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$L\omega H$-closedness in $L\omega$-spaces

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ABSTRACT

In this paper, the concepts of $L\omega H$-sets and $L\omega H$-closed spaces are proposed in $L\omega$-spaces by means of $(\alpha\omega)$−remote neighborhood family. The characterizations of $L\omega H$-sets and $L\omega H$-closed spaces are systematically discussed. Some important properties of $L\omega H$-closed spaces, such as the $L\omega H$-closed spaces is ω-regular closed hereditary, arbitrarily multiplicative and preserving invariance under almost (ω_1, ω_2)-continuous mappings are proved.

Key words: $(\alpha\omega)$−-remote neighborhood family, α-net, $\omega\theta$-cluster points, ω-Hausdorff space, $L\omega H$-closed spaces, $L\omega H$-set, almost (ω_1, ω_2)-continuous mapping.

INTRODUCTION

As we all know, H-closedness is one of the most important notions in general topology. In 1968, N. V. Velicko generalized the H-closedness and introduced the concept of H-sets in topological spaces [9]. In 1992, Chen introduced the concept of L-fuzzy H-sets in L-fuzzy topological spaces and established the theory of L-fuzzy H-closedness[1]. In this paper, the concepts of $L\omega H$-sets and $L\omega H$-closed spaces will be proposed in $L\omega$-spaces[2,3]. The theory of $L\omega H$-closedness, which is generalization of the theory of L-fuzzy H-closedness, will be set up in $L\omega$-spaces.

2. PRELIMINARIES

Throughout this paper, L denotes a fuzzy lattice, Let X and Y be nonempty crisp sets, and M denotes the set consisting of all nonzero $\vee$-irreducible elements of L. 0 and 1 denote the least and greatest elements of L respectively. Let L^X be the set of all L-fuzzy sets (briefly, L-sets) on X and $M^*(L^X)$ the set of all nonzero $\vee$-irreducible elements (i.e., so-called molecules[10] or points for short) of L^X. The least and the greatest elements of L^X will be denoted by 0_X and 1_X respectively. For any $\alpha\in M$, $\beta(\alpha)$ is called the greatest minimal set of α[7], and $\beta^*(\alpha)=\beta(\alpha)\cap M$ is said to be the standard minimal set of α[10].

Definition 2.1.[2] Let X be a nonempty crisp set.
(i) An operator ω: $L^X \to L^X$ is said to be an ω-operator if (1) for all $A, B\in L^X$ and $A \ B$, $\omega(A) \ \omega(B)$; (2) for all $A\in L^X$, $A \ \omega(A)$.
(ii) An L-set $A\in L^X$ is called an ω-set if $\omega(A)=A$.
(iii) Put $\Omega=\{A\in L^X \mid A=\omega(A)\}$, and call the pair (L^X, Ω) an $L\omega$-space.

Definition 2.2.[2] Let (L^X, Ω) be an $L\omega$-space, $A\in L^X$ and $x_\alpha\in M^*(L^X)$. If there exists a $Q\in\Omega$ such that $x_\alpha \ Q$ and $P \ Q$, then call P an ω-remote neighborhood (briefly, ωR-neighborhood) of x_α. The collection of all ωR-neighborhood of x_α is denoted by $\omega\eta(x_\alpha)$. If $A \ P$ for each $P\in\omega\eta(x_\alpha)$, then x_α is said to be an ω-adherence point of A, and the union of all ω-adherence points of A is called the ω-closure of A and denoted by $\omega\mathrm{cl}(A)$. If $A=\omega\mathrm{cl}(A)$, then call A an ω-closed set, and call A' is an ω-open set. If P is an ω-closed set and $x_\alpha \ P$, then P is said to be an

ω-closed remote neighborhood (briefly, ωCR-neighborhood) of x_α, and the collection of all ωCR-neighborhoods of x_α is denoted by $\omega\eta^-(x_\alpha)$. Note $\omega\mathrm{C}(L^X)$ and $\omega\mathrm{O}(L^X)$ be the family of all ω-closed sets and all ω-open sets in L^X respectively.

Definition 2.3.[8] Let (L^X, Ω) be an $L\omega$-space, N be a molecular net in L^X and $x_\alpha \in M^*(L^X)$. If N is eventually not in $\omega\mathrm{int}(P)$ for each $P\in\omega\eta^-(x_\alpha)$, then x_α is said to be an $\omega\theta$-limit point of N or N $\omega\theta$-converges to x_α. If N is frequently not in $\omega\mathrm{int}(P)$ for each $P\in\omega\eta^-(x_\alpha)$, then x_α is said to be an $\omega\theta$-cluster point of N or N $\omega\theta$-accumulates to x_α. The union of all $\omega\theta$-limit points ($\omega\theta$-cluster points) of N is written by $\omega\theta$-lim$N$ ($\omega\theta$-ad$N$).

