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Research Article

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Meshing features of involute arc teeth cylindrical gears

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ABSTRACT

This paper intended to derive the equation of the teeth surface of the involute arc cylindrical gear. Based on the forming principles of the teeth surfaces of the spur cylindrical gear and the helical cylindrical gear, an arc line parallel to the axis of the cylinder was firstly drawn in the generating plane, and then the scanning surface of the arc line should be the teeth surface of the involute arc cylindrical gear. After that, the equation of the arc line was derived and then the teeth surface. Finally the simulations of those equations were described with the instruments of proeWildfire 5.0 and MATLAB R2010a. Thus, the correctness of the equations was further proved.

Key words: involute; arc teeth; equation of the arc teeth surface

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INTRODUCTION

Cylindrical gear transmission has found a widespread application in the industry. Currently, it mainly includes three modes of transmission: spur cylindrical gear，helical cylindrical gear and double helical cylindrical gear[1]. However, there are some drawbacks in structures of the above-mentioned gears[2].

When spur cylindrical gears mesh, the instantaneous contact line of two teeth profile surfaces of the gears is parallel to their axes. Thus the teeth of the two gears throw into or out of the meshing along their teeth width in the process of transmission. Under this circumstance, the gear teeth are suddenly loaded or unloaded. Hence, a spur gear transmission might be poor of stability, susceptible to shock and vibrate and high likely to make noise. Spur gears are also possible to manufacture bigger error. In addition, due to the overlapping coefficient of spur gear transmission is small; the load on the gears is also limited.

Helical cylindrical gear has the following defects. Due to the unidirectional sloping teeth, the transmission drive will produce axial thrust. This thrust then cause that shafting structure of the gears is complex and friction losses is further increased. Thus, the transmission efficiency is reduced and the spiral angle is too some degree narrowed.

The third kind of gear, that is, double helical cylindrical gear, can overcome the above shortcomings; however, it also has such disadvantages as difficult and high-cost manufacturing, complicated structures and heavy weight.

Therefore, In order to overcome the above-mentioned problems, a new type of cylindrical gear-an arc teeth cylindrical gear is then proposed. Its main characteristic is: the teeth of the cylinder are arc-shaped. As the above gears, the teeth profile curves of this gear can be either involute or other forms. Its appearance is shown in fig. 1.

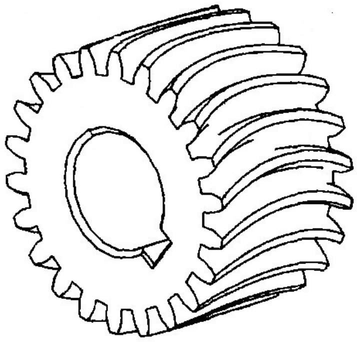

Fig. 1: Appearance of the arc teeth cylindrical gear

At present, most of the researches are concentrated on the arc teeth cylindrical gear with involute teeth profile. The equations of its teeth surface are mainly set up based on the following two methods.

As for the first kind of method, the teeth surface is believed to be formed through scanning of an involute line along an arch line[1]. The equation of the teeth surface is set up through the geometric transformation. That is, this equation is derived from the plane meshing theory, but there is no equation of the arc line on the cylinder.

The second method suggests that the equation of the teeth surface should be derived based on the principle of processing and the movement rules between workpieces and cutters[2-4]. However, this kind of method has no solid theoretical foundation; in addition, the equation might be different according to different methods of processing.

Due to the drawbacks of the above methods, the application of arc teeth cylindrical gears has been greatly restricted.

This paper firstly proposes the forming principles of the teeth surface of involute arc teeth cylindrical gears through borrowing some principles of involute spur gears and involute helical gears. Based on those forming principles, the equation of the arc line on the cylinder is derived and then the equation of the teeth surface. Finally the correctness of those equations is proved by drawing equation figures through the instruments of proeWildfire 5.0 and MATLAB R2010a.

FORMING PRINCIPLES OF THE TEETH SURFACE OF INVOLUTE ARC CYLINDRICAL GEAR

Let the radius of arc line CGD be R_t and its string line CD, which is parallel to the axis of the base cylinder; Then let curve C 'G' D ' be the imprinted arc line of CGD on the cylinder and the track surface of CGD the arc teeth surface of the involute arc cylindrical gear. If the teeth width is B, then R_t should be longer than half of B. Provided that, when R_t is $+\infty$, the involute arc cylindrical gear shall become an involute spur cylindrical gear. The forming principle of the teeth surface of the involute arc cylindrical gear is illustrated in fig. 2.

