



## Monte carlo experimental research on table tennis size affects paddler experience sense and audience ornamental

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### ABSTRACT

*Different diameters' table tennis bring out paddler different experience qualities, meanwhile it gives audience different ornamental quality. Take paddler universal serve strength 40N as striking force, according to information, it works out that best visual table tennis shifting effect is 13.09 m/s and table tennis best radius is 42mm. Then consider from paddler hitting percentage, table tennis collides the table, the bigger original route deviation angle is, the more difficult paddle receive would be, so that fewer times of back and forth would be. Based on this theory, with the same strength, calculate diameter 40mm table tennis collides table deviation angle is 16.1°, diameter 38mm table tennis collides table deviation angle is 32.1°, so that define 40mm table tennis has an advantage over 38mm table tennis. Finally by Monte Carlo simulation experiment defines that best table tennis radius is 42mm.*

**Key words:** Experience quality, deviation angle, table tennis diameter, Monte Carlo Simulation

### INTRODUCTION

Any one sport event should have higher ornamental. Otherwise, it will lose audience. And sport events that lose audience are no vitality. With regard to table tennis' audience has decreasing tendency since 1970s, table tennis technical ornamental research has already become an important subject in table tennis theoretical cycle. In order to increase table tennis air resistance in flying, slow down its running speed in competition so that achieve further increasing and enriching table tennis professional paddlers' striking techniques and skills, finally increase table tennis competitions overall ornamental, international table tennis federation let international table tennis professional tournament official table tennis adopted diameter increase from 38mm to 40mm in 2000. However, the coming of "large ball era" has always suffered table tennis coaches and paddlers disputes. To solve the problem, Chinese and foreign scholars always adopt investigation method, carry out simple statistical analysis of results appeared by paddler using two kinds of table tennis [1-3]. Though it can solve these problems, shortcoming still exist on theoretical aspects.

National associated scientific committee researchers have made "different diameters and weights table tennis affect striking speed and rotation experiment". Experiment conclusions are big diameter table tennis' speed is slower than that of small diameter one, while its rotation is weaker than that of small diameter one; for same diameter table tennis, large weight and elastic force one has quicker speed and stronger rotation than small weight and elastic force one. In Feb.23rd, 2000, international table tennis federation extraordinary general meeting and congress have passed 40mm large ball reform scheme in Kuala Lumpur, decided that table tennis competition would use diameter 40mm, weight 2.7g large ball to substitute 38mm small ball since Oct.1<sup>st</sup>, 2000 that was also after Sydney Olympic Games [4-6].

To further improve table tennis paddler experiment quality and audience ornamental quality, analyze table tennis features from mechanics perspective, utilize computer making simulation on table tennis dynamic process so that find best table tennis diameter.

**MODEL DESIGN**

In order to find out 38mm and 40mm such two table tennis differences in paddler experience sense and ornamental, convert problems into researching two table tennis in speed, rotation as well as its dropping to opponent table and bounce generates angle offset problems. Adopt mechanics principle, research table tennis initial movement situation and rebound movement trajectory, and establish two table tennis models based on that as table tennis initial movement model and rebound trajectory model [7-9].

**Preparation before model establishment**

In international formal competitions, international table tennis federation normally adopt length 274cm, width 152.5cm, height 76cm, net height 15.25 cm table, as Figure 1 shows [1]. When serving table tennis, it should first through its own table, then bounce to opponent table through net. In continuously striking, table tennis should directly drop to opponent table without through its own table. These are basic rules.

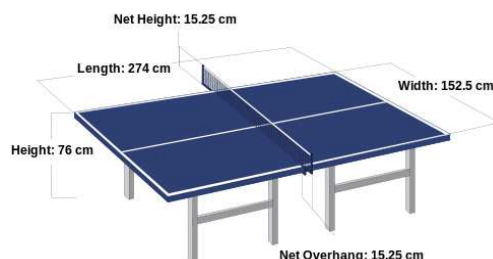


Figure 1: Table tennis table schematic figure

**Conditional assumption in researching:**

- Except table tennis diameter is variable, other conditions not changing that table tennis competition table tennis table, net, rule and competition environment all achieve current international table tennis federation stipulation;
- Table tennis hardness owns rigidity, and mass distribution is evenly.

