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Long blade vibration model for turbine-generator shafts torsional vibration analysis

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ABSTRACT

The umbrella type vibration of the last stage blade disk of the low pressure turbine can be induced by the external excitation which caused by the torsional vibration of the turbine-generator shafts. However, there is no applicable model for analyzing the blade disk vibration due to shift torsional vibration. According to the characteristic of umbrella type blade disc vibration, a blade model for analyzing the vibration of the long blades during the torsional vibration of the shafts is given in this paper. A parameter adjusting method based on sensitivity analysis is presented so as to make the inherent characteristic of the blade disc model close to that of the real blade disc.

Keywords: Blade disc vibration model; Parameter Adjustment; Turbine-Generator; Torsional vibration.

INTRODUCTION

At present, simplified model for shaft-blade combined vibration is to consider the shafts of the steam turbine generator unit as a mass-spring model. In this case, the turbine blades are considered as the branch structures of the shafts model and their radial movements are ignored. To account the interaction of the blades during vibration, the influence of the shroud has been analyzed [1-5], and the shroud is considered as springs [6]; or considered as damping [7-8]. During shaft lateral vibration, blades in different position have different vibration states, therefore, these kinds of model can effectively simulate the interactions among different blades during lateral vibration. However, during torsional vibration, blades in different position receive the same external torque from the turbine shift. Therefore, the characteristic of the last stage blade disk vibration due to shaft torsional vibration is umbrella type vibration without radius. There is no relative movement among the blades in these two models under umbrella type vibration. Hence, these two models cannot stimulate the impact caused by the shroud on the blade during shaft vibration. Therefore, it is necessary to establish a new vibration model for simulating the vibration of the long blades during the torsional vibration of turbine-generator shafts.

Long Blade Vibration Model

To analyze the impact caused by the blade shroud on the long blades during torsional vibration analysis, a mass-spring model can be established according to the following methods: during shaft torsional vibration, the blade vibration can be considered as lateral vibration relative to the blade root. Then, each blade can be considered as a mass-spring model with one end fixed on the shaft, which has only x direction vibration. The mass-spring model can be shown in the following Fig.1.

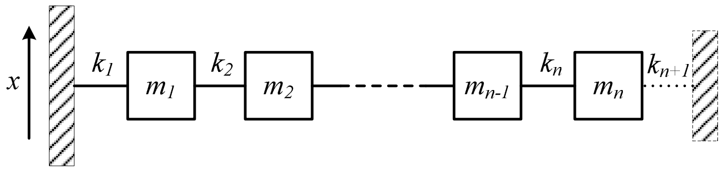

Fig.1 Blade vibration equivalent model

Parameter Adjustment. Turbine blades can be modeled to lumped mass torsional vibration model with n degrees of freedom, the un-damped free torsional vibration differential equation is as follows:

$$\boldsymbol{I}\ddot{\boldsymbol{\theta}} + \boldsymbol{K}\boldsymbol{\theta} = \boldsymbol{0} \tag{1}$$

Where *I* and *K* are the moment of inertia matrix and stiffness matrix, respectively; $\boldsymbol{\theta}$ and $\ddot{\boldsymbol{\theta}}$ are mass displacement matrix and acceleration matrix, respectively.

Substituting $\theta=\varphi\sin(\omega t+\psi)$ into Eq. (1), the natural frequency ω_i and vibration modes φ_i can be obtained from:

$$\left(\boldsymbol{K} - \omega^2 \boldsymbol{I}\right)\phi = \boldsymbol{0} \tag{2}$$

Based on vibration mechanics, the relationships between natural frequency and vibration modes are as follows:

$$\left(\boldsymbol{K} - \omega_i^2 \boldsymbol{I}\right)\phi_i = \boldsymbol{0} \tag{3}$$

$$\phi_i^T [I] \phi_i = 1 \tag{4}$$

Where ω_i and φ_i are the natural frequency and vibration mode after regularization of the *i*tch order, respectively. Multiply Eq. (3) by φ_i^T from left and differentiating it, following relations are obtained:

$$\frac{\partial \varphi_i^T}{\partial x_j}(\mathbf{K} - \omega_i^2 \mathbf{I})\varphi_i + \varphi_i^T\left(\frac{\partial \mathbf{K}}{\partial x_j} - \omega_i^2 \frac{\partial \mathbf{I}}{\partial x_j} - 2\omega_i \mathbf{I}\frac{\partial \omega_i}{\partial x_j}\right)\varphi_i + \varphi_i^T(\mathbf{K} - \omega_i^2 \mathbf{I})\frac{\partial \varphi_i}{\partial x_j} = \mathbf{0} \tag{5}$$

