q}_i, \bar{q}_{i+1}\}$ and $S_2 = \{\bar{q}_i, \bar{q}_{i+1}, \bar{q}_{i+2}\}$

The choice of stencil is shown in Figure 1:

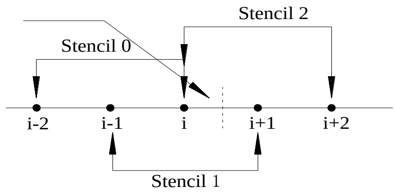

Fig.1 Stencil option of WENO scheme

The expressions for $q^{(r)}_{i+1/2}$ are:

$$q^{(0)}_{i+1/2}=\frac{1}{3}\bar{q}_{i-2}-\frac{7}{6}\bar{q}_{i-1}+\frac{11}{6}\bar{q}_{i},\quad q^{(1)}_{i+1/2}=-\frac{1}{6}\bar{q}_{i-1}+\frac{5}{6}\bar{q}_{i}+\frac{1}{3}\bar{q}_{i+1},\quad q^{(2)}_{i+1/2}=\frac{1}{3}\bar{q}_{i}+\frac{5}{6}\bar{q}_{i+1}-\frac{1}{6}\bar{q}_{i+2}. \quad (5\text{ a, b, c})$$

The definitions of coefficients are in literature [7].

These optimized schemes for all the k candidate stencils are then convexly combined to obtain the WENO schemes. The WENO schemes use all the values of $q^{(r)}_{i+1/2}$ to combine convexly to compute $q^{L}_{i+1/2}$.

$$q^{L}_{i+1/2}=\sum_{r=0}^{k-1}\omega_r\bar{q}^{(r)}_{i+1/2} \quad (6)$$

in which ω_r is the weight.

Determination of Weight ω_r

In order to satisfy the stability and the compatibility, we request $\omega_r\geq 0$ and $\sum_{r=0}^{k-1}\omega_r=1$. Further said that if the function $q(x)$ in all stencils is a smooth function, then the constant d_r exists and satisfies:

$$q^{L}_{i+1/2}=\sum_{r=0}^{k-1}d_r q^{(r)}_{i+1/2}=q\left(x_{i+1/2}\right)+O\left(\Delta^{2k-1}\right) \quad (7)$$

After the simple algebraic operation, these coefficients in Eq.(8) may be determined and satisfies $\sum_{k=0}^{r-1}d^{r}_{k}=1$.

Under the smooth function situation, we have $\omega_r=d_r+O\left(\Delta x^{k-1}\right)$,$r$=0,⋯,$k$-1. In order to make the calculation more effectively, we may use the following form the weight $\omega_r=\alpha_r\Big/\sum_{s=0}^{k-1}\alpha_s$ $\omega_r=\alpha_r\Big/\sum_{s=0}^{k-1}\alpha_s$, $r=0,\cdots,k-1$, $\alpha_r=d_r/(\varepsilon+\beta_r)^2$, in which ε is a real number, generally takes $\varepsilon=10^{-6}$, to avoid the denominator being zero. β_k is called "the smooth factor" as a measure of smooth of the kth possible interpolation region, and its formula is given in Literature [8](Wei, 2001) .

The third-order mathematical expression for the computation of $q^{L}_{i+1/2}$ is:

$$q^{L}_{i+1/2}=\omega_0 * q^{(0)}_{i+1/2}+\omega_1 * q^{(1)}_{i+1/2}+\omega_2 * q^{(2)}_{i+1/2} \quad (8)$$

$q^{R}_{i+1/2}$ is computed by the similar solution method.

After the solutions of $q^{L}_{i+1/2}$ and $q^{R}_{i+1/2}$, the normal fluxes may be obtained by the Roe's average flux- differencing-splitting method.

Time processing

Time processing may use the Rugge-Kutta method with the nature of TVD, for example, regarding the time for *mth* order, $q^{(i)}=\sum_{k=0}^{i=1}\left(\alpha_{ik}q^{(k)}+\Delta t\beta_{ik}L\left(q^{(k)}\right)\right), i=1,\cdots,m$. At initial time step $q^{(0)}=q^n$, after finishing one time-step calculation, $q^{(m)}=q^{n+1}$.

One-Dimensional Dam-Break Simulation

The scenario considered here was the total and instantaneous dam-failure on a flat and frictionless bed. This provides an ideal benchmark test case for shock-capturing schemes since analytical solution has been known. Figure 2 shows the illustration of the dam-break problem, where the initial upstream water depth was h_1 =10 m, and the downstream water depth was h_0 =1.The length of the computational region is 200m, and the dam was located at *x*=200 m. The grid spacing was 1 m. The time step was 1 s. Figure 3 shows the water surface position, and Figure 4 shows the velocity distribution, 7.0s after dam failure by using the model, where the solid line represents the analytical solution and the circle points illustrate the predicted results. It can be seen that the shallower the downstream water depth, then the faster the flood wave travels. The agreement between the analytical and numerical solutions is satisfactory.

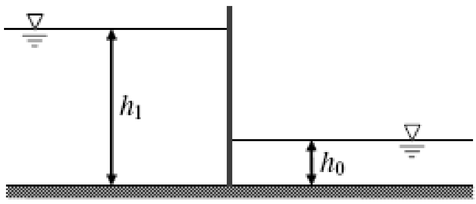

Fig.2 Illustration of the dam-break problem

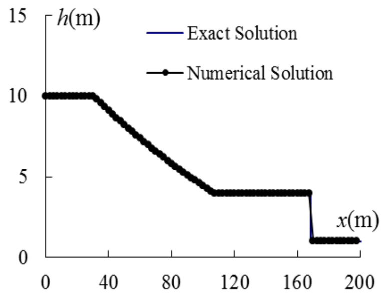

Fig. 3 Water surface position

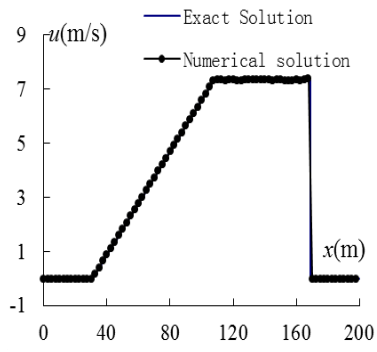

Fig. 4 Velocity distribution

CONCLUSION

The WENO scheme can be employed for the solution of one-dimensional flow equations written in conservative form, and can effectively simulate the rapidly varying water waves. The proposed mathematical model can effectively simulate 1D flow accompanied with a dam-break. The proposed method can also be expanded to 2D the flows.

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