



Research Article

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Multiattribute decision making models and methods using interval-valued fuzzy sets

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ABSTRACT

The concept of interval-valued fuzzy sets is the generalization of the concept of fuzzy sets. The theory of interval-valued fuzzy sets is well suited to dealing with vagueness. Recently, interval-valued fuzzy sets have been used to build soft decision making models that can accommodate imprecise information. However, it seems that there is little investigation on multicriteria decision making using interval-valued fuzzy sets with multiple criteria being explicitly taken into account. In this paper, multiattribute decision making using interval-valued fuzzy sets is investigated, in which multiple criteria are explicitly considered, several linear programming models are constructed to generate optimal weights for attributes, and the corresponding decision-making methods have also been proposed. Feasibility and effectiveness of the proposed method are illustrated using a numerical example.

Key words: Fuzzy set; Interval-valued fuzzy set; Multiattribute decision making; Linear programming model

INTRODUCTION

The theory of fuzzy sets proposed by Zadeh [1] has attracted wide attentions in various fields, especially where conventional mathematical techniques are of limited effectiveness, including biological and social sciences, linguistic, psychology, economics, and more generally soft sciences. In such fields, variables are difficult to quantify and dependencies among variables are so ill-defined that precise characterization in terms of algebraic, difference or differential equations becomes almost impossible. Even in fields where dependencies between variables are well defined, it might be necessary or advantageous to employ fuzzy rather than crisp algorithms to arrive at a solution.

Out of several higher-order fuzzy sets, interval-valued fuzzy sets introduced by Zadeh [2-3] and intuitionistic fuzzy sets introduced by Atanassov [4-5] have been found to be well suited to dealing with vagueness. The concept of an interval-valued fuzzy set can be viewed as an alternative approach to define a fuzzy set in cases where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. In general, the theory of interval-valued fuzzy sets is the generalization of fuzzy sets. Therefore, it is expected that interval-valued fuzzy sets could be used to simulate human decision-making processes and any activities requiring human expertise and knowledge, which are inevitably imprecise or not totally reliable [6-8].

In this paper, multiattribute decision making using interval-valued fuzzy sets is investigated, in which attributes are explicitly considered, several corresponding linear programming models are constructed to generate optimal weights of attributes, and the corresponding decision-making methods are also proposed. This paper is organized as follows. The definitions and properties of interval-valued fuzzy sets are briefly introduced in Section 2. Multiattribute decision-making models with interval-valued fuzzy values are then proposed, and the corresponding linear programming models and methods are established in Section 3. A numerical example and a short conclusion are given in Section 4 and 5, respectively.

DEFINITIONS AND PROPERTIES OF INTERVAL-VALUED FUZZY SETS

Definition1. (Zadeh [1]) Let $X = \{x_1, x_2, \dots, x_n\}$ be a finite set for our considerations. By an interval-valued fuzzy

set A in X , we mean: $A = \{x_i, [A^-(x_i), A^+(x_i)], x_i \in X\}$ where $A^-, A^+ : X \rightarrow [0, 1]$. $[A^-(x_i), A^+(x_i)]$ is the interval of membership function of an element x_i to the set A , while the condition $0 \leq A^-(x_i) \leq A^+(x_i) \leq 1, x_i \in X$ is fulfilled.

The difference $\pi_A(x_i) = A^+(x_i) - A^-(x_i)$ is called an interval-valued fuzzy index and the number $\pi_A(x_i) \in [0, 1]$ should be treated as a hesitancy margin connected with the evaluation degree of each element x_i to a set A . It is one of the most important and original idea distinguishing the interval-valued fuzzy sets theory from the fuzzy sets theory. The family of all interval-valued fuzzy sets in X is denoted by $IVFS(X)$.

Distance between interval-valued fuzzy sets was first introduced by Zadeh [2]. Here, we introduce a normalized Hamming distance, which will be employed in Section 3.