Use symbol explanation in researching process, refer to Table 1.

Table 1: Symbol convention

$m$	Table tennis mass	$R$	Table tennis diameter
$v$	Table tennis running speed	$\omega$	Table tennis angular speed
$I$	refer to rotational inertia	$r$	Table tennis radius
$\mu$	Friction coefficient	$f$	Friction force
$J$	impulse	$\vartheta$	deflection angle
$U_2$	swinging speed	$F$	Table tennis suffered joint force

**Table tennis ornamental sense model establishment and solution**

Because racket affects table tennis mechanical analysis is extremely complicated, in order to simplify problems, this paper ignores all very little action force, now assume that racket effects on table tennis are two forces effects that are vertical to racket force  $F$  as well as table tennis and racket tangential friction force  $f$ , and table tennis initial speed and angular speed are 0. Let table tennis mass to be  $m$ , diameter as  $R$ . As Figure 2 shows, let racket and horizontal included angle to be  $\theta$ , time that racket effects on table tennis as  $t$ , swinging speed as  $v$  [2, 10].

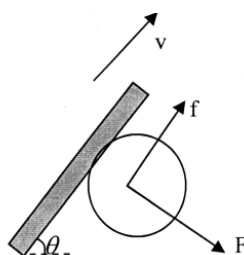


Figure 2: Racket initial parameter analysis figure

Force  $f$  impulse in the time  $t_0$  is  $\int_0^{t_0} f dt$  that expressed by  $I_f$ ,  $F$  impulse in the time  $t_0$  is  $\int_0^{t_0} F dt$  that

expressed by  $I_F \cdot F$  goes through table tennis mass center and tangents with  $f$  and table tennis, therefore friction force  $f$  generates table tennis a torque  $\frac{R}{2}$ ,  $f$  torque's impulsive moment in the time  $t_0$  is  $\frac{R}{2} \int_0^{t_0} f dt$ , that is  $\frac{R}{2} I_f$ . Table tennis mass center rotational inertia is  $I_c = \frac{2}{3} m r^2 (r = \frac{R}{2})$ , end speed along  $F$  direction is  $v_F$ , end speed along  $f$  direction is  $v_f$ , end angular speed is  $\omega$ . According to momentum theorem and angular momentum theorem, it can get following formula:

$$\begin{cases} I_F = m v_F - 0 \\ I_f = m v_f - 0 \\ \frac{R}{2} I_f = I_c \omega - 0 \end{cases} \quad (1)$$

Solve:

$$v_F = \frac{I_F}{m} \quad (2)$$

$$v_f = \frac{I_f}{m} \quad (3)$$

$$\omega = \frac{R I_f}{2 I_c} = \frac{3 I_f}{m R} \quad (4)$$

Further deduce formula (4), it gets:

$$\omega = \frac{D I_f}{2 I_c} = \frac{3 I_f}{m R} = \frac{3}{m R} \int_0^{t_0} f dt = \frac{3 t_0}{m R} f = \frac{3 t_0}{m R} \mu F \quad (5)$$

In formula,  $\mu$  expresses friction coefficient ( $\mu$  value normally is 0.5~0.8 [3], According to the model, this paper values  $\mu = 0.65$ ),  $t_0 = \frac{1}{1000}$  second [4].

**Table tennis competition appreciation perspective analysis:** Table tennis competitions' ornamental mainly are related to technical quality, tactics changes, paddler instrument and stroking motion aesthetic perception as well as competition times and other factors. But for research problems, it mainly analyzes from table tennis technical mass that is to evaluate 38mm and 40mm two different diameters table tennis ornamental through analyzing table tennis speed and turns.

