



Research Article

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## Mathematical model based on the product sales market forecast of markov forecasting and application

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### ABSTRACT

Markov forecasting is a prediction method which the application in probability theory of Markov chain theory and method to study the change rules of the economic phenomenon and in order to predict the future situation. The purpose of this paper is to apply Markov chain theory to the actual market share analysis, it established Markov forecasting model of market share. This model was applied to municipal automobile sales market forecast and had carried on the empirical research.

**Key words:** Market forecast; market share; Markov chain

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### INTRODUCTION

Under the condition of market economy, it is important to the enterprise development that understanding the market dynamics in time and mastering all kinds of products market share. Therefore a reasonable prediction and analysis of the product sales market has an important significance. Market share of the congeneric products is a random process changing over time, and the current market share is only related to the previous market share but the association with long dated is small, this is in accordance with the Markov property. Therefore we can apply Markov theory to the analysis and forecast of market share.

Markov forecasting is named after Russian mathematician A.A.Markov. It is a prediction method which the application in probability theory of Markov chain theory and method to study the change rules of the economic phenomenon and in order to predict the future situation. It is an effective method for prediction of stochastic process, which is based on the way such as market research to obtain materials, using effective mathematical methods such as mathematical statistics, system engineering, etc, to realize the deep combination of quantitative modeling and qualitative evaluation. Markov method is usually used to predict the short-term market conditions, but it also can be used to predict long-term market conditions under the stable conditions. Currently it is widely used in product sales status, market share, product expected profits, equipment updating, new product development and securities investment, etc.

In view of random variability of the market share, this paper based on Markov process, using effective mathematical methods such as mathematical statistics, system engineering etc, analyzing relevant samples, to build Markov matrix of transition probability and forecasting model, and then make calculation and analysis. Last, we select the car market data of a city, and carry on the empirical research, calculate the corresponding market share, analysis of the data to organize production scientifically, reduce blindness, in order to improve the market competitiveness of enterprise and the market share of its products.

## 1. MARKOV PREDICTION OF THE PRODUCT MARKET SHARE

Markov chain is a special kind of random process; it shows the state of things from the past to now, from now to future, just like a chain, a ring by a ring. Its characteristic is "no aftereffect". That is a time series, what it is in the future, what it will take which value associated with the status and the number selection of now, but it has nothing to do with the status or the number selection of past. Generally, the final state of  $n$ -th only associated with increase or decrease of the  $n$ -th period change and the final state of  $n-1$ -th, and has nothing to do with the previous state of  $(n-1)$ -th. This feature is a priority condition that Markov chain is applied into economic forecasting.

Product market share is the percentage of enterprise actual sales quantity in the actual industry sales. It is a comprehensive indicator in evaluation of product market competition ability. The changes of development trend of the market share heralds the increase or decrease of the product competition ability, reflects the enterprise's competition situation and operation prospects. So, we must grasp the market dynamic at any time, understand the enterprise product market share, and predict the development trend of the share, take appropriate and effective measures to adjust measures, to make the enterprise products in an efficient benign condition forever in the market marketing situation.

The change of product market share to a certain extent follows the same rule: the condition of the next time final market share is based on the current final market share, next period through market competition, that is customer transfer between each seller to change increase and decrease, forming a temporary market sales trend, it has no direct relation with the sales status of previous period even more previous.

Using the Markov chain to predict the market share basically has the following steps:

- (1) First of all, we through the market survey, make a statistical calculation of the necessary data.
- (2) Secondly using the obtained data, we calculate the initial market share of each manufacturer product.
- (3) Again, through the situation of the customer flowing between each manufacturer to calculate the transition probability matrix.
- (4) Finally, carry on the forecast.

## 2. THE CONSTRUCTION OF A MODEL

### 2.1 MARKOV PROCESS AND BUILD UP THE FORECAST MODEL

Markov chain is the generic terms of a number of Markov process of the things in a continuous period of time, it shows the state of things from the past to the present, from now to future, it like a chain, a ring after a ring. In the field of prediction, people first study of the initial distribution of each state prediction object and the transition probability between each state, and describe the change trend of state and then predict the future. The characteristics of the Markov prediction method is that it don't need a lot of statistical data, only the limited recent data is needed, then the quantitative prediction can be realized. And on the basis of suitable for short-term forecasts Markov prediction method can be applied to long-term forecast at the same time, as long as the state transition matrix rolling times is enough. But the market is stable, and there aren't any big changes in a certain period of time.

1、 Clear the system status and the current initial distribution of system status

For an enterprise, knowing that, can ever victorious. Therefore, the enterprise should not only forecast product market share of the enterprise, at the same time should also be aware of the change of main competitors. For example, at the second quarter of a city each brand car market share in 2004 (top 5) as follows).