Let A and B be two interval-valued fuzzy sets in the set X . Namely,

$$A = \{x_i, [A^-(x_i), A^+(x_i)], x_i \in X\}, \text{ and } B = \{x_i, [B^-(x_i), B^+(x_i)], x_i \in X\}$$

The normalized Hamming distance between A and B is defined as follows

$$D(A, B) = \frac{1}{2n} \sum_{i=1}^n (|A^-(x_i) - B^-(x_i)| + |A^+(x_i) - B^+(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|) \quad (1)$$

Where $p_A(x_i) = A^+(x_i) - A^-(x_i)$ and $p_B(x_i) = B^+(x_i) - B^-(x_i)$.

Theorem 1. D defined by Eq. (1) is a metric.

Proof. Evidently, D is symmetric and $D(A, A) = 0$.

Conversely, if $D(A, B) = 0$, according to Eq. (1), we must have

$$A^-(x_i) = B^-(x_i), \quad A^+(x_i) = B^+(x_i) \text{ and } p_A(x_i) = p_B(x_i) \text{ for all } x_i \in X.$$

Hence, it follows that $A = B$ according to Definition 1. Thus D is positive definite.

For any interval-valued fuzzy sets A, B and C , where $C = \{x_i, [C^-(x_i), C^+(x_i)], x_i \in X\}$. Using Eq. (1), we have

$$\begin{aligned} D(A, B) &= \frac{1}{2n} \sum_{i=1}^n (|A^-(x_i) - B^-(x_i)| + |A^+(x_i) - B^+(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|) \\ &\leq \frac{1}{2n} \sum_{i=1}^n (|A^-(x_i) - C^-(x_i)| + |A^+(x_i) - C^+(x_i)| + |\pi_A(x_i) - \pi_C(x_i)|) \\ &\quad + \frac{1}{2n} \sum_{i=1}^n (|C^-(x_i) - B^-(x_i)| + |C^+(x_i) - B^+(x_i)| + |\pi_C(x_i) - \pi_B(x_i)|) \\ &= D(A, C) + D(C, B) \end{aligned}$$

i.e., $D(A, B) \leq D(A, C) + D(C, B)$.

So D is triangular. Hence, we have completed the proof of Theorem 1.

If A and B are conventional fuzzy sets, i.e., $A = \{x_i, [A(x_i), A(x_i)], x_i \in X\}$ and $B = \{x_i,$

$$[B(x_i), B(x_i)], x_i \in X\}, D(A, B) \text{ defined by Eq. (1) becomes } D(A, B) = \frac{1}{n} \sum_{i=1}^n |A(x_i) - B(x_i)|$$

If A and B are crisp sets, i.e., $A = \{x_i, [A(x_i), A(x_i)], x_i \in X\}$ and $B = \{x_i, [B(x_i), B(x_i)], x_i \in X\}$, where

$$A(x_i) = \begin{cases} 1 & \text{if } x_i \in X \\ 0 & \text{otherwise} \end{cases} \text{ and } B(x_i) = \begin{cases} 1 & \text{if } x_i \in X \\ 0 & \text{otherwise} \end{cases}$$

$D(A, B)$ is the cardinality of the symmetric difference of A and B , i.e., the set-theoretic difference between their union and intersection.

MODELS AND METHODS FOR MULTIATTRIBUTE DECISION MAKING USING INTERBAL-VALUED FUZZY VALUES

Presentation of multiattribute decision-making problems under interval-valued fuzzy environment

Suppose there exists an alternative set $X = \{x_1, x_2, \dots, x_n\}$ which consists of n $x_i \in X$ non-inferior decision-making alternatives from which a most preferred alternative is to be selected. Each alternative is assessed on m attributes. Denote the set of all attributes $A = \{a_1, a_2, \dots, a_m\}$. Assume that $[A_{ij}^-, A_{ij}^+]$ is the interval membership degree of the alternative $x_i \in X$ with respect to the attribute $a_j \in A$ to the fuzzy concept "excellence", respectively, where $0 \leq A_{ij}^- \leq A_{ij}^+ \leq 1$. In other words, the evaluation of the alternative $x_i \in X$ with respect to the attribute $a_j \in A$ is an interval-valued fuzzy set. The interval-valued indices $\pi_{ij} = A_{ij}^+ - A_{ij}^-$ are such that the larger π_{ij} the higher a hesitation margin of the decision maker as to the "excellence" of the alternative $x_i \in X$ with respect to the attribute $a_j \in A$. Interval-valued indices allow us to calculate the best final result (and the worst one) we can expect in a process leading to a final optimal decision. During the process the decision maker can change his evaluations in the following way. He can increase his evaluation by adding the value of the interval-valued index.