Because most of audience selects to watch table tennis competitions by TV, only consider competition ornamental watching by TV, and audience best translational visual angle is  $30^\circ/s$  [5], While viewing angle that watches TV is not changing, root cause of changing is camera converted certain included angle that moves with table tennis movement, therefore regard camera as human eyes, take  $30^\circ$  rotation as best point, according to data, it indicates that general camera distance from competition court is around 25meters [6]. Therefore best audience sense table tennis movement speed is:  $\frac{30^\circ}{360^\circ} \times 2\pi \times 25 = 13.09 m/s$ .

Analysis when diameter is 38mm ( $m = 2.5 \times 10^{-3} kg$ ): Assume that outside world vertical force on table tennis is  $F = 40N$ , so  $v_F = \frac{I_F}{m} = \frac{F t_0}{m} = \frac{40}{2.5 \times 10^{-3}} \times \frac{1}{1000} = 16 m/s$

According to formula (5), it gets: 
$$\omega = \frac{3}{2.5 \times 10^{-3} \times 38 \times 10^{-3}} \times \frac{1}{1000} \times 0.65 \times 40 = 821.05 \text{ rad / s}$$

Analysis when diameter is 40mm ( $m = 2.79 \times 10^{-3} \text{ kg}$ ): Similarly assume  $F = 40\text{N}$ ,  
 so 
$$v_F = \frac{I_F}{m} = \frac{Ft_0}{m} = \frac{40}{2.79 \times 10^{-3}} \times \frac{1}{1000} = 14.3369 \text{ m / s}$$

$$\omega = \frac{3}{2.79 \times 10^{-3} \times 40 \times 10^{-3}} \times \frac{1}{1000} \times 0.65 \times 40 = 698.93 \text{ rad / s}$$

Through comparing 38mm and 40mm  $v_F$  with 13.09m/s, it is obviously that adopt 40mm diameter table tennis is more ornamental.

Judge competition is wonderful or not, it tends to check whether there are many two paddlers back and forth times or not, according to data indication, it is clear that applause would appear when paddlers have 7 to 8 back and forth times [6], therefore evaluate audience ornamental can be up to paddlers back and forth times, the more back and forth times are, the happier audience would be. Turns are normally related to paddlers' strength and table tennis speed as well as deflection angle. The faster speed is, the fewer turns would be; the larger table tennis deflection angle is, the larger difficulties receive would be, the fewer turns would be. Through 4.2.2.1 analysis as well as following deflection angle analysis, it shows that 40mm diameter table tennis speed is slower than 38mm diameter table tennis, and deflection angle is also smaller than 38mm diameter table tennis, therefore 40mm diameter table tennis would let competition turns increasing, and increase ornamental.

**Paddler experience sense and best diameter model establishment and solution**

Through above model analysis, it gets that 40mm diameter table tennis is really better than 38mm diameter table tennis in ornamental perspective, but it doesn't solve paddler experience sense and best table tennis diameter problems.

**Table tennis running trajectory analysis:** In movement process, table tennis mainly suffers three forces influences that are respectively gravity, air resistance and Magnus force, as Figure 3 shows.

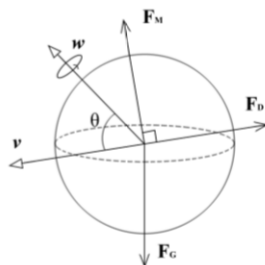


Figure 3: Table tennis force dissolution figure

Among them, G shows gravity,  $F_M$  shows Magnus force,  $F_D$  shows air resistance,  $v$  shows table tennis movement speed,  $\omega$  shows table tennis angular speed.

$$F = G + F_M + F_D \tag{6}$$

$$F_G = -mg\hat{k} \tag{7}$$

$$F_D = -\frac{1}{2} \rho C_D (\pi R)^2 \|v\|v \tag{8}$$

In formula,  $\rho$  shows air density,  $C_D$  shows resistance coefficient,  $R$  shows table tennis radius. According to Magnus formulas,  $F_M = \rho(2\pi R)^2(\omega \times v)$ .