**Table 1 at the second quarter of a city each brand car market share in 2007**

brand	effective sale (quantity)	market share (%)
Guangzhou Honda	3748	6.7
FAW-TOYOTA	3140	5.6
Shanghai General Motor	2814	5.0
Dongfeng Nissan	2882	5.1
Guangzhou Toyota	2045	3.6
others	41431	73.9

According to table 1 we can divide the system into the following six states ,Guangzhou Honda、 FAW-TOYOTA、 Shanghai General Motor、 Dongfeng Nissan、 Guangzhou Toyota、 other brands, get the initial distribution of six system states at the second quarter of the market share in 2007:

$$S(0) = (0.067, 0.056, 0.050, 0.051, 0.036, 0.739)$$

2、 Do market research, and establish the Markov transition matrix

The key of using the Markov chain to forecast is to establish the state transition probability matrix (It refers to the system state with the corresponding to a conditional probability when the situation at time  $t$  is change into that at time  $t + 1$ ). Therefore, the key of the forecast of market share also lies in determining the transition distribution of product which is bought by users in the forecast period by the market survey.

Transition matrix: if the probability from state  $S_i$  to state  $S_j$  is  $S_{ij}$ , then  $S_{ij}$  is the first-order transition probability from the state  $S_i$  transfer to state  $S_j$  after a period, where  $S_{ii}, S_{jj}$  is transition probability at the same state, it is also known as the reserve probability. In general, transfer matrix is composed of transition probability is:

$$P(1) = \begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ P_{n1} & P_{n2} & \cdots & P_{nn} \end{pmatrix}, P(k) = \begin{pmatrix} P_{11}(K) & P_{12}(K) & \cdots & P_{1n}(K) \\ P_{21}(K) & P_{22}(K) & \cdots & P_{2n}(K) \\ \cdots & \cdots & \cdots & \cdots \\ P_{n1}(K) & P_{n2}(K) & \cdots & P_{nn}(K) \end{pmatrix}$$

$$(1) P_{ij} \geq 0; (2) \sum_{j=1}^n P_{ij} = 1, \text{ the sum of each column is 1.}$$

$P(1)$  is the first step transition probability matrix,  $P(k)$  is the  $k$  step transition probability matrix, it is the result of the transfer once again on the basis of step  $k - 1$ ,  $P(k) = P(k - 1) \cdot P(1) = P^k(1)$ .

## 2.2 STRUCTURE PREDICTION MODEL, CALCULATION AND ANALYSIS

Markov chain only depends on the initial state and transition probability of the system, the model is as follows: when the initial system  $k = 0$  (base) is known, after  $k$  transfer in state  $S_i(k)$ , and  $\sum S_i(k)$ .

By Chapman Karl Mo Ge Lyapunov equation:

$$S_j(k + 1) = S_j(k) \cdot P_{ij} \quad (k = 0, 1, 2, \dots)$$

In vector:

$$S(k + 1) = S(k) \cdot P$$

Can get recurrence formula:  $S(k + 1) = S(0) \cdot P^{(k+1)}$ , ( $k = 0, 1, 2, \dots$ ), it can be used as a prediction model of dynamic evolution system implementation.

According to the basic principle of Markov chain prediction: assume that the base of the market share is  $S(0) = [S_A(0), S_B(0), S_C(0)]$ . Therefore, a city car market shares is:

$$S(1) = S(0) \begin{pmatrix} P_{AA} & P_{AB} & P_{AC} \\ P_{BA} & P_{BB} & P_{BC} \\ P_{CA} & P_{CB} & P_{CC} \end{pmatrix} \\ = [S_A(1), S_B(1), S_C(1)]$$

Establish a mathematic model of the application market forecast.

For the current forecast results:

$$P_A(1) = P_{AA} \cdot S_A(0) + P_{BA} \cdot S_B(0) + P_{CA} \cdot S_C(0)$$

$$S_B(1) = P_{AB} \cdot S_A(0) + P_{BB} \cdot S_B(0) + P_{CB} \cdot S_C(0)$$

$$S_C(1) = P_{AC} \cdot S_A(0) + P_{BC} \cdot S_B(0) + P_{CC} \cdot S_C(0)$$

Predicted results for next time:

$$S_A(2) = P_{AA} \cdot S_A(1) + P_{BA} \cdot S_B(1) + P_{CA} \cdot S_C(1)$$

$$S_B(2) = P_{AB} \cdot S_A(1) + P_{BB} \cdot S_B(1) + P_{CB} \cdot S_C(1)$$

$$S_C(2) = P_{AC} \cdot S_A(1) + P_{BC} \cdot S_B(1) + P_{CC} \cdot S_C(1)$$

...

By analogy, we can get the predicted state probability after some time (period).