Similarly, assume that $[r_j^-, r_j^+]$ is the interval membership degree of the attribute $a_j \in A$ to the fuzzy concept "importance", respectively, where $0 \leq r_j^- \leq r_j^+ \leq 1$. The interval-valued indices are such that the larger h_j the higher a hesitation margin of decision maker as to the "importance" of the attribute $a_j \in A$. Interval-valued indices allow us to calculate the biggest weight (and the smallest one) we can expect in a process leading to a final decision. During the process the decision maker can change his evaluating weights in the following way. He can increase his evaluating weights by adding the value of the interval-valued index. In addition, in this paper assume that

$$\sum_{j=1}^m r_j^- \leq 1 \text{ and } \sum_{j=1}^m r_j^+ \leq 1 \text{ in order to find weights } r_j \in [0, 1] \text{ (} j = 1, 2, \dots, m \text{) satisfying } r_j^- \leq r_j \leq r_j^+$$

$$\text{and } \sum_{j=1}^m r_j = 1$$

Optimization model of multiattribute decision making under interval-valued fuzzy environment

For each alternative $x_i \in X$, its optimal comprehensive value can be computed via the following programming

$$\max \{z_i = \sum_{j=1}^m b_{ij} r_j\}$$

$$\begin{aligned} & \min \{ z_i^- = \sum_{j=1}^m A_{ij}^- r_j^- \} \\ & \text{s.t. } \begin{cases} r_j^- \leq r_j \leq r_j^+ & (j = 1, 2, \dots, m) \\ \sum_{j=1}^m r_j = 1 \end{cases} \end{aligned} \tag{2}$$

for each $i = 1, 2, \dots, n$.

To solve Eq. (2), we can solve the following two linear programmings

$$\begin{aligned} & \min \{ z_i^- = \sum_{j=1}^m A_{ij}^- r_j^- \} \\ & \text{s.t. } \begin{cases} r_j^- \leq r_j \leq r_j^+ & (j = 1, 2, \dots, m) \\ \sum_{j=1}^m r_j = 1 \end{cases} \end{aligned} \tag{3}$$

for each $i = 1, 2, \dots, n$.

$$\text{and } \max \{ z_i^+ = \sum_{j=1}^m A_{ij}^+ r_j^+ \}$$

$$\begin{aligned} & \max \{ z_i^+ = \sum_{j=1}^m A_{ij}^+ r_j^+ \} \\ & \text{s.t. } \begin{cases} r_j^- \leq r_j \leq r_j^+ & (j = 1, 2, \dots, m) \\ \sum_{j=1}^m r_j = 1 \end{cases} \end{aligned} \tag{4}$$

for each $i = 1, 2, \dots, n$.

Solving Eqs. (3) and (4) by Simplex method, we can obtain their optimal solutions $\bar{r}^i = (\bar{r}_1^i, \bar{r}_2^i, \dots, \bar{r}_m^i)$ and $\underline{r}^i = (\underline{r}_1^i, \underline{r}_2^i, \dots, \underline{r}_m^i)$ respectively. In total, $2n$ linear programmings need to be solved since there are n alternatives in the set X .