Definition 2.4.[8] Let N be a α-net in $A(\alpha\in M)$, $\lambda\in\beta^*(\alpha)$, if N is frequently not in $\omega\mathrm{int}(P)$ for each $P\in\omega\eta^-(x_\lambda)$, then x_λ is said to be an $\omega\theta$-cluster point of N with hight λ.

Definition 2.5.[4] Suppose that (L^X, Ω) be an $L\omega$-space, $A\in L^X$, $\alpha\in M$ and $\Phi\subseteq\omega\mathrm{C}(L^X)$.
(1) If there exists a $P\in\Phi$ such that $P\in\omega\eta^-(x_\alpha)$ for each molecule x_α in A, then Φ is called an $\alpha\omega$-remote neighborhood family (briefly, $\alpha\omega$-RF) of A, in symbol $\wedge\Phi < A(\alpha\omega)$. If there exists a nonzero $\vee$-irreducible element $\lambda\in\beta^*(\alpha)$ with $\wedge\Phi < A(\lambda\omega)$, then Φ is said to be an $(\alpha\omega)^-$-RF, in symbol $\wedge\Phi \ll A(\alpha\omega)$.

(2) If there exists a $P\in\Phi$ such that x_α $\omega\mathrm{int}(P)$ for each molecule x_α in A, then Φ is called an almost $\alpha\omega$-remote neighborhood family (briefly, almost $\alpha\omega$-RF) of A, in symbol $(\wedge\Phi)^* < A(\alpha\omega)$. If there exists a nonzero $\vee$-irreducible element $\lambda\in\beta^*(\alpha)$ with $(\wedge\Phi)^* < A(\lambda\omega)$, then Φ is said to be an almost $(\alpha\omega)^-$-RF, in symbol $(\wedge\Phi)^* \ll A(\alpha\omega)$.

Definition 2.6. [4, 5] Assume (L^X, Ω) be an $L\omega$-space, $A\in L^X$, $\gamma\in M$ and $\Gamma\subseteq\omega\mathrm{O}(L^X)$.
(1) If there is a $B\in\Gamma$ such that $B(x)$ γ for each $x\in\tau_{\gamma'}(A)=\{x\in X \mid A(x)$ $\gamma'\}$, then Γ is known as a $\gamma\omega$-cover. If there exists a prime element $t\in\alpha^*(\gamma)$ such that Γ is a $t\omega$-cover of A, then Γ is said to be a $(\gamma\omega)^+$-cover of A, where $\alpha^*(\gamma)$ is the standard maximal set of γ.

(2) If there is a $B\in\Gamma$ such that $\omega\mathrm{cl}(B)(x)$ γ for each $x\in\tau_{\gamma'}(A)=\{x\in X \mid A(x)$ $\gamma'\}$, then Γ is known as an almost $\gamma\omega$-cover. If there exists a prime element $t\in\alpha^*(\gamma)$ such that Γ is an almost $t\omega$-cover of A, then Γ is said to be an almost $(\gamma\omega)^+$-cover of A, where $\alpha^*(\gamma)$ is the standard maximal set of γ.

Definition2.7. [3, 5] Assume (L^X, Ω_i) be an $L\omega_i$-space (i=1, 2) and $f : (L^X, \Omega_1)\to(L^Y, \Omega_2)$ an L-valued Zadeh's type function.
(1) If $f^{\leftarrow}(B)\in\omega_1\mathrm{O}(L^X)$ for each $B\in\omega_2\mathrm{O}(L^Y)$, then call f (ω_1, ω_2)-continuous.

(2) If $f^{\leftarrow}(\omega_2\mathrm{cl}(\omega_2\mathrm{int}(B)))\in\omega_1 C(L^X)$ for each $B\in L^Y$, then call f almost (ω_1,ω_2)-continuous.

3. *LωH*-SET AND ITS CHARACTERISTICS

In this section, we will introduce the concepts of $L\omega H$-sets by making use of $\alpha\omega$-remote neighborhood family and $\gamma\omega$-cover in an $L\omega$-space first, give the equivalent characterizations of $L\omega H$-set by means of α-net, α-filter and α-ideal next, and then discuss the characteristics of $L\omega H$-set.

Definition 3.1. Assume (L^X, Ω) be an ω-Hausdorff space[6] and $A\in L^X$. If every $(\alpha\omega)^-$-RF Φ of A has a finite subfamily Ψ such that Ψ is an almost $(\alpha\omega)^-$-RF, where $\alpha\in M$, then call A an $L\omega H$-set.

