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Non-stationary long memory parameter estimate based on wavelet

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ABSTRACT

In this paper, based on the theory of wavelet transform, presents several new estimation method of time varying long memory parameters, and gives the consistency and asymptotic properties of estimators of these new methods. Finally, this paper studies the behavior of small samples of non-stationary long memory process, gives the reference solution of some factors selected which can affect the accuracy of estimate such as wavelet scale et al. At the same time, this paper compares the advantages and disadvantages of various estimation methods, and proves the effectiveness and robustness of the new methods.

Key words: Wavelet transform; Long memory parameter; Wavelet scale; Estimation method

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INTRODUCTION

There exists much non-stationary and long memory in the economic and financial time series. Because wavelet is able to reveal the characteristics of the data both time domain and frequency domain, it has become an important tool for the research of this kind of time series.This paper, mainly based on the theory of wavelet transform, studies the property of non-stationary long memory time series,first reviews all the existing long memory parameter estimation methods and classifies and analyzes these methods, and points out the advantages and limitations of these estimate methods in the application.Then the local wavelet estimation method of time varying long memory parameter is improved and promoted, and on the basis of this method, this paper puts forward a new estimate method of time varying long memory parameter for non-stationary time series, applying the theory of wavelet transform. At the same time, this paper also proves the consistency and asymptotic property of the estimator obtained by the new method.

Wavelet transform

Wavelet transform is an effective mathematical method developed in recent years. Because it can reveal data characteristics in the time domain and frequency domain,and has the ability of depicting local characteristics of events in the time domain, wavelet transform becomes an ideal tool for studying the non-stationary time series.Wavelet transform is a localized analysis method in the time(or space) and frequency domain, it can finally achieve time segment in high frequency and frequency segment in low frequency through stretch and translation operations to multi-scale refinement of random process.It can automatically adapt to the requirement of time-frequency signal analysis on any details of the signal, and wavelet transform is also called "mathematics microscope".

2.1 Continuous wavelet transform
For any square integrable function$\Psi(t)$, if it satisfies admissibility condition (2.1)

Where, the Fourier transform$\Psi(t)$ of$\Psi(f)$ is a function of frequency f, so$\Psi(t)$ is a basic wavelet or mother wavelet function. And$\Psi(t)$must satisfy the following conditions:
1) , that is, wavelet function has a unit of energy.

2) , that is, wavelet function is integrable and bounded function.
3), that is, wavelet function has zero mean.

The continuous wavelet transform of function(or signal) x(t) is a binary function, which can be defined by the formula

Where, is displacement and scale stretch of the mother wavelet function w , referred to as the wavelet basis function, also called as wavelet. And s is a scale factor(s>0), used for stretching the mother wavelet function, u is time-displacement factor, can be positive or negative,for the displacement of mother wavelet function, and in the formula, t, s and u are continuous variables, therefore, the transformation is called continuous wavelet transform.

2.2 Discrete wavelet transform
Continuous wavelet transform takes a function transformation into binary function, so it contains a large number of additional information in the analysis of a function. The data processed by continuous wavelet transform is extremely huge, which is not conducive to practical application.In order to overcome this shortcoming, continuous wavelet transform needs to be discretized. Although the discrete wavelet transform can also be derived without continuous wavelet transform, we still can use it as "discretization" process of continuous wavelet transform through sampling specific scale corresponding to continuous wavelet transform coefficient[1].It not only reduces the redundancy of continuous wavelet transform coefficients, and is also good to maintain the branch, smoothness and symmetry of the continuous wavelet transform.Discretization of continuous wavelet transform firstly needs to discretize the scale factor s and time-displacement factor u.

The scale factor s and time-displacement factor u is discretized:

So, discrete wavelet transform is:

The discrete wavelet transform of function(or signal) x(t) is:

In the practical application, in order to improve the effectiveness of the wavelet transform, wavelet function of orthogonality is usually constructed. Because the wavelet with the orthogonality can eliminate the correlation caused by redundancy, and can also reduce the calculation error in the process of the transformation results which can accurately reflect the nature of the original signal itself.

2.3 Commonly used wavelets
Haar wavelet (shown as figure 2.1) is derived by the Haar function which is a set of functions orthogonal to each other put forward by a mathematician A.Haar[2] in 1910.Haar wavelet is a filter whose length L is 2, and it can be defined by scale filter (low-pass filter):

According to orthogonal mirror relationship, the wavelet can also be defined by the wavelet filter

(high-pass filter) and .Haar function meets compactly supported and orthogonal wavelet system[3] of Daubechies condition, and it is the simplest wavelet of Daubechies wavelet department(N=1, N denotes the highest vanishing moment of Daubechies wavelet function).They can reach the same regularity Daubechies wavelet of N = 2, but still do not have continuity.Although the Haar wavelet is easy to understand and implement,but because it is a poor approximation of an ideal band-pass filter,it cannot be full close to the application of the real world.