After generating the corresponding optimal weight vectors, the optimal comprehensive value of alternative $x_i \in X$ can be computed as an interval $[z_i^-, z_i^+]$, where

$$z_i^- = \sum_{j=1}^m A_{ij}^- \bar{r}_j^i \tag{5} \quad \text{and} \quad z_i^+ = \sum_{j=1}^m A_{ij}^+ \underline{r}_j^i \tag{6}$$

for each $i = 1, 2, \dots, n$. That is, the optimal comprehensive value of the alternative $x_i \in X$ is an interval-valued fuzzy set $A_i = \{x_i, [\sum_{j=1}^m A_{ij}^- \bar{r}_j^i, \sum_{j=1}^m A_{ij}^+ \underline{r}_j^i]\}$. (7)

however, optimal solutions of Eqs. (3) and (4) are different in general, i.e., the weight vectors $\bar{r}_j^i, \underline{r}_j^i$ for all alternatives for all $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, m$. Therefore, the comprehensive values of all n alternatives $x_i \in X$ cannot be compared.

Since X is a non-inferior alternative set, there exists no evident preference on some alternatives. Hence, for each alternative $x_i \in X$, its objective function z_i^- in Eq. (3) should be assigned an equal weight $1/n$. Eq. (3) is then aggregated into the following linear programming:

$$\begin{aligned} \min\{z_0^- &= (\sum_{i=1}^n \sum_{j=1}^m \bar{a}_{ij}^- r_j^-) / n\} \\ \text{s.t.} \quad & r_j^- \leq r_j \leq r_j^+ \quad (j = 1, 2, \dots, m) \\ & \sum_{j=1}^m \bar{a}_{ij}^- r_j = 1 \end{aligned} \tag{8}$$

In a similar way, Eq. (4) is aggregated into the following linear programming

$$\begin{aligned} \min\{z_0^+ &= (\sum_{i=1}^n \sum_{j=1}^m \bar{a}_{ij}^+ r_j^+) / n\} \\ \text{s.t.} \quad & r_j^- \leq r_j \leq r_j^+ \quad (j = 1, 2, \dots, m) \\ & \sum_{j=1}^m \bar{a}_{ij}^+ r_j = 1 \end{aligned} \tag{9}$$

Solving Eqs. (8) and (9) by Simplex method, we can obtain their optimal solutions

$$\bar{r}^0 = (\bar{r}_1^0, \bar{r}_2^0, \dots, \bar{r}_m^0) \text{ and } \overline{\bar{r}}^0 = (\overline{\bar{r}}_1^0, \overline{\bar{r}}_2^0, \dots, \overline{\bar{r}}_m^0) \text{ respectively.}$$

After generating the corresponding optimal weight vectors, the optimal comprehensive value of the alternative $x_i \in X$ can be computed as an interval $[z_i^{0-}, z_i^{0+}]$ where

$$z_i^{0-} = \sum_{j=1}^m \bar{a}_{ij}^- \bar{r}_j^0 \tag{10} \quad \text{and} \quad z_i^{0+} = \sum_{j=1}^m \bar{a}_{ij}^+ \overline{\bar{r}}_j^0 \tag{11}$$

for each $i = 1, 2, \dots, n$. That is, the optimal comprehensive value of the alternative $x_i \in X$ is an interval-valued fuzzy set given by

$$\hat{A}_i = \{x_i, [\sum_{j=1}^m \bar{a}_{ij}^- \bar{r}_j^0, \sum_{j=1}^m \bar{a}_{ij}^+ \overline{\bar{r}}_j^0]\}. \tag{12}$$

In generating the above-interval-valued fuzzy set only two linear programmings (i.e. Eqs. (8) and (9)) need to be solved. However, the optimal solutions of Eqs. (8) and (9) are normally different, so $\bar{r}_j^0 \neq \overline{\bar{r}}_j^0$ in general, or $\bar{r}_j^0 < \overline{\bar{r}}_j^0$ for all $j = 1, 2, \dots, m$. Therefore, it is possible that $z_i^{0-} > z_i^{0+}$. If this is the case, it follows that the interval-valued index is negative.