Theorem 3.1. Let (L^X, Ω) be an ω-Hausdorff space and $A\in L^X$. Then A is an $L\omega H$-set if and only if for each $\gamma'\in M$, every $(\gamma\omega)^+$-cover Γ of A has a finite subfamily μ such that μ is an almost $(\gamma\omega)^+$-cover of A.

Proof. *Necessity.* Suppose that A is an $L\omega H$-set and Γ is any $(\gamma\omega)^+$-cover of A ($\gamma'\in M$). Put $\Phi=\Gamma' =\{B \mid B'\in\Gamma\}$ and $\alpha=\gamma'$. Then $\alpha\in M$ and Φ is an $(\alpha\omega)^-$-RF of A. In reality, $\Phi\subseteq\omega C(L^X)$ because of $\Gamma\subseteq\omega O(L^X)$. Since Γ is any $(\gamma\omega)^+$-cover of A, i.e. there exists $t\in\alpha^*(\gamma)$ such that for each $x\in\tau_{t'}(A)$ we can take an ω-closed set $P=B'\in\Phi$ with $B(x)$ t, equivalently, t' $B'(x)=P(x)$. Let $\lambda=t'$, since $t\in\alpha^*(\gamma)$, we have $\lambda\in M$, then $P\in\omega\eta^-(x_\lambda)$.This implies that Φ is a $(\alpha\omega)^-$-RF of A. Thus Φ has a finite subfamily Ψ which is an almost $(\alpha\omega)^-$-RF of A, that is, there exists a $s'\in\beta^*(\gamma')$ such that for each $x\in\tau_{\gamma'}(A)$ we can take an $P\in\Psi$ with s $\omega\mathrm{int}(P(x))$. In other words, there are $s\in\alpha^*(\gamma)$ and $B=P'\in\Psi'=\mu$ with $\omega\mathrm{cl}(B(x))=\omega\mathrm{cl}(P'(x))=(\omega\mathrm{int}(P(x)))'$ s for each $x\in\tau_{\gamma'}(A)$. This means that μ is a finite subfamily of Γ and an almost $(\gamma\omega)^+$-cover of A.
Sufficiency. Assume that every $(\gamma\omega)^+$-cover of A has a finite subfamily which is an almost $(\gamma\omega)^+$-cover of A ($\gamma'\in M$). If Φ is a $(\alpha\omega)^-$-RF of A ($\alpha\in M$), then $\Gamma=\Phi'=\{P' \mid P\in\Phi\}$ is a $(\gamma\omega)^+$-cover of A where $\gamma=\alpha'$, $\gamma'\in M$. Hence Γ has a

finite subfamily μ which is an almost $(\gamma\omega)^{+}$-cover of A by the hypothesis, that is, there exists a $t\in\alpha^{*}(\gamma)$ such that for each $x\in\tau_{\gamma'}(A)$ we can take $P=B'\in\mu'=\Psi$ with $\omega\mathrm{cl}(B(x))\ \ t$. In other words, there are $t'\in\beta^{*}(\gamma)$ and $P\in\Psi$ with $\omega\mathrm{int}(P)\in\omega\eta^{-}(x_{t'})$ for each $x\in\tau_{\gamma'}(A)$. This means that Ψ is a finite subfamily of Φ and an almost $(\alpha\omega)^{-}$-*RF* of A. Therefore A is an $L\omega H$-set.

Definition 3.2. Assume (L^{X}, Ω) be an $L\omega$-space, $\Phi\subseteq\omega C(L^{X})$ and $\alpha\in M$. Φ is said to has $(\alpha\omega)^{*}$-finite intersection property for A, if $\vee_{x\in X}(A\wedge(\wedge\Psi^{\circ}))(x)\ \ \alpha$ for each $\Psi\in 2^{(\Phi)}$, where $\Psi^{\circ}=\{\omega\mathrm{int}(P)\mid P\in\Psi\}$.

Theorem 3.2. Let (L^{X}, Ω) be an ω-Hausdorff space[6] and $A\in L^{X}$. Then A is an $L\omega H$-set if and only if for each $\alpha\in M$ and each $\Phi\subseteq\omega C(L^{X})$ having $(\alpha\omega)^{*}$-finite intersection property for A, there exists a molecule $x_{\alpha}\ \ A$ with $x_{\alpha}\ \ \wedge\Phi$.