**Table tennis rebound model establishment and solution:** It mainly researches movement speed, angular speed, linear speed as well as impulse change situations before, during and after table tennis drops to table to explain table tennis deflection angle. As Figure 4 variable symbols, it shows each variable change before and after table tennis drops to table.

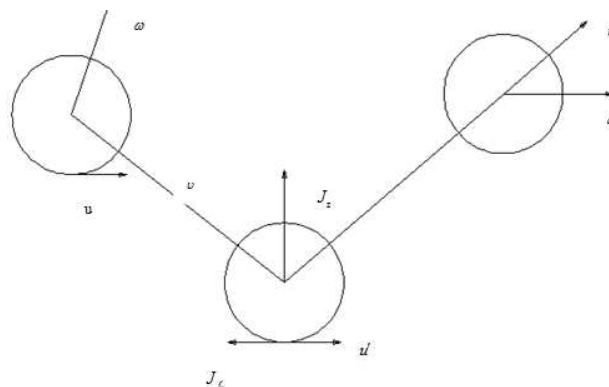


Figure 4: Variable change situations before and after table tennis drops to table

In Figure 4,  $v$  and  $v'$  shows table tennis speed,  $\omega$  and  $\omega'$  shows table tennis angular speed,  $u$  and  $u'$  shows table tennis and table relative speeds;  $J_{fr}$  shows impulse caused by table friction,  $J_z$  shows upwards impulse.

When table tennis is stroking from one party to another party's table and fast rebounds, table tennis movement speed  $v$  direction and angular speed  $\omega$  direction will occur to deflection, which would lead to table tennis running trajectory shifts comparing to original running trajectory. While different diameters' table tennis caused table tennis running trajectory offset angles  $\phi$  different. By researching different diameters table tennis deflection angles, to define paddlers' experience sense and audience ornamental brought by 38mm and 40mm two table tennis and define best table tennis diameter. Corresponding table tennis and racket angle offset model has already proved by experiment [8]. This paper based on the model; establish table tennis and table angle offset models.

According to previous analyzed table tennis running movement speed  $v$  and angular speed  $\omega$ , it gets table tennis parallel to table speed  $u$  when takes movement, its expression is:

$$u = v_{xy} + (\omega \times r) \tag{9}$$

In formula,  $v_{xy}$  represents  $v$  projection in coordinate  $xOy$  plane.

According to table tennis rigidity hypothesis, table tennis speed  $v$  vertical direction component expression is

$$v'_z = -\epsilon v_z \tag{10}$$

In formula,  $v'_z$  represents after table tennis rebounding  $v$  vertical direction speed,  $v_z$  represents before rebounding  $v$  vertical direction speed,  $\epsilon$  represents elastic deformation coefficient.

Table tennis impulse in coordinate  $xOy$  plane has connections with impulse in  $z$  Positive coordinate axis direction that can express with following formula:

$$J_{xy} = -\mu J_z \frac{u}{\|u\|} \tag{11}$$

In formula (11),  $J_{xy}$  represents impulse in Coordinate  $xOy$  plane,  $J_z$  represents impulse in  $z$  axis positive direction,  $\mu$  and represents table tennis and table sliding friction coefficient.

According to impulse-momentum theorem:

$$mv' - mv = J \tag{12}$$

As well as angular momentum and rotational inertia principle:

$$I\omega' - I\omega = r \times J \tag{13}$$

Get two equations as:

$$J_z = -m(1 + \epsilon)v_z \tag{14}$$

$$u' = \mu J_2 \left( \frac{1}{m} + \frac{r^2}{I} \right) \frac{u}{\|u\|} + u \tag{15}$$

In formula (15),  $u'$  represents speed when table tennis touches table.