However, this is not permitted by Definition 1. Note that Eq. (8) is equivalent to the following linear programming

$$\max\{z_0^- = - (\sum_{i=1}^n \sum_{j=1}^m \bar{a}_{ij}^- r_j^-) / n\}$$

$$\begin{aligned} & \tilde{r}_j^- \leq r_j \leq \tilde{r}_j^+ \quad (j = 1, 2, \dots, m) \\ \text{s.t.} & \sum_{j=1}^m \overset{\circ}{a}_{ij} r_j = 1 \end{aligned} \tag{13}$$

Since Eqs. (9) and (13) have the same constraints, they can be combined to formulate the following linear programming

$$\begin{aligned} \max \{z &= (\sum_{i=1}^n \sum_{j=1}^m (\overset{\circ}{a}_{ij}^+ - \overset{\circ}{a}_{ij}^-) r_j) / n\} \\ & \tilde{r}_j^- \leq r_j \leq \tilde{r}_j^+ \quad (j = 1, 2, \dots, m) \\ \text{s.t.} & \sum_{j=1}^m \overset{\circ}{a}_{ij} r_j = 1 \end{aligned} \tag{14}$$

Normally, Eqs. (9) and (13) are not equivalent to Eq. (14). However, Some of solutions of Eqs. (9) and (13) can be generated by solving Eq. (14). Eq. (14) can be rewritten as follows

$$\begin{aligned} \max \{z &= (\sum_{i=1}^n \sum_{j=1}^m p_{ij} r_j) / n\} \\ & \tilde{r}_j^- \leq r_j \leq \tilde{r}_j^+ \quad (j = 1, 2, \dots, m) \\ \text{s.t.} & \sum_{j=1}^m \overset{\circ}{a}_{ij} r_j = 1 \end{aligned} \tag{15}$$

The optimal solution $r^0 = (r_1^0, r_2^0, \dots, r_m^0)^T$ can be obtained solving Eq. (14) or Eq. (15) by Simplex method. Then, the optimal comprehensive value of the alternative $x_i \hat{I} X$ can be computed as an interval $[z_i^{0-}, z_i^{0+}]$, where

$$z_i^{0-} = \sum_{j=1}^m \overset{\circ}{a}_{ij}^- r_j^0 \tag{16} \quad \text{and} \quad z_i^{0+} = \sum_{j=1}^m \overset{\circ}{a}_{ij}^+ r_j^0 \tag{17}$$

for each $i = 1, 2, \dots, n$. That is, the optimal comprehensive value of the alternative $x_i \hat{I} X$ is an interval-valued fuzzy set given by $A_i^0 = \{x_i, [z_i^{0-}, z_i^{0+}]\}$ (18)

Multiaattribute decision-making method under an interval-valued fuzzy environment

Using the above Eq. (14) or Eq. (15), n optimal comprehensive values of A_i^0 all alternatives $x_i \hat{I} X$ ($i = 1, 2, \dots, n$) can be obtained. Now, we are interested in how a final best compromise alternative or the final ranking order of the alternative set X can be generated.

In a similar way to the TOPSIS method proposed by Hwang and Yoon [11], we define the following index for each

alternative $x_i \hat{I} X$,
$$x_i = \frac{D(A_i^0, B)}{D(A_i^0, B) + D(A_i^0, G)} \tag{19}$$

where $A_i^0 = \{x_i, [z_i^{0-}, z_i^{0+}]\} = \{x_i, [\sum_{j=1}^m \overset{\circ}{a}_{ij}^- r_j^0, \sum_{j=1}^m \overset{\circ}{a}_{ij}^+ r_j^0]\}$ given by Eq. (18) is an interval-valued fuzzy set

corresponding to the optimal comprehensive value of the alternative $x_i \in X$. $G = \{g, [1, 1]\}$ is an interval-valued fuzzy set corresponding to the evaluation of the ideal alternative g . $B = \{b, [0, 0]\}$ is an interval-valued fuzzy set corresponding to the evaluation of the negative ideal alternative b . Obviously, normally $g \in X$ and $b \in X$. $D(A_i^0, B)$ is a distance measure between the interval-valued fuzzy sets A_i^0 and B . $D(A_i^0, G)$ is a distance measure between the interval-valued fuzzy sets A_i^0 and G . There are several distance formulae between interval-valued fuzzy sets [3]. In this paper, we choose the distance formula given by Eq. (1) in Section 2. Obviously, for each alternative $x_i \in X$, we have $0 \leq x_i \leq 1$.