Proof. *Necessity*. Grant that A is an $L\omega H$-set, $\Phi\subseteq\omega C(L^{X})$ and Φ has $(\alpha\omega)^{*}$-finite intersection property for A $(\alpha\in M)$. If $\lambda\in\beta^{*}(\alpha)$, $x_{\alpha}\ \ \wedge\Phi$ for each $x_{\alpha}\ \ A$, then Φ is an $(\alpha\omega)^{-}$-*RF* of A by the hypothesis of Φ. Hence Φ has a finite subfamily Ψ which is an almost $(\alpha\omega)^{-}$-*RF* of A, i.e, there is a $\lambda\in\beta^{*}(\alpha)$ satisfying $x_{\lambda}\ \ \wedge\Psi^{\circ}$ for each $x_{\lambda}\ \ A$, in other words, $\vee_{x\in X}(A\wedge(\wedge\Psi))(x)\ \ \lambda$. It contradicts the fact that Φ has $(\alpha\omega)^{*}$-finite intersection property for A. Hence the necessity is proved.

Sufficiency. Assume that the condition holds and that Φ is an $(\alpha\omega)^{-}$-*RF* of A. If for any finite subfamily Ψ of Φ, Ψ is not an almost $(\alpha\omega)^{-}$-*RF* of A, then for each $\lambda\in\beta^{*}(\alpha)$ there exists a molecule $x_{\lambda}\ \ A$ with $x_{\lambda}\ \ \wedge\Psi$, i.e, $\vee_{x\in X}(A\wedge(\wedge\Psi))(x)\ \ \lambda$. This shows that Φ has $(\alpha\omega)^{*}$-finite intersection property for A. By the assumption we have $\lambda\in\beta^{*}(\alpha)$, $x_{\alpha}\ \ A$ satisfying $x_{\alpha}\ \ \wedge\Psi$. It contradicts that Φ is an $(\alpha\omega)^{-}$-*RF* of A. Therefore Φ has a finite subfamily Ψ which is an almost $(\alpha\omega)^{-}$-*RF* of A, and hence A is an $L\omega H$-set.

Theorem 3.3. Let (L^{X}, Ω) be an ω-Hausdorff space and $A\in L^{X}$. Then A is an $L\omega H$-set if and only if for each $\alpha\in M$ and $\lambda\in\beta^{*}(\alpha)$, every α-net N in A has an $\omega\theta$-cluster point in A with hight λ.

Proof. *Necessity*. Suppose that A is an $L\omega H$-set and that $N=\{N(n)\mid n\in D\}$ is an α-net in A. If for each $\lambda\in\beta^{*}(\alpha)$, N has not any $\omega\theta$-cluster point in A with hight λ, then there exists a $P[x]\in\omega\eta^{-}(x_{\lambda})$ such that N is eventually in $\omega\mathrm{int}(P[x])$ for each $x_{\lambda}\ \ A$, that is, there is a $n(x)\in D$ with $N(n)\ \ \omega\mathrm{int}(P[x])$ whenever $n\ \ n(x)$. Write $\Phi=\{P[x]\mid x_{\lambda}\ \ A\}$. Obviously, Φ is an $(\alpha\omega)^{-}$-*RF* of A, so Φ has a finite subfamily $\Psi=\{P[x_i]\mid i=1, 2, \ldots, m\}$ which is an almost $(\alpha\omega)^{-}$-*RF* of A, i.e., there is an $i\in\{1, 2, \ldots, m\}$ with $y_t\ \ \omega\mathrm{int}(P[x_i])$ for some $t\in\beta^{*}(\alpha)$, and each $y_t\ \ A$. Take $P=\wedge_{i=1}^{m}P[x_i]$. Then $y_t\ \ \omega\mathrm{int}(P)$ for each $y_t\ \ A$. Since D is a directed set, there is a $n_0\in D$, such that $n_0\ \ n(x_i)$ and $N(n)\ \ \omega\mathrm{int}(P[x_i])$ $(i=1, 2, \ldots, m)$ whenever $n\ \ n_0$ and so $N(n)\ \ \omega\mathrm{int}(P)$. This shows that for each $y_t\ \ A$, $\vee(N(n))\ \ t$ as long as $n\ \ n_0$. It contradicts the fact that N is an α-net. Therefore N has at least an $\omega\theta$-cluster point in A with hight λ.