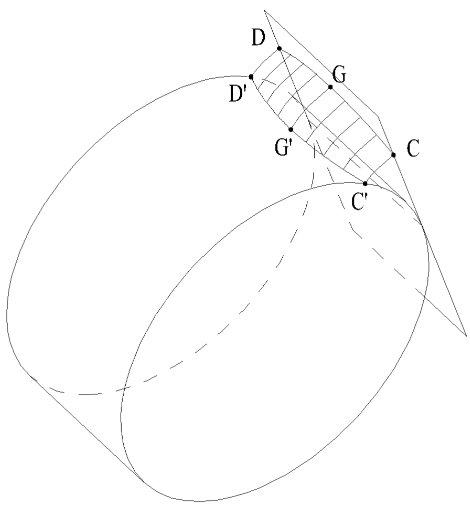

Fig. 2: Forming principle of the teeth surface of the involute arc cylindrical gear

EQUATION OF ARC LINE ON THE CYLINDER

If the axis of the cylinder is axis z and the radial cross-section is xoy coordinate plane, then the coordinate system o – xyz can be established on the rack of No. 1 gear. If the axis of the cylinder is axis z_1 and the radial cross-section is $x_1o_1y_1$ coordinate plane, then the coordinate system o_1-$x_1y_1z_1$ can be established on No. 1 gear. Now provided that axis y_1 and axis y is located at the same position on No.1 gear, when the coordinate system o_1-$x_1y_1z_1$ is moved h (the distance) in the positive direction, the coordinate system o_h-$x_hy_hz_h$ can be established on No. 1 gear. Thus the middle point of arc line CGD is located in axis y_1. O_t is the center of the arc line CGD. And string line CD is parallel to axis Z_1. All the above coordinate systems are shown in fig. 3.

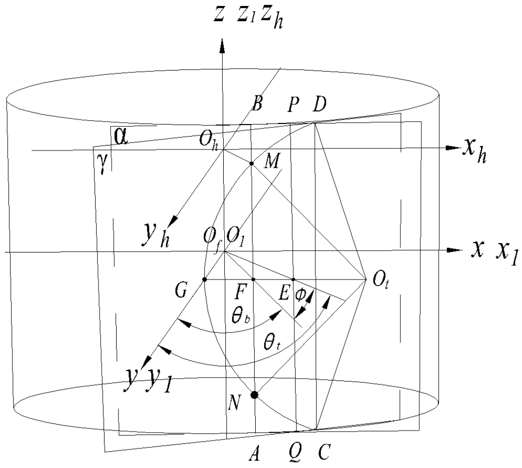

Fig. 3: Coordinate systems on the cylinder

Now if the generating plane rotates on the cylinder, then the curve of the straight line O_tG, which is imprinted on the cylinder, shall fall in the intersection of the plane of the coordinate $x_1o_1y_1$ and the surface of the cylinder. Thus, Provided that the generating plane rotates toward any position (plane α), the turning angle of the plane is θ_b, the tangent line between the generating plane and the cylinder surface is AB, the point of intersection between CGD and AB is M and N, and the point of intersection between GO_t and AB is F, then

$$\theta_b = GF / R_b \tag{1}$$

If the generating plane keeps rotating, then Point M and N shall begin to make an involute movement. Provided that the generating plane rotates until the plane γ, the turning angle is θ_t,; the abduction angle between M and N is Φ, the tangent line is PQ, and the point of intersection between GO_t and PQ is E, then

$$GE = R_b\theta_t \tag{2}$$

And

$$\phi + \theta_b = \theta_t \tag{3}$$

Then in the plane right-angled triangle O_tFM, if the distance between point M and straight line O_tG is h, then h shall be between -B/2 and B/2. According to Pythagorean Theorem, we have

$$O_tF = \sqrt{R_t^2 - h^2} \tag{4}$$

And

$$GF = O_tG - O_tF = R_t - \sqrt{R_t^2 - h^2} \tag{5}$$

Now if we bring the formula (5) into (1), we have

$$\theta_b = GF / R_b = (R_t - \sqrt{R_t^2 - h^2}) / R_b \tag{6}$$

Therefore, the equation of the curve of the arc line CGD shall be

$$\begin{cases} x_1 = R_b \sin\theta_b \\ y_1 = R_b \cos\theta_b \\ z_1 = h \end{cases} \tag{7}$$

If R_t is $+\infty$, then arc line CGD shall become a straight line; Its curve on the base cylinder shall become a straight line parallel to the axis of the cylinder and the turning angle shall become zero. At this situation, CGD shall become an involute teeth surface of a spur gear.