Furthermore, $x_i = 0$ if $A_i^0 = B$ (or x_i is the negative ideal alternative b); $x_i = 1$ if $A_i^0 = G$ (or x_i is the ideal alternative g). It is easy to see that the higher x_i the better the alternative x_i .

According to Eq. (1), $D(A_i^0, B)$ and $D(A_i^0, G)$ are reduced into the following formulae

$$D(A_i^0, B) = \frac{|z_i^{0-} - 0| + |1 - z_i^{0+} - 1| + |1 - z_i^{0-} - (1 - z_i^{0+}) - 0|}{2}$$

$$= \frac{z_i^{0-} + z_i^{0+} + (z_i^{0+} - z_i^{0-})}{2} = z_i^{0+} \quad (20)$$

and $D(A_i^0, G) = 1 - z_i^{0-} \quad (21)$

Hence, Eq. (23) can be simply written as follows $x_i = \frac{z_i^{0-}}{1 + z_i^{0+} - z_i^{0-}} \quad (22)$

AN NUMERICAL EXAMPLE

Consider an air-condition system selection problem. Suppose there exist three air-condition systems x_1, x_2 and x_3 . Denote the alternative set by $X = \{x_1, x_2, x_3\}$. Suppose three attributes a_1 (economical), a_2 (function) and a_3 (being operative) are taken into consideration in the selection problem. Denote the set of all attributes by $T = \{a_1, a_2, a_3\}$. Using statistical methods, the interval membership degrees $[A_{ij}^-, A_{ij}^+]$ for the alternative $x_i \in X$ with respect to the attribute $a_j \in A$ to the fuzzy concept “excellence” can be obtained, respectively. Namely,

	x_1	x_2	x_3	$\bar{0}$
$([A_{ij}^-, A_{ij}^+])_{3 \times 3}$	a_1 [0.75, 0.90]	[0.80, 0.85]	[0.40, 0.55]	$\bar{1}$
	a_2 [0.60, 0.75]	[0.68, 0.80]	[0.75, 0.95]	$\bar{1}$
	a_3 [0.80, 0.80]	[0.45, 0.50]	[0.60, 0.70]	$\bar{0}$

In a similar way, the interval membership degrees $[r_i^-, r_i^+]$ for the three attributes $a_j \in A$ to the fuzzy concept “importance” can be obtained, respectively. Namely,

$$([r_i^-, r_i^+])_{1 \times 3} = ([0.25, 0.75] \quad [0.35, 0.60] \quad [0.30, 0.35])'$$

According to Eq. (14) or Eq. (15), the following linear programming can be obtained

$$\max\{z = \frac{0.35r_1 + 0.47r_2 + 0.15r_3}{3}\},$$

$$\begin{array}{l} \text{s.t.} \\ 0.25 \leq r_1 \leq 0.75 \\ 0.35 \leq r_2 \leq 0.60 \\ 0.30 \leq r_3 \leq 0.35 \\ r_1 + r_2 + r_3 = 1 \end{array}$$

Solving the above linear programming, its optimal solution can be obtained as follows

$$r^0 = (0.25, 0.40, 0.35)^T.$$

Using Eqs. (16) and (17), the optimal comprehensive value of the alternative $x_i \in X$ can be computed as follows:

$$z_1^{0-} = 0.7075, z_2^{0-} = 0.6295, z_3^{0-} = 0.6100 \text{ and}$$

$$z_1^{0+} = 0.8050, z_2^{0+} = 0.7075, z_3^{0+} = 0.7625$$

Thus, the optimal comprehensive value of the alternative $x_i \in X$ can be expressed as an interval-valued fuzzy set

$A_1^0 = \{x_1, [0.7075, 0.8050]\}$, $A_2^0 = \{x_2, [0.6295, 0.7075]\}$, and $A_3^0 = \{x_3, [0.6100, 0.7625]\}$, respectively.