Sufficiency. Assume that every α-net in A has at least an $\omega\theta$-cluster point with hight λ for each $\alpha\in M$ and $\lambda\in\beta^{*}(\alpha)$, Φ is an $(\alpha\omega)^{-}$-*RF* of A and $2^{(\Phi)}$ is the set of all finite subfamily of Φ. If for each $\lambda\in\beta^{*}(\alpha)$ and each $\Psi\in 2^{(\Phi)}$, Ψ is not an almost $(\alpha\omega)^{-}$-*RF* of A, i.e., there exists a molecule $N(\lambda, \Psi)\ \ A$ satisfying $N(\lambda, \Psi)\ \ \wedge\Psi$ for each $\lambda\in\beta^{*}(\alpha)$. In $\beta^{*}(\alpha)\times 2^{(\Phi)}$, we define the relation as follows: $(\lambda_1, \Psi_1)\ \ (\lambda_2, \Psi_2)$ if and only if $\lambda_1\ \ \lambda_2$, $\Psi_1\ \ \Psi_2$, then $\beta^{*}(\alpha)\times 2^{(\Phi)}$ is a directed set with the relation " ". Let $N=\{N(\lambda, \Psi)\mid(\lambda, \Psi)\in\beta^{*}(\alpha)\times 2^{(\Phi)}\}$. One can easily see that N is an α-net in A. We assert that N has not any $\omega\theta$-cluster point in A with hight α. In fact, for some $\lambda\in\beta^{*}(\alpha)$ and each $x_{\lambda}\ \ A$, we can choose an ω-closed set $Q\in\Phi$ with $Q\in\omega\eta^{-}(x_{\lambda})$, specially, $\omega\mathrm{int}(Q)\in\omega\eta^{-}(x_{\lambda})$ by the definition of Φ. Taking $\lambda_1\in\beta^{*}(\alpha)$ and $\Psi\in 2^{(\Phi)}$, we have $Q\in\Psi$ according to $(\lambda, \Psi)\ \ (\lambda_1, \{Q\})$, and hence $N(\lambda, \Psi)\ \ \omega\mathrm{int}(Q)$. This implies that N is eventually in $\omega\mathrm{int}(Q)$, and thus x_{λ} is not an $\omega\theta$-cluster point of N. It is in contradiction with the hypothesis of sufficiency. Consequently, A is an $L\omega H$-set.

Theorem 3.4. Let (L^{X}, Ω) be an $L\omega$-space, $A\in L^{X}$, and N be an α-net in $A(\alpha\in M)$. Then $N\propto_{\omega\theta}x_{\alpha}$ if and only if there is an α-subnet T of N with $T\rightarrow_{\omega\theta}x_{\alpha}$.

Proof. *Necessity*. Suppose that $N=\{N(n)\mid n\in D\}$ is an α-net in A, and $N\propto_{\omega\theta}x_{\alpha}$. Then N is eventually in $\omega\mathrm{int}(P)$ for each $P\in\omega\eta^{-}(x_{\alpha})$, that is, for each $n_0\in D$ there exists an $n\in D$ with $n\ \ n_0$ satisfying $N(n)\ \ \omega\mathrm{int}(P)$. Let $E=\{(n, P)\mid n\in D, P\in\omega\eta^{-}(x_{\alpha}), N(n)\ \ \omega\mathrm{int}(P)\}$. For each $(n_1, P_1), (n_2, P_2)\in E$, we define the relation as follows: $(n_1, P_1)\ \ (n_2, P_2)$ if and only if $n_1\ \ n_2$, $P_1\ \ P_2$, then E is a directed set with the relation " ". Assume that $\phi: E\rightarrow D$ with $\phi(n, P)=n$. Let $T(n, P)=N(\phi(n, P))=N(n)$ for each $(n, P)\in E$, then T is an α-subnet of N. Now we just need to prove that $T\rightarrow_{\omega\theta}x_{\alpha}$. In fact, for each $P\in\omega\eta^{-}(x_{\alpha})$, taking $(n_0, P)\in E$, when $(n, Q)\ \ (n_0, P)$, we have $T(n, Q)\ \ \omega\mathrm{int}(P)$ according to $T(n, Q)=N(n)\ \ \omega\mathrm{int}(Q)$ and $Q\ \ P$. This implies that $T\rightarrow_{\omega\theta}x_{\alpha}$.

Theorem 3.5. Let (L^X, Ω) be an ω-Hausdorff space and $A\in L^X$. Then A is an $L\omega H$-set if and only if for each $\alpha\in M$, $\lambda\in\beta^*(\alpha)$ and each α-net in $A(\alpha\in M)$, every α-subnet T of N has an $\omega\theta$-cluster point in A with hight λ.

Proof. By Theorem 3.3 and Theorem 3.4, it is proved.

Definition 3.3. Let (L^X, Ω) be an ω-Hausdorff space, $\alpha\in M$ and $\lambda\in\beta^*(\alpha)$, Δ be an α-filter in L^X and $x_\lambda\in M^*(L^X)$. If $F \quad \omega\mathrm{int}(P)$ and for each $P\in \omega\eta^-(x_\lambda)$ and each $F\in\Delta$, then x_λ is called an $\omega\theta$-cluster point of Δ with hight λ.