Due to table tennis and table touching time is quite short, we assume that  $v'_{xy}$  isn't changing when table tennis touches table. Therefore establish  $u'$  and  $v'$  proportion relationship, the expression is:

$$u' = cv' \quad (c \geq 0) \tag{16}$$

Through formula (10) (15) and (16), it can get:

$$c = 1 - \frac{5}{2} \mu (1 + \epsilon) \frac{|v_z|}{\|u\|} = 1 - \frac{5}{2} \alpha \tag{17}$$

$$\alpha = \mu (1 + \epsilon) \frac{|v_z|}{\|u\|}$$

Among them,

Through combining formula (12) (13) and (17), it can get  $v$  and  $v'$  relationship:

$$v' = A_v v + B_v \omega \tag{18}$$

$$\omega' = A_\omega v + B_\omega \omega \tag{19}$$

In physics, when  $c > 0$  shows it is sliding friction status, but this paper don't get involved in physical constraints of  $c$  sizes.

$$A_v = \begin{pmatrix} 1-\alpha & 0 & 0 \\ 0 & 1-\alpha & 0 \\ 0 & 0 & \epsilon \end{pmatrix}, \quad B_v = \begin{pmatrix} 0 & \alpha r & 0 \\ -\alpha r & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

And

$$A_\omega = \begin{pmatrix} 0 & -\frac{3\alpha}{2r} & 0 \\ \frac{3\alpha}{2r} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad B_\omega = \begin{pmatrix} 1-\frac{3\alpha}{2} & 0 & 0 \\ 0 & 1-\frac{3\alpha}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In physics,  $c \leq 0$  indicates it is rolling friction status. When  $c = 0$ ,  $u' = 0$ , table tennis is fully rolling. At this

time,  $\alpha = \frac{2}{5}$ . Since rolling status is zero sliding constraint status, so:

$$A_v = \begin{pmatrix} \frac{3}{5} & 0 & 0 \\ 0 & \frac{3}{5} & 0 \\ 0 & 0 & -\varepsilon \end{pmatrix}, \quad B_v = \begin{pmatrix} 0 & \frac{2}{5}r & 0 \\ -\frac{2}{5}r & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \text{And:} \quad A_\omega = \begin{pmatrix} 0 & -\frac{3}{5r} & 0 \\ \frac{3}{5r} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad B_\omega = \begin{pmatrix} \frac{2}{5} & 0 & 0 \\ 0 & \frac{2}{5} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$v' = \begin{pmatrix} \frac{3}{5}\|v\|\sin\phi_1\cos\theta_1 + \frac{2}{5}r\|\omega\|\sin\phi_2\cos\theta_2 \\ \frac{3}{5}\|v\|\sin\phi_1\cos\theta_1 - \frac{2}{5}r\|\omega\|\sin\phi_2\cos\theta_2 \\ -\varepsilon v_z \end{pmatrix}$$

From (16) and (17), it gets:

Among them,  $\theta_1, \phi_1, \theta_2, \phi_2$  respectively shows speed  $v$  and z axis included angle, speed  $v$  projection in xy plane and x axis included angle, expresses angular speed  $\omega$  and z axis included angle, angular speed  $\omega$  projection in xy plane and x axis included angle, as Figure 5.

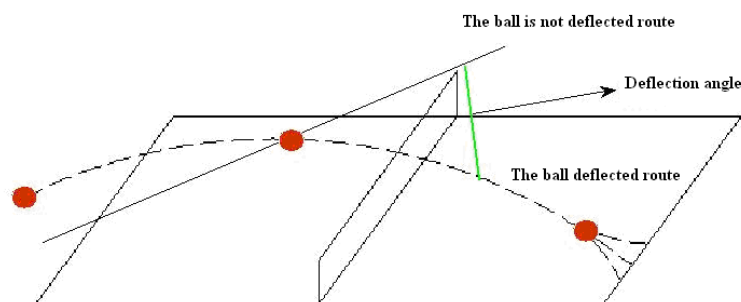


Figure 5: Deflection angle (serve)