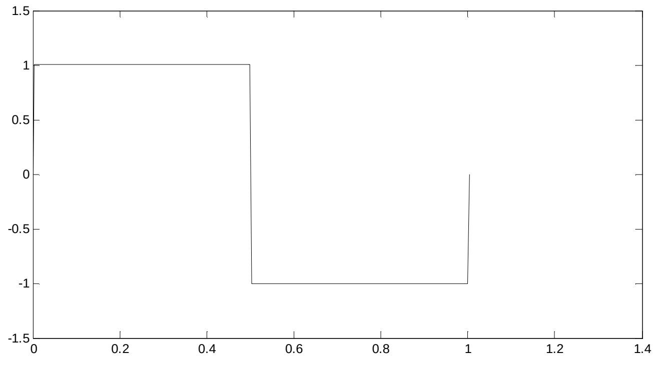

Figure 2.1 Haar wavelet filter

Daubechies wavelet is derived by compactly supported function of the highest vanishing moment, put forward by the famous mathematician Daubechies.Usually these wavelets have not analytic expression, or only a simple analytical expression, which is usually just indicated by the filter array.The simplest wavelet of Daubechies wavelet department is Haar wavelet whose scale filter and the wavelet filter is discontinuous, and it is also the only one discrete wavelet of Daubechies wavelet department. And others are continuous and compactly supported wavelets.With the increase of vanishing moment wavelet is becoming more and more smooth.

Extreme phase wavelet serieD (L) and minimum asymmetric wavelet LA (L) are two kinds of different Daubechies wavelet by choosing different decomposition way, in which L is the length of the filter.Shanii and Yenins[4], at the same time, give theextremum stage and the accurate value of the minimum asymmetric wavelet filter.And longer extremum and the least asymmetry wavelet filter have not a precise form[5].

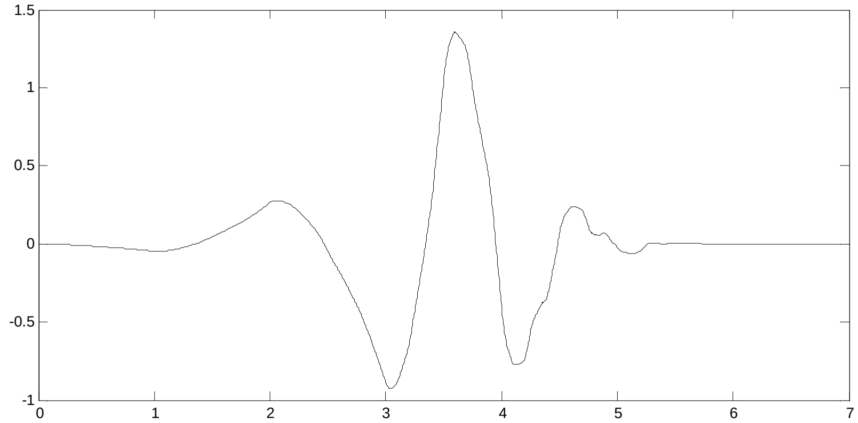

Figure 2.2 db4 wavelet

3 Non-stationary long memory parameter estimate

We find that there are a lot of literature researches on smooth time series model, however,in the actual application, time series in general are not smooth, such as the mean value function of time sequence or variance functions which are time-varying, or other covariance function is a function of time.So many researchers stabilize the non-stationary time series with some simple smooth transforms, for example, difference disposal is applied to time series, subtracting a polynomial trend or a trigonometric function trend, et al.But not all of the non-stationary time series can be stabilized by transform or in some cases a stabilized the original data no longer has any real meaning.Therefore, it is very meaningful to study the non-stationary time series model.

In general, non-stationary long memory model is divided into two types: One is that assuming long memory parameter is a constant but it is beyond the scope of stationary, that is $|d|\geq 2$, this kind of constant non-stationary long memory parameter estimate is usually estimated by the estimate method based on Whittle method. Another kind is to assume that long memory parameter d is a function d(t) about time t and $-1/2<d(t)<1/2$, for this type of the parameters, the study is based on the application of wavelet transform theory.In the telecommunications network, physical signal, seismic survey, and economic and financial aspects, time series has often the characteristic of time varying longer memory,so it is not reasonable to take long memory parameter as constant.In order to obtain the long memory characteristic of data, in this paper, the study is focused on the non-stationary long memory model of the time-varying parameter.