Theorem 3.6. Let (L^X, Ω) be an ω-Hausdorff space and $A\in L^X$. Then A is an $L\omega H$-set if and only if for each $\alpha\in M$ and $\lambda\in\beta^*(\alpha)$, every α-filter containing A has an $\omega\theta$-cluster point in A with hight λ.

Proof. *Necessity*. Grant that A is an $L\omega H$-set and that Δ is an α-filter containing A. Then $F\wedge A\in\Delta$ for each $F\in\Delta$ and $\vee_{x\in X}(F\wedge A)(x) \quad \alpha$, and thus there exists a molecule $N(F, \lambda) \quad F\wedge A$ with hight λ for each $\lambda\in\beta^*(\alpha)$. Define $N(\Delta)=\{N(F, \lambda) \quad F\wedge A \mid (F, \lambda)\in\Delta\times\beta^*(\alpha)\}$ and define a relation in $\Delta\times\beta^*(\alpha)$ as follows:

$$(F_1, \lambda_1) \quad (F_2, \lambda_2) \text{ if and only if } F_1 \quad F_2 \text{ and } \lambda_1 \quad \lambda_2.$$

Evidently, $\Delta\times\beta^*(\alpha)$ is a directed set with the relation " ", and then $N(\Delta)$ is an α-net in A. Because A is an $L\omega H$-set, so $N(\Delta)$ has an $\omega\theta$-cluster point in A with hight λ, say x_λ. We assert that x_λ is also an $\omega\theta$-cluster point of Δ. In reality, $N(\Delta)$ is frequently not in $\omega\mathrm{int}(P)$ for each $\lambda\in\beta^*(\alpha)$ and $P\in\omega\eta^-(x_\lambda)$, i.e, for each $F\in\Delta$ there exists a $F_1\in\Delta$ with $F_1 \quad F$ and some $r\in\beta^*(\alpha)$ satisfying $N(F_1, r) \quad \omega\mathrm{int}(P)$. Hence we have $F \quad P$ by virtue of the fact that $N(F_1, r) \quad F\wedge A \quad F_1 \quad F$. This means that x_λ is an $\omega\theta$-cluster point of Δ. Therefore the necessity is proved.

Sufficiency. Suppose that every α-filter containing A has an ω-cluster point in A with hight λ for each $\alpha\in M$ and $\lambda\in\beta^*(\alpha)$. Grant that $N=\{N(n) \mid n\in D\}$ is an α-net in A. Let $F_m=\vee\{N(n) \mid n \quad m\}$, $\Delta(N)=\{F\in L^X \mid$ there is a F_m such that $F \quad F_m\}$ for each $m\in D$. By virtue of the fact that N is α-net, we have $\vee_{x\in X}F_m(x)=\vee_{x\in X}\{\vee\{N(n) \mid n \quad m\}\}(x) \quad \lambda$ for each $\lambda\in\beta^*(\alpha)$ and $m\in D$. Then $\vee_{x\in X}F_m(x) \quad \alpha$ according to the arbitrariness of λ in $\beta^*(\alpha)$. By the definition of $\Delta(N)$, there exists $F_m \quad F$ for each $F\in\Delta$, so $\vee_{x\in X}F_m(x) \quad \alpha$. This implies that $\Delta(N)$ is an α-filter containing A, and hence $\Delta(N)$ has an $\omega\theta$-cluster point in A with hight λ for each $\lambda\in\beta^*(\alpha)$ by the supposition, say x_λ. Then we have $F \quad \omega\mathrm{int}(P)$ for each $P\in\omega\eta^-(x_\lambda)$ and $F\in\Delta(N)$, specially, $F_m \quad \omega\mathrm{int}(P)$ for each $F_m(m\in D)$. By the definition of F_m, we have $N(n) \quad \omega\mathrm{int}(P)$ for some $n \quad m$. This implies that N is frequently not in $\omega\mathrm{int}(P)$, in other words, x_λ is an $\omega\theta$-cluster point of N in A with hight λ. In accordance with Theorem 3.3, A is an $L\omega H$-set.

Definition 3.4. Assume (L^X, Ω) be an $L\omega$-space, I be an α-ideal in L^X $(\alpha\in M)$, $\lambda\in\beta^*(\alpha)$, $x_\lambda\in M^*(L^X)$. If $B\vee\omega\mathrm{int}(P)\neq 1_X$ for each $P\in\omega\eta^-(x_\lambda)$ and $B\in I$, then I is called an $\omega\theta$-cluster point of I with hight λ.