$$\Delta\vartheta = \frac{v'_y}{v'_x}$$

Then deflection angle expression is

In case that  $\theta_1, \phi_1, \theta_2, \phi_2$  are the same,  $\Delta\vartheta$  becomes bigger, then it indicates when table tennis touches table, it is prone to get deflection, and it is more difficult to predict remote curve after table tennis collides with table, then paddler is also more difficult to stroke the table tennis, back and forth times would be more and more fewer, audience ornamental quality not high; Since it is difficult to stroke expected received table tennis, paddler experience mass is also not high. For 38mm and 40mm comparison, with regard to  $\theta_1, \phi_1, \theta_2, \phi_2$  these four variables, since paddlers' stroking ways are different, table tennis and table colliding angles are also different, we first take random values of  $\theta_1, \phi_1, \theta_2, \phi_2$  respectively as  $30^\circ, 50^\circ, 30^\circ, 20^\circ$ ,  $\Delta\vartheta_{38}=35.1^\circ$ ,  $\Delta\vartheta_{40}=16.1^\circ$ . It is clear that 38mm deflection angle is larger than that with 40mm, so that is to say 38mm table tennis is more difficult to stroke than 40mm table tennis.

**Best radius selection:** Just as previous deflection angle problems, when  $\theta_1, \phi_1, \theta_2, \phi_2$  these four variables still value as  $30^\circ, 50^\circ, 30^\circ, 20^\circ$ , change table tennis radius, then it can get following one tendency figure that deflection angles changing followed by radius, figure as following Figure 6:

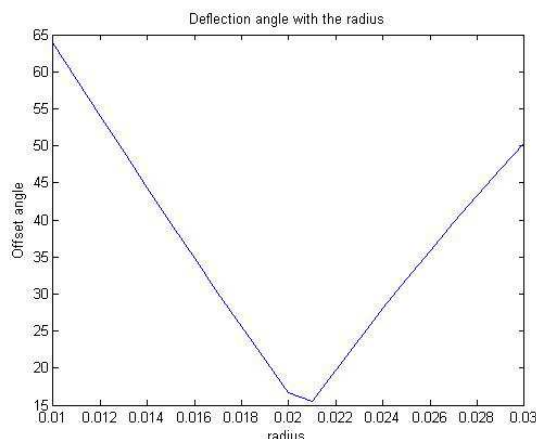


Figure 6: Table tennis best radius selection figure

It is found when  $r=21\text{mm}$ , that is to say when table tennis diameter changes into  $42\text{mm}$ , table tennis deflection angle is the minimum one. It is also thought when table tennis diameter changes into  $42\text{mm}$ ; it is paddler best experience radius. Because best visual angular speed is  $13.09\text{ m/s}$ , similarly can get best running radius is  $42\text{mm}$ .

**MODEL TEST RESEARCH**

Above result is defined in case angle is fixed. In order to get more precise calculation result and provide above four angles size from  $0$  to  $90^\circ$  at random so that define  $\theta_1, \phi_1, \theta_2, \phi_2$  different situations best radius, when test times is 100, running result is  $42\text{mm}$  diameter table tennis is best table tennis that accounts for 65%, is far higher than other types table tennis. Calculation result is as following Figure 7 shows.

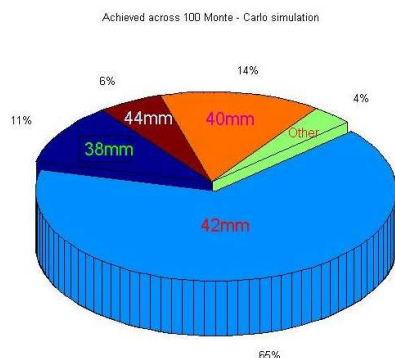


Figure 7: 100 times testing Monte Carlo simulation result

When testing times is 500, simulation result still indicates that  $42\text{mm}$  diameter table tennis is the best table tennis that accounts for 68%, is far higher than other types table tennis. Running result is as following Figure 8 shows.