For alternatives x_1 , x_2 and x_3 , the following index for each alternative can be generated using Eq. (22):

$$x_1 = \frac{z_1^{0-}}{1 + z_1^{0+} - z_1^{0-}} = \frac{0.8050}{1 + 0.8050 - 0.7075} = 0.7335$$

$$x_2 = \frac{z_2^{0-}}{1 + z_2^{0+} - z_2^{0-}} = \frac{0.7075}{1 + 0.7075 - 0.6295} = 0.6563$$

$$\text{and } x_3 = \frac{z_3^{0-}}{1 + z_3^{0+} - z_3^{0-}} = \frac{0.7625}{1 + 0.7625 - 0.6100} = 0.6616$$

Then, the best alternative is x_1 . The optimal ranking order of the alternatives is given by $x_1 \succ x_3 \succ x_2$.

CONCLUSION

In the above analysis, we have proposed several linear programming models and methods for multiattribute decision making under "interval-valued fuzziness". In such decision situations, attributes are explicitly considered and are not compound, which differ from of the ways used by Szmidt and Kaeprzyk [9-11], Shyi-Ming Chen, Li-Wei Lee [12], Deng-Feng Li, Shu-Ping Wan [13]. Moreover, the evaluations of each alternative with respect to each attribute on a fuzzy concept "excellence" are given using interval-valued fuzzy sets, and the weights of each attribute are also given using interval-valued fuzzy sets. This allows us to use flexible ways to simulate real decision situations, thereby building more realistic scenarios describing possible future events. In conclusion, multiattribute decision-making models using interval-valued fuzzy sets can represent a wide spectrum of possibilities, which enables the explicit consideration of the best and the worst results one can expect.

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REFERENCES

- [1] L. A. Zadeh. *Information and Control*, vol8, pp.338–356, 1965.

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- [2] L.A.Zadeh. *IEEE Trans Syst Man Cyber*, vol SMC-3 (1) , pp.28-44 , **1973**.
- [3] L.A.Zadeh. *Inform. Sci.* vol8 , pp. 199–249, **1975**.
- [4] K. Atanassov. *Fuzzy Sets and Systems*, vol 20, pp. 87–96, **1986**.
- [5] K. Atanassov. *Intuitionistic Fuzzy Sets: Theory and Applications*, Physical-Verlag, Heidelberg, New York, **1999**.
- [6] Shyi-Ming Chen, Jiann-Mean Tan. *Fuzzy Sets Systems*, vol67, pp. 163–172, **1994**.
- [7] Dug Hun Hong, Chang-Hwan Choi. *Fuzzy Sets Systems*, vol 114, pp.103–113, **2000**.
- [8] Deng-Feng Li. *J. Comput. System Sci.*, vol70, pp. 73–85, **2005**.
- [9] E. Szmidt, J. Kacprzyk., *NIFS*, vol2 (1), pp. 15–32, **1996**..
- [10] E. Szmidt, J. Kacprzyk, *NIFS*, vol 2 (3), pp. 22–31, **1996**.
- [11] E. Szmidt, J. Kacprzyk, *Intuitionistic fuzzy sets for more realistic group decision making, International Conference on Transition to Advanced Market Institutions and Economies, Warsaw*, June 18–21, pp. 430–433, **1997**.
- [12] Shyi-Ming Chen, Li-Wei Lee, Hsiang-Chuan Liu, Szu-Wei Yang. *Expert Systems with Applications*, Volume 39, Issue 12, pp. 10343-10351, 15 September **2012**.
- [13] Deng-Feng Li, Shu-Ping Wan. *Applied Soft Computing*, Volume 13, Issue 11, pp.4333-4348, November **2013**.