EQUATION OF THE TEETH SURFACE OF THE INVOLUTE ARCH CYLINDRICAL GEAR

In fig. 3, the cylinder is cut apart through point M, and its cross-section is shown in fig. 4. In this figure, if M´ is the corresponding point of M on the surface of the cylinder; then the curve MM´ shall be the involute teeth profile on the cross-section and the straight line PM shall be the section line of the generating plane. Now if the involute abduction angle at point M is Φ, then the value of Φ shall be

$$\phi = EF / R_b = (EG - FG) / R_b = (R_b\theta_t + \sqrt{R_t^2 - h^2} - R_t) / R_b \tag{8}$$

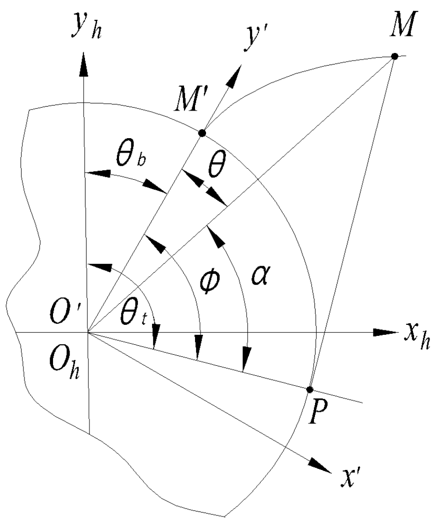

Fig. 4: Section across Point M

Now if we add formula (6) and (8), formula (3) shall be derived. Similarly, if Rt is $+\infty$ and θ_b is zero, then the turning angle of the generating plane (θ_t) shall be Φ. Thus, the equation of the involute MM´ in the coordinate system o´-x'y'z' shall be

$$\begin{cases} x' = R_b(\sin\phi - \phi\cos\phi) \\ y' = R_b(\cos\phi + \phi\sin\phi) \\ z' = 0 \end{cases} \tag{9}$$

Now if we let the coordinate system o_h-$x_h y_h$zh rotate θ_b clockwise along axis z, then the following formula (formula 10) shall be derived.

$$\begin{cases} x_h = x^{'} \cos\theta_b + y^{'} \sin\theta_b \\ y_h = -x^{'} \sin\theta_b + y^{'} \cos\theta_b \mathrm{L}\ (10) \\ z_h = z^{'} \end{cases}$$

Now bring formula (9) into formula (10), then

$$\begin{cases} x_h = R_b(\sin\theta_t - \phi\cos\theta_t) \\ y_h = R_b(\cos\theta_t + \phi\sin\theta_t)\mathrm{L}\ 11 \\ z_h = 0 \end{cases}$$

Now if we let the coordinate system o_1-$x_1y_1z_1$ move h along axis z in the positive direction, then the equation of the involute teeth surface of the arch cylindrical gear in the coordinate system o_1-$x_1y_1z_1$ shall be

$$\begin{cases} x_1 = R_b(\sin\theta_t - \phi\cos\theta_t) \\ y_1 = R_b(\cos\theta_t + \phi\sin\theta_t)\mathrm{L}\ 12 \\ z_1 = h \end{cases}$$

As gear 1 rotates along the other gear, if we let the coordinate system o_1-$x_1y_1z_1$ on gear 1 and the coordinate system o-xyz on the rack of gear 1 rotate β1, as shown in fig. 5;

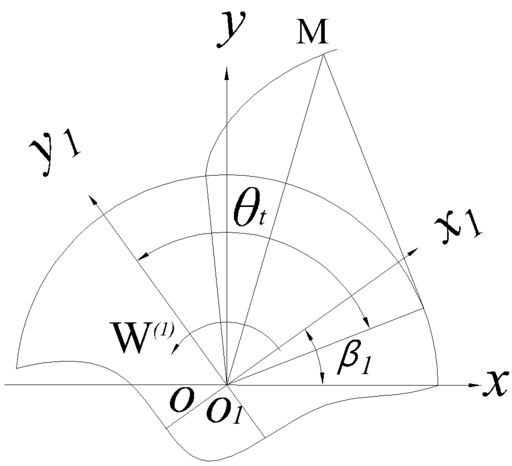

Fig. 5: Rotation of gear 1

Then the following formula shall be derived.

$$\begin{cases} x = x_1\cos\beta_1 - y_1\sin\beta_1 \\ y = x_1\sin\beta_1 + y_1\cos\beta_1 \\ z = z_1 \end{cases} \tag{13}$$