### 3. EXAMPLE

Predict mainly brand car market share of a city (Guangzhou Honda, FAW-TOYOTA, Shanghai General Motor, Dongfeng Nissan, Guangzhou Toyota, others). First of all, according to each brand at the same time to sign the contract number and transfer (lose) contract number of sampling investigation, the survey results are shown in table 2 (survey statistical process is omitted). In period of the previous period for the survey period (Like the period of the previous period for April 1, 2007 to June 30, 2007), late to forecast the car market share (this period, the next period, the next second phase...). Assume that market research raw data in table 2 as follows:

Table 2 cities each brand car market investigation data analysis table

	On April 1, sign a intent contract number	turn to (quantity: vehicles)							On June 30, the actual sales (vehicles)	
		→ lose								
			A	B	C	D	E	F	total	
Guangzhou Honda <i>A</i>	3711	↓ For taking	0	108	89	123	87	245	652	3748
FAW-TOYOTA <i>B</i>	3585		115	0	101	107	77	498	898	3140
Shanghai General Motor <i>C</i>	3029		850	87	0	153	78	388	791	2814
Dongfeng Nissan <i>D</i>	2600		54	42	84	0	124	124	428	2882
Guangzhou Toyota <i>E</i>	2027		114	58	112	114	0	165	563	2045
others <i>F</i>	32743		156	126	113	123	121	0	639	41431
total		524	421	499	620	487	1420	56060		
	direct acquisition	165	32	77	90	94	7907			

We can see the data from table 2: brand *A* of the previous period (on April 1) to sign the intent contract number is 3711, there are 625 customers to buy other brands at the same time, (remainder) for  $3748 - 3711 - 542 + 652 = 165$ ; keep original intention the probability of order user number is  $(3784 - 652)/3711 = 0.824$ ; the probability of turn to brand *B* order users is  $108/3711 = 0.029$ ; the probability of turn to brand *C* order users is  $89/3711 = 0.024$ ; the probability of turn to brand *D* order users is  $123/3711 = 0.033$ ; the probability of turn to brand *E* order users is  $87/3711 = 0.023$ ; the probability of turn to other order users is  $245/3711 = 0.066$ . In the same method we can calculate to the probability of other brands of keep and turn to(omit). The calculated results column of a table in sequence, get Markov transition matrix—step transition probability matrix, it is:

$$P = \begin{pmatrix} 0.824 & 0.029 & 0.024 & 0.033 & 0.023 & 0.066 \\ 0.032 & 0.750 & 0.028 & 0.030 & 0.021 & 0.139 \\ 0.028 & 0.029 & 0.739 & 0.051 & 0.026 & 0.128 \\ 0.021 & 0.016 & 0.032 & 0.835 & 0.048 & 0.048 \\ 0.056 & 0.029 & 0.055 & 0.056 & 0.722 & 0.081 \\ 0.005 & 0.004 & 0.003 & 0.004 & 0.004 & 0.980 \end{pmatrix}$$

If the market is stable, in a certain period of time without big changes, we can forecast.

The market share of base period is:  $A: 3748/56060 = 0.067$ ,  $B: 3140/56060 = 0.056$ ,  
 $C: 2814/56060 = 0.050$ ,  $D: 2882/56060 = 0.051$ ,  $E: 2045/56060 = 0.036$ ,

$$F : 41431/56060 = 0.739$$

So, the state probability of base period is

$$S(0) = (0.067, 0.056, 0.050, 0.051, 0.036, 0.739).$$

This opening market share forecast is

$$S(1) = S(0) \cdot P = (0.067 \ 0.56 \ 0.050 \ 0.050 \ 0.051 \ 0.036 \ 0.739) \times$$

$$\begin{pmatrix} 0.824 & 0.029 & 0.024 & 0.033 & 0.023 & 0.066 \\ 0.032 & 0.750 & 0.028 & 0.030 & 0.021 & 0.139 \\ 0.028 & 0.029 & 0.739 & 0.051 & 0.026 & 0.128 \\ 0.021 & 0.016 & 0.032 & 0.835 & 0.048 & 0.048 \\ 0.056 & 0.029 & 0.055 & 0.056 & 0.722 & 0.081 \\ 0.005 & 0.004 & 0.003 & 0.004 & 0.004 & 0.980 \end{pmatrix} =$$

$$(0.065 \ 0.050 \ 0.046 \ 0.054 \ 0.035 \ 0.748)$$

That is brand *A* car sales planning in current period market share is  $S_A(1) = 0.065$ ; brand *B* car sales planning in current period market share is  $S_B(1) = 0.050$ ; brand *C* car sales planning in current period market share is  $S_C(1) = 0.046$ ; brand *D* car sales planning in current period market share is  $S_D(1) = 0.054$ ; brand *E* car sales planning in current period market share is  $S_E(1) = 0.035$ ; brand *F* car sales planning in current period market share is  $S_F(1) = 0.748$ .