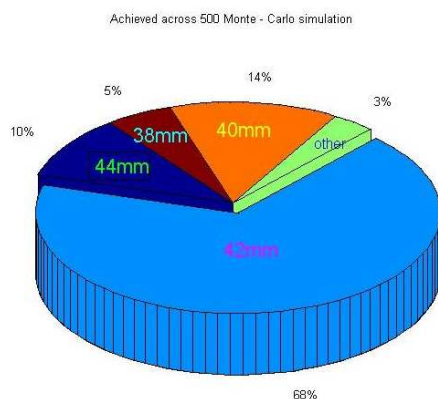


Figure 8: 500 times testing Monte Carlo simulation result

When testing times is 1000, simulation result still indicates that  $42\text{mm}$  diameter table tennis is the best table tennis



that accounts for 70%, is far higher than other types table tennis. Running result is as following Figure 9 shows.

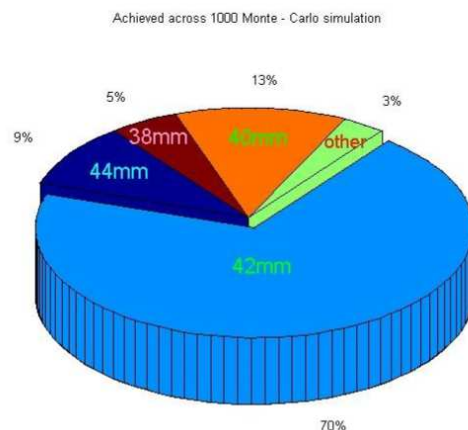


Figure 9: 1000 times testing Monte Carlo simulation result

When testing times is 1500, simulation result still indicates that 42mm diameter table tennis is the best table tennis that accounts for 70%, is far higher than other types table tennis. Running result is as following Figure 10 shows.

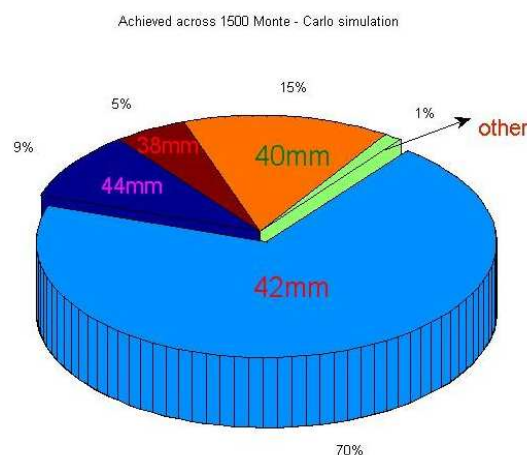


Figure 10: 1500 times testing Monte Carlo simulation result

When simulation times arrive more than 1000, there is 70% probability that best radius is 42mm, for other 30% probability why it is not 42mm, we think  $\theta_1, \phi_1, \theta_2, \phi_2$ . values between  $0$  and  $90^\circ$ , some might not value but it still takes, such as table tennis might not vertical to table and stroke upwards, but to sum up it gets best radius is 42mm.

## CONCLUSION

Through applying table tennis movement model, analyze model movement speed and angle, and took people best ornamental table tennis running speed ( $13.09 m/s$ ) as measurement criterion, it got that running speed at  $14.3369 m/s$  diameter 40mm table tennis ornamental result had an advantage over that running speed at  $16 m/s$  diameter 38mm table tennis. Meanwhile, only took speed as measurement criterion, it was obvious that 40mm table tennis speed was slower that was more helpful for paddler experiencing. By establishing table tennis rebound trajectory model to measure different diameters table tennis rebound angle offsets, it got that table tennis with 42mm diameter had the lowest offset that can improve paddler experience sense more. And through different diameters table tennis Monte Carlo simulation, it got that 42mm diameter table tennis accounted for largest percentage, therefore 42mm diameter table tennis was the best table tennis.

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