Inserting Eqs. (3) And Eq. (4) into Eq. (5), the natural frequency sensitivity of structure parameter is obtained by:

$$\frac{\partial \omega_i}{\partial x_j} = \frac{\phi_i^T\left(\dfrac{\partial \boldsymbol{K}}{\partial x_j} - \omega_i^2 \dfrac{\partial \boldsymbol{I}}{\partial x_j}\right)\phi_i}{2\omega_i} \tag{6}$$

Where x_j stands for the elastic stiffness K_j.

The sensitivity of ω_i to the *jth* spring is given by:

$$\frac{\partial \omega_i}{\partial K_j} = \frac{\phi_i^T\left(\dfrac{\partial \boldsymbol{K}}{\partial K_j}\right)\phi_i}{2\omega_i} = \frac{\left[(\phi_i)_j - (\phi_i)_{j+1}\right]^2}{2\omega_i} \tag{7}$$

All the sensitivities of each order natural frequencies can be calculated based on Eq.(7). The value of the sensitivity reflects the rate of change of natural frequency with the torsional rigidity variation.

The natural frequency of torsional vibration ω_i can be expressed by the moment of inertia I_j and torsional stiffness K_j as follows:

$$\omega_i = f\left(I_1 \cdots I_j, K_1 \cdots K_j\right) \tag{8}$$

With Taylor series expansion Eq. (8) can be written when ignoring the secondary and its above modified terms:

$$\omega_i - \omega_{io} = \Delta\omega_i = \sum_{j=1}^{n} \frac{\partial \omega_i}{\partial I_j}\Delta I_j + \sum_{j=1}^{n} \frac{\partial \omega_i}{\partial K_j}\Delta K_j \tag{9}$$

Eq. (9) describe the relationship between the variation of torsional vibration natural frequency $\Delta\omega_i$ and structure parameters ΔI_j、ΔK_j. The relationship is considered to be linear due to the the variation of structure parameters are relatively small. So the secondary and secondary above modified terms in the Taylor expansion were ignored. The torsional moment of inertia has been molded accurately enough and the damping coefficient has little influence on the torsional vibration inherent characteristics when adjusting the torsional vibration model. Therefore, only considering the torsional rigidity variation Eq. (9) can be simplified as:

$$\Delta\omega_i = \sum_{j=1}^{n} \frac{\partial \omega_i}{\partial K_j} \Delta K_j \tag{10}$$

Torsional vibration natural frequency deviation $\Delta\omega_i$ can be achieved based on the actual value ω_i by monitoring and analysis and the calculation result ω_{io} by the torsional vibration model. And then the torsional rigidity of adaptive adjustment quantity ΔK_j can be solved by an equation set based on Eq. (10), which can make the original vibration model become more accurate. The equation set can be written as matrix expression [A] [X] = [B]:

$$\boldsymbol{B} = \left(\Delta\omega_1 \cdots \Delta\omega_i\right)^T \tag{11}$$

$$\boldsymbol{X} = \left(\Delta K_1 \cdots \Delta K_j\right)^T \tag{12}$$

$$\mathbf{A} = \begin{pmatrix} \frac{\partial \omega_1}{\partial K_1} & \cdots & \frac{\partial \omega_1}{\partial K_j} \\ \vdots & \cdots & \vdots \\ \frac{\partial \omega_i}{\partial K_1} & \cdots & \frac{\partial \omega_i}{\partial K_j} \end{pmatrix} \tag{13}$$

The solution of this equation set which is torsional rigidity adjustment quantity ΔK_j desired to know can be obtained from matrix transformation equations, that is ***X***=***A***/***B***.

From the above, the parameter adjusting procedure is shown in Fig.2.