In the state of the automobile marketing system relatively stable situation, after the sales planning the second marketing planning can continue to solve the next market share, by  $S(2) = S(1) \cdot P$ , we can get:

$$S(2) = (0.065 \ 0.050 \ 0.046 \ 0.054 \ 0.035 \ 0.748) \times$$

$$\begin{pmatrix} 0.824 & 0.029 & 0.024 & 0.033 & 0.023 & 0.066 \\ 0.032 & 0.750 & 0.028 & 0.030 & 0.021 & 0.139 \\ 0.028 & 0.029 & 0.739 & 0.051 & 0.026 & 0.128 \\ 0.021 & 0.016 & 0.032 & 0.835 & 0.048 & 0.048 \\ 0.056 & 0.029 & 0.055 & 0.056 & 0.722 & 0.081 \\ 0.005 & 0.004 & 0.003 & 0.004 & 0.004 & 0.980 \end{pmatrix}$$

$$= (0.063 \ 0.046 \ 0.043 \ 0.056 \ 0.034 \ 0.756)$$

So, the next stage marketing planning of each brand market share respectively is

$$S_A(2) = 0.063, S_B(2) = 0.046, S_C(2) = 0.043, S_D(2) = 0.056, S_E(2) = 0.034, S_F(2) = 0.756$$

Each brand car predict market share with graph as shown in figure 1:

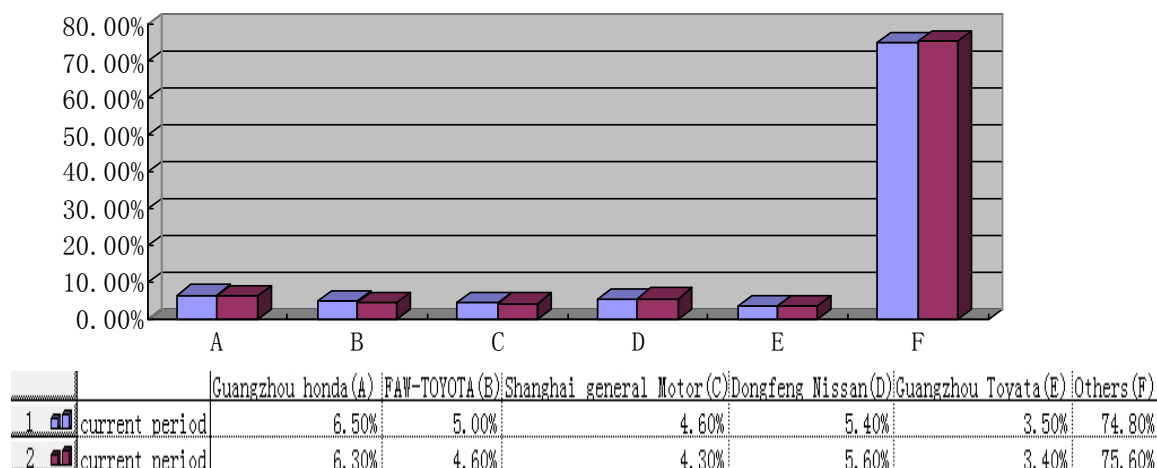


Figure 1 the market share charts

The prediction error analysis: according to the actual survey results, we get the current actual market share of each brand car, table 3, it is compared with the predicted value, get the following results.

Table 3 the error between this prediction (%) and the actual value (%)

project	Guangzhou Honda	FAW-TOYOTA	Shanghai General Motor	Dongfeng Nissan	Guangzhou Honda	others
predicted value	6.50	5.00	4.60	5.40	3.50	74.80
actual value	6.54	4.98	4.65	5.64	3.56	74.73
error	0.04	0.02	0.05	0.24	0.06	0.07

As we can see from table 3, the predicted error is smaller. It can be thought of that using Markov matrix method to forecast market share of the car is very feasible.

## CONCLUSION

Enterprise is a dynamic changing system, some of the variables and factors with the passage of time and continuous random variation, the market share is one of the variables. Facing the increasingly fierce market competition, who can accurately grasp the future market trend in time, he can gain the initiative in the market. However, with the general prediction method to predict the market share is very difficult to get the accurate results, such as long-term trend prediction method, which is based on the changing rule of the historical data to predict future market conditions, but it is not suitable for the variables that lack of the change rule of the market share. In this paper, we present the Markov prediction method, which overcomes the deficiency of the traditional prediction methods, it provides a more effective method for the product market sales forecast. Therefore, this article constructs the mathematical model based on Markov prediction, and using this model has carried on the empirical research to a city car sales in the market, and make a calculation and analysis, thus improve the market competitiveness of enterprises.

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