Thus, the equation of the involute teeth surface of arch cylindrical gear in the coordinate system o-xyz shall be

$$\begin{cases} x = R_b[\sin(\theta_t - \beta_1) - \phi\cos(\theta_t - \beta_1)] \\ y = R_b[\cos(\theta_t - \beta_1) + \phi\sin(\theta_t - \beta_1)] \\ z = h \end{cases} \tag{14}$$

SIMULATION OF EQUATIONS

Case 1: Simulation of the arc line on the base cylinder

According to formula (7) and formula (6), the simulation of the arc line can be described with the instrument of proeWildfire 5.0. Now if the radius of the cylinder (r) is 20mm and the radius of the arc line (R_t) is 12mm and t is between 0 and 1, then the formula of the equation of the arc line shall be

$$\begin{cases} r = 20 \\ theta = ((12-(12\wedge 2 - z\wedge 2)\wedge 0.5)/20) \\ *(180/pi) \\ z = 10*t \end{cases} \tag{15}$$

Now provided that h≤0, let us make a curve with the instrument of proeWildfire 5.0, its appearance (Default) is shown in fig.6 below.

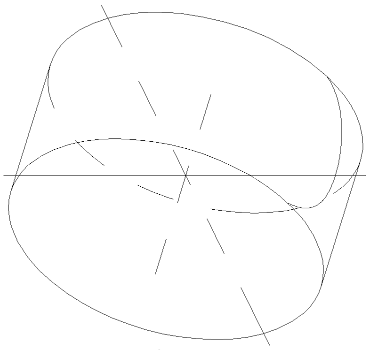

Fig. 6: Default description

Then its appearance (Front) is shown in fig. 7. In this figure, the curve made according to formula (7) does fall on the cylinder.

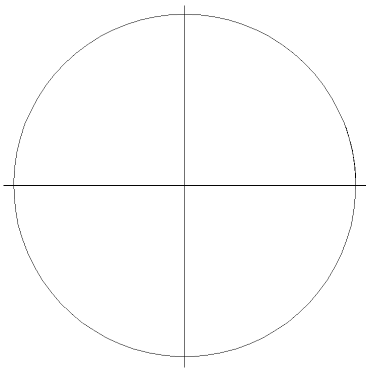

Fig. 7: Front description

Case 2: Simulation of the teeth surface of the arc cylindrical gear

According to formula (12), the simulation of the teeth surface can be described with the instrument of MATLAB R2010a. Now if the radius of the cylinder (r) is 20mm and R_t is 12mm and θ_t is 1, then the formula of the equation of the teeth surface shall be %% Drawing cylndrical surface code

```
clear
clc
close
h=10; %% h is half of the height of the cylndrical surface
d=0.12;
[A,Z]=meshgrid(0:2*pi/fix(2*pi/(1.2*d)):2*pi,-h:h/fix(h/d):h);
LX=20*cos(A);
LY=20*sin(A);
C=mesh(LX,LY,Z);
axis equal
hold on
```

```
clear all
%% Drawing tooth surface code
h=[-10:0.3:10];
a=[0:0.1:1]; %% alphabet a take the place of θt
[U,V]=meshgrid(h,a);
%%Rt=12 Rb=20
MX=20.*(sin(V)-((sqrt(12.^2-U.^2)-12+20*V)/20).*cos(V));
MY=20.*(cos(V)+((sqrt(12.^2-U.^2)-12+20*V)/20).*sin(V));
surf(MX, MY, U);
```

Now the appearance of the teeth surface is shown in figure 8.

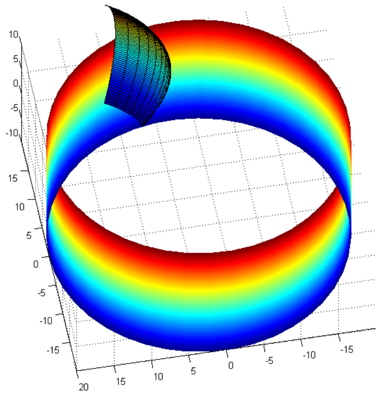

Fig. 8: Simulation of the arc teeth surface

CONCLUSION

The teeth surface of the involute arc cylindrical gear has been established based on the forming principles of the teeth surfaces of the involute spur cylindrical gear and the involute helical cylindrical gear.

The equation of the imprinted curve of the arc line has been derived.

The equation of the teeth surface of the arc cylindrical gear has been derived.

The simulation figures of those equations have been made with the instruments of proeWildfire 5.0 and MATLAB R2010a

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