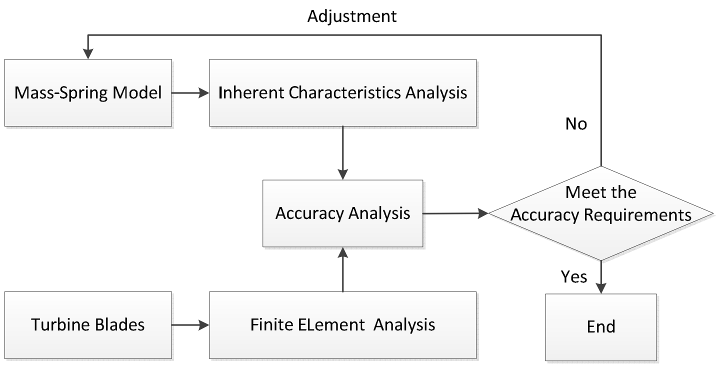

Fig.2 The logic scheme of torsional vibration model online adaptive adjustment

Analysis and Application

Take a last stage blade in the low pressure turbine of a 1000MW turbine-generator unit as a research object. Assume the acting stress at the blade tip imposed by the shroud were proportional to the displacement of blade tip relative to its root, shown in Fig.3. Based on finite element analysis results of the nature characteristics of blade vibration, the nature characteristics of mass-spring model can be shown in Fig.4 and table 1.

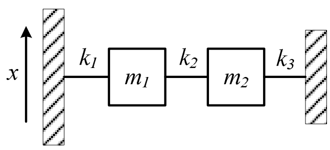

Fig.3 Mass-spring equivalent model

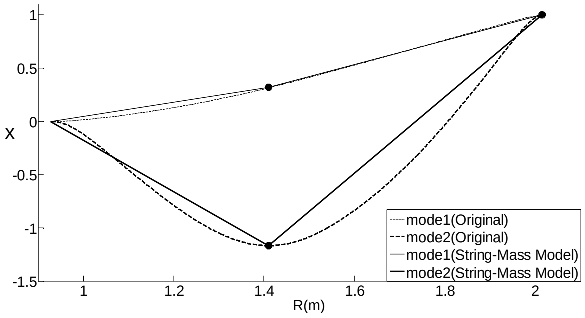

Fig.4 Mass-spring model vibration model

Table 1 Mass-spring vibration model nature characteristics

ω_1	ω_2	ω_2 / ω_1	$(\mathbf{X}_1)_1$	$(\mathbf{X}_1)_2$	$(\mathbf{X}_2)_1$	$(\mathbf{X}_2)_2$
107.743Hz	403.342Hz	3.7436	0.3198	1	-1.1667	1

Let $m_1 = 1kg$ $k_1 = 1 \times 10^6 N/m$ calculate mass-spring model parameters through parameter modification method shown in the fig. And the calculation value is shown in the following table 2.

Table 2 mass-spring model calculation value

m_2	k_2	k_3	ω_2 / ω_1	$(\mathbf{X}_1)_2$	$(\mathbf{X}_2)_2$	Δm_2	Δm_2	Δm_3
2	2	0.5	3.7321	0.3660	-1.3660	0.2193	-0.1326	-0.0240
2.2193	1.8674	0.5240	4.1175	0.3775	-1.1936	0.1821	0.1949	-0.0293
2.4014	2.0623	0.5533	4.1586	0.3526	-1.1811	0.1944	0.2393	-0.0127
2.5958	2.3016	0.5660	3.9224	0.3299	-1.1678	0.0996	0.1393	-0.0005
2.6954	2.4409	0.5665	3.7365	0.3192	-1.1623	—	—	—

It can be seen from the table, after 4 times of calculation, the model nature frequency ratio model is almost the same with blade vibration characteristics. And then, it is only required to adjust m_1 and k_1 to make the model rotational inertia relative to shaft centerline be the same with that of actual blade. And the 1st phase nature frequency of the model is the same with that of blade. So, the mass parameter and rigidity parameter of single blade model can be calculated. In-addition, one circle has 85 blades. The parameters of the whole circle blade mass-spring model can be calculated. The mass and rigidity values of single blade and one circle blades is shown in the following table 3.

Table 3 Mass-spring model parameter

m_1	m_2	k_1	k_2	k_3
3.35kg	9.02kg	1.3431×107N/m	3.2783×107N/m	7.6087×106N/m
m_1'	m_2'	k_1'	k_2'	k_3'
284.75kg	766.70kg	1.1416×109N/m	2.7866×109N/m	6.4674×108N/m

CONCLUSION

It has been proved that the inherent characteristics of the long blade umbrella type vibration due to torsional vibration of turbine-generator shafts can be accurately simulated using the blade modeling method and the parameter adjusting method. Which can insure the accuracy of the blade vibration response analysis.

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