Theorem 3.7. Let (L^X, Ω) be an ω-Hausdorff space and $A\in L^X$. Then A is $L\omega H$-set if and only if every α-ideal which does not contain A has an $\omega\theta$-cluster point in A with hight λ for each $\alpha\in M$ and $\lambda\in\beta^*(\alpha)$.

Proof. *Necessity*. Assume that A is an $L\omega H$-set, I is an α-ideal which does not contain A and $N(I)=\{N(I)((b, B))=b \quad A \mid (b, B)\in D(I)\}$ where $D(I)=\{(b, B) \mid b\in M^*(L^X), B\in I \text{ and } b \quad B\}$. Then $N(I)$ is an α-net in A. Hence $N(I)$ has an $\omega\theta$-cluster point in A with hight λ by Theorem 3.3, say x_λ. Now we will prove that x_λ is also an $\omega\theta$-cluster point of I. In reality, x_λ is an $\omega\theta$-cluster point of $N(I)$ in A with hight λ for each $\lambda\in\beta^*(\alpha)$, then for each $(b_0, B_0)\in D(I)$ there exists a $(b, B)\in D(I)$ with $(b, B) \quad (b_0, B_0)$ satisfying $N(I)((b, B))=b \quad \omega\mathrm{int}(P)$. Hence we have $B' \quad \omega\mathrm{int}(P)$ by virtue of the fact that $b \quad B$, equivalently, $B\vee\omega\mathrm{int}(P)\neq 1_X$. So x_α is also an $\omega\theta$-cluster point of I. Consequently, the necessity is proved.

Sufficiency. Grant that every α-ideal which does not contain A has an $\omega\theta$-cluster point in A with hight λ for each $\lambda\in\beta^*(\alpha)$ $(\alpha\in M)$ and Δ is an α-filter containing A. Let $I=\{F'\in L^X \mid F\in\Delta\}$. Evidently, I is an α-ideal which does not contain A. Now we will prove that Δ has an $\omega\theta$-cluster point in A with hight λ for each $\lambda\in\beta^*(\alpha)$. Actually, by the hypothesis we know that I has an $\omega\theta$-cluster point in A with hight λ for each $\lambda\in\beta^*(\alpha)$, say x_λ, i.e., $F'\vee P\neq 1_X$, equivalently, $F \quad P$, for each $F\in\Delta$ and each $P\in\omega\eta^-(x_\lambda)$. Therefore x_λ is an $\omega\theta$-cluster point of Δ in line with Definition 3.3, and hence A is an $L\omega H$-set by Theorem 3.6. This implies that the sufficiency holds.

4. SOME IMPORTANT PROPERTIES OF *LωH*-SETS

In this section, we will further deliberate the properties of $L\omega H$-sets in an $L\omega$-space.

Definition 4.1. Assume (L^X, Ω) be an ω-Hausdorff space. If the largest $L\omega$-set 1_X is $L\omega H$-set, then call (L^X, Ω) an $L\omega H$-closed space.

Theorem 4.1. Let (L^X, Ω) be an ω-Hausdorff space and $A, B\in L^X$. If A is an $L\omega H$-set and B is an ω-regular closed set, then $A\wedge B$ is an $L\omega H$-set.

Proof. Assume that Φ is an $(\alpha\omega)^{-}$-RF of $A\wedge B$ $(\alpha\in M)$. Let $\Phi^*=\Phi\cup\{B\}$, we asset that Φ^* is an $(\alpha\omega)^{-}$-RF of A. Actually, for some $\lambda\in\beta^*(\alpha)$ and for each x_λ A, if x_λ B, then we have x_λ $A\wedge B$. Since $\wedge\Phi<(A\wedge B)(\lambda)$, there exists a $P\in\Phi\subset$ with $P\in\omega\eta^{-}(x_\lambda)$. If x_λ B, then we have $B\in\omega\eta^{-}(x_\lambda)$ and $B\in$. Since A is an $L\omega H$-set, there exist $t\in\beta^*(\alpha)$ and $\Psi^*\in 2^{(\Phi^*)}$, such that Ψ^* is an almost $t\omega$-RF of A. Let $\Psi=\Psi^*\backslash\{B\}$, then $\Psi\in 2^{(\Phi)}$ and $x_t\in A$ if x_t $A\wedge B$. By the property of Ψ^*, there exists $P\in\Psi^*$ such that $\omega\mathrm{int}(P)\in\omega\eta^{-}(x_t)$. But x_t B, so $P\neq B$, i.e. $P\in\Psi$, then we have $(\wedge\Psi)^*<(A\wedge B)(t)$. This implies that Ψ is an almost $(\alpha\omega)^{-}$-RF of Φ, and hence $A\wedge B$ is an $L\omega H$-set.

This theorem shows that $L\omega H$-set is hereditary with respect to ω-regular closed sets.