We will create a couple of estimators of time varying long memory parameter d(t), and gives the specific procedures and the consistency identification of these estimators of these estimation methods on the basis of wavelet transform.In the application of the theory of wavelet transform to estimate non-stationary long memory parameters, we find that the selection of wavelet scale has big influence on the estimate precision, so we study a series of sample behaviors in order to provide the best selection of wavelet scales.

This paper proposes the following algorithm to estimate the local self-similar parameter H(t):

1) The maximum overlap discrete wavelet transform is used to , we get a series of wavelet coefficients , where .
2) The sample space [0, 1) is divided into non-overlapping and equilong range, where, l is determined by the length of the sample data.The intervals are
, h=0, 1,, .
3) Choosing an appropriate , makes is the largest scale in the estimate process, which means that wavelet

coefficient conforming to is applied to estimate the self-similar parameter H(t).
4) We splits wavelet coefficient () into parts, and in turn each wavelet coefficient is corresponding to the interval .
5) In any child range, we consider binary data set

In each child range , we obtain local estimate of a parameter H(t) with the ordinary least-squares regression.As a result, we get local estimates of the self-similar parameter.

6) In order to avoid boundary effect, we remove the beginning and the end of the local estimates.H (t) time index is corresponding to the midpoint of each child range , that is ,and using locally weighted regression scatter smoothing smoothes these local estimation points.Then, we get the smooth curve of approximate self-similar parameters.According to the Cavanaugh' proof[6], we can also get the consistency of new estimator : If and . But the limitation of the method is that sample size must be expressed in a binary number.

This paper studies local finite sample behavior of self-similar process with simulation test, and the grid search method is used to provide recommendations for selecting these two quantities.In addition, with the aid of deviation and root mean square error, we compare the two methods.

Table 1 The selection of optimal l and J according to different types of H (t) and different sample size and the root mean square error

		Sample Size					
H(t)	**Method**						
Constant	**Method 1**	l=4, J=7	l=4, J=6	l=4, J=7	l=4, J=8	l=6, J=9	l=6, J=9
		0.1763	0.1651	0.1115	0.0797	0.0659	0.0662
	Method 2	l=4,J=7	l=4, J=6	l=5, J=6	l=7, J=7	l=7, J=8	l=4, J=7
		0.1732	0.1556	0.1254	0.00945	0.0754	0.0581
Linear	**Method 1**	l=4, J=7	l=4, J=6	l=4, J=7	l=4, J=8	l=4, J=9	l=4, J=9
		0.2016	0.1864	0.1433	0.1147	0.0920	0.0933
	Method 2	l=4, J=7	l=5, J=6	l=4,J=4	l=4, J=5	l=6, J=5	l=6, J=6
		0.1716	0.1450	0.1629	0.1198	0.0887	0.0698
Logarithmic	**Method 1**	l=4, J=7	l=4, J=6	l=4, J=7	l=4, J=8	l=4, J=9	l=4, J=9
		0.2027	0.2069	0.1572	0.1252	0.1001	0.1045
	Method 2	l=4, J=7	l=4, J=6	l=4,J=4	l=4, J=5	l=4, J=6	l=5, J=6
		0.1782	0.1704	0.1818	0.1319	0.1057	0.0859
Exponential	**Method 1**	l=4, J=7	l=4, J=6	l=4, J=7	l=4, J=6	l=4, J=9	l=4, J=9
		0.1941	0.0828	0.1367	0.1225	0.0844	0.0828
	Method 2	l=4, J=5	l=4, J=6	l=4, J=4	l=4, J=5	l=4, J=5	l=4, J=6
		0.1738	0.1374	0.1538	0.1158	0.0844	0.0648

From table 1, in most cases, the sample range will be divided into (l=4) child ranges,whichis the optimal choice of making estimation error minimum.At the same time we also notice that ensuring the minimum error of estimate do not need all wavelet coefficients of wavelet scales to participate in the regression calculation.In fact, if the number of non-overlapping equilong child ranges is determined, choosing the number of different wavelet scales has obvious effects to the accuracy of estimate results.The only special case is that the sample size is , with the new estimation method proposed in this paper or the estimation method proposed by Cavanaugh, for the majority of sample ranges equally split, we always choose all of wavelet coefficients corresponding to wavelet scale to participate in the regression in order to achieve the best effect of estimates.It also illustrates the research on the best selection of wavelet scales under different sample sizes is meaningful.

CONCLUSION

This paper analyzes deeply the previous parameter estimation methods,points out the advantages and limitations of these estimate methods in the application, and studies how to apply these methods improve the accuracy of estimates.What’s more, this paper puts forward a new estimation method of time varying long memory parameter. At the same time, we also prove the consistency and asymptotic property of the estimators obtained by the new method.

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