Theorem 4.2. Let (L^X, Ω_1) be an $L\omega_1H$-closed space, and (L^Y, Ω_2) be ω_2-Hausdorff space. If $f: L^X\to L^Y$ is a fully, strata-preserving and inverted strata-preserving almost (ω_1, ω_2)-continuous fuzzy mapping, then (L^Y, Ω_2) is an $L\omega_2H$-closed space.

Proof. Assume that $N=\{N(n) \mid n\in D\}$ is an α-net in $L^Y(\alpha\in M)$. By the inverted strata-preserving of f, there exists a $r\in M$ such that $f^{\leftarrow}(N)=\{f^{\leftarrow}(N(n) \mid n\in D\}$ is an r-net in L^X. Since L^X is an $L\omega_1H$-closed space, $f^{\leftarrow}(N)$ has an $\omega_1\theta$-cluster point in L^X with hight t for each $t\in\beta^*(r)$, say x_t. By the strata-preserving of f, there exists a $\lambda\in M$ such that $f(x_t)=y_\lambda\in M^*(L^Y)$. Now we will prove that y_λ is an $\omega_2\theta$-cluster point of N. In reality, $f^{\leftarrow}(\omega_2\mathrm{cl}(\omega_2\mathrm{int}(B)))\in\omega\eta^{-}(x_t)$ for each $B\in\omega_2\eta^{-}(y_\lambda)$. Then there exists a $n\in D$ such that $f^{\leftarrow}(N(n))$ $\omega_2\mathrm{int}(f^{\leftarrow}(\omega_2\mathrm{cl}(\omega_2\mathrm{int}(B)))$. Since

$$f^{\leftarrow}(\omega_2\mathrm{int}(B)) \quad \omega_2\mathrm{int}(f^{\leftarrow}(\omega_2\mathrm{int}(\omega_2\mathrm{cl}(\omega_2\mathrm{int}(B)))) \quad \omega_2\mathrm{int}(f^{\leftarrow}(\omega_2\mathrm{cl}(\omega_2\mathrm{int}(B)))),$$

we have $f^{\leftarrow}(N(n)$ $f^{\leftarrow}(\omega_2\mathrm{int}(B))$, i.e. $N(n)$ $\omega_2\mathrm{int}(B)$. This implies that y_λ is an $\omega_2\theta$-cluster point of N, and hence (L^Y, Ω_2) is an $L\omega_2H$-closed space.

This theorem means that the $L\omega H$-closed space is topological variant under the strata-preserving and inverted strata-preserving almost (ω_1, ω_2)-continuous mappings.

Theorem 4.3. Let (L^X, Ω_1) and (L^Y, Ω_2) be an $L\omega_1$-space and an $L\omega_2$-space respectively, and $f: L^X\to L^Y$ be an (ω_1, ω_2)-continuous L-valued Zadeh's type function. If A is an $L\omega_1$-compact set in (L^X, Ω_1), then $f^{\to}(A)$ is an $L\omega_2$-compact set in (L^Y, Ω_2).

Proof. Since L-valued Zadeh's type function is also strata-preserving and inverted strata-preserving order-homomorphism, the theorem is hold in line with Theorem 4.2.

This theorem means that the $L\omega H$-closedness is topological variant under almost (ω_1, ω_2)-continuous L-valued Zadeh's type functions.

Theorem 4.4. *Let (L^X, Ω) be the product space of a collection of $L\omega$-spaces $\{(L^{X_t}, \Omega_t) \mid t\in\Gamma\}$. If for each $t\in\Gamma$, (L^{X_t}, Ω_t) is an $L\omega_tH$-closed space, then (L^X, Ω) is an $L\omega H$-closed space.*

Proof. Assume that N is an α-net in A. Since projection mapping $p_t: L^X\to L^{X_t}$ $(t\in\Gamma)$ is a ω-continuous order-homomorphism, $p_t(N)$ is an α-net in (L^{X_t}, Ω_t). By the hypothesis, we know that (L^{X_t}, Ω_t) is an $L\omega_tH$- closed space for each $t\in\Gamma$, so there is an $\omega_t\theta$-cluster point of $p_t(N)$ in L^{X_t} with hight λ_t for each $\lambda_t\in\beta^*(\alpha)$, say x_{λ_t}. Let $x_\lambda=(x_{\lambda_t})_{t\in\Gamma}$, then x_λ is an $\omega\theta$-cluster point of N in L^X with hight λ for each $\lambda\in\beta^*(\alpha)$. This implies that (L^X, Ω) is an $L\omega H$-closed space .

This theorem means that the $L\omega H$-closedness is arbitrarily multiplicative.

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