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LINGO optimization model-based gymnastics team competition best starting line-up research

Yu Peng and Guoqing Zhang

Institute of Physical Education, Hunan Institute of Science and Technology, Yueyang, Hunan, China

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ABSTRACT

Women's gymnastics team competition is composed of high-low bars, balance beam, and vaulting horse and floor exercises these four items. In practical competitions, athletes strength surely decides team total score result, but a good starting line-up also has very important impacts on competition results. The paper goes deeper into exploring women's gymnastics team's team competitions forming problems, firstly according to known conditions ,it provides each athlete score in the most pessimistic case, establishes objective functions to let total score to be as high as possible, combines with athlete participation rules, it gets constraint conditions, adopts $0\sim1$ *integer programming to establish model, applies* $lingo$ *software to solve the model, and gets best starting line-up under pessimistic model; finally applies probability statistical theory and normal distribution knowledge to further deepen problem, by calculating, it gets variance* D_{ij} *and expectation* $C_{i,j}$ *, and inputs them into standard normal programming functions so that can get objective functions, applies* $lingo$ *software to solve best starting line-up.*

Key words: best line-up, 0~1 programming, probability statistics, normal distribution

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INTRODUCTION

Gymnastics world cup is one of gymnastics top competitions that just second to Olympic Games and world championships, is formal listed into international gymnastics federation competitions yearbook. In Athens, 2004, in case that delegation overfulfilling task, Chinese gymnastics team that started with seven gold medals became the team with the biggest failure [1-3]. After returned, Chinese gymnastics team started from negative, made every effort, grasped from bit, from every motion and life details to coaches group division adjustment, as well as analyzed every athlete battlefield performing status, made comprehensive layout and adjust members starting line-up [4-6]. Till October, 2006, in Aahus, Demark, Chinese gymnastics that started from negative finally shocked the world with eight gold medals. In two years, Chinese gymnastics from negative to "shock", it succeeded in line-up, new staff used strength to prove that Chinese gymnastics team is an excellent fighting collective [7-9]. That's not just talk, but if no assessing situations, no well arranging on line-up, then the whole gymnastics team performing and even fighting for champions will be greatly impacted, so grasp every players scoring information, and arrange reasonable starting line-up is crucial to team performance [10-12].

Zhang Yu-bao, Cheng Zai-kuan researched on the 29th Olympic Games men's competitive gymnastics competition tactics features (2009) [13]. Kong Ling-Feng, Hu Hai-Xu, Du Chang-Liang (2010)organized discussion and analysis in vertically and horizontally, total score, inside and outside series on men's gymnastics team finals top six performance in 2008 Olympic Games, summarized same type international high level men's gymnastics team finals comprehensive competitive performance mapped competition tactics winning features.Zhang Wei, Yuan Chi, Ye Yan-Qing, Jia Xiang, Dai Qi-Wei (2012)analyzed problem about how to organize gymnastics team competition athletes participation line-up on the premise of known statistical data.Wang Zhi-Rui, Tan Lin (2013)studied on gymnastics team competition tactics by documents literature, it found that gymnastics tactics were not only applied in competitions, but also applied in preparation period training, pre-competition adaptation training as well as competitions [14].

The paper goes into deeper exploration on women's gymnastics team forming problem, the whole paper overall adapts optimization thought and targeted at different problems, respectively applies integer programming model and probability statistical theory to establish corresponding model, combines with *lingo* 、*matlab* and other software programming to solve, and gets best line-up arrangement way in different cases.

PESSIMISTIC MODEL ESTABLISHMENT AND SOLUTION

In probability theory, random variable $X_{1,}X_2,\cdots,X_n$ are pair wise independent, if $Y = X_1 + X_2 + \cdots X_{n,}$ then:

Introduce expectation: $E(Y) = E(X_1 + X_2 + \cdots X_{n,}) = E(X_1) + E(X_2) + \cdots E(X_n)$

Introduce variance: $D(Y) = D(X_1 + X_2 + \cdots X_n) = D(X_1) + D(X_2) + \cdots D(X_n)$

Introduce normal distribution formula: $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Table 1: Athlete each item score and probability distribution table

Item	1	2	3	4	5
High-low bars	8.4~0.15 9.2~0.25 9.4~0.1 9.5~0.5	9.3~0.1 9.5~0.1 9.6~0.6 9.8~0.2	8.4~0.1 8.8~0.2 9.0~0.6 10~0.1	8.1~0.1 9.1~0.5 9.3~0.3 9.5~0.1	8.4~0.15 9.5~0.5 9.2~0.25 9.4~0.1
Balance beam	8.4~0.1 8.8~0.2 9.0~0.6 10~0.1	8.4~0.15 9.0~0.5 9.2~0.25 9.4~0.1	8.1~0.1 9.1~0.5 9.3~0.3 9.5~0.1	8.7~0.1 8.9~0.2 9.1~0.6 9.9~0.1	9.0~0.1 9.2~0.1 9.4~0.6 9.7~0.2
Vaulting horse	9.1~0.1 9.3~0.1 9.5~0.6 9.8~0.2	8.4~0.1 8.8~0.2 9.0~0.6 10~0.1	8.4~0.15 9.5~0.5 9.2~0.25 9.4~0.1	9.0~0.1 9.4~0.1 9.5~0.5 9.7~0.3	8.3~0.1 8.7~0.1 8.9~0.6 9.3~0.2
Floor exercises	8.7~0.1 8.9~0.2 9.1~0.6 9.9~0.1	8.9~0.1 9.1~0.1 9.3~0.6 9.6~0.2	9.5~0.1 9.7~0.1 9.8~0.6 10~0.2	8.4~0.1 8.8~0.2 9.0~0.6 10~0.1	9.4~0.1 9.6~0.1 9.7~0.6 9.9~0.2
Item	6	7	8	9	10
High-low bars	9.4~0.1 9.6~0.1 9.7~0.6 9.9~0.2	9.5~0.1 9.7~0.1 9.8~0.6 10~0.2	8.4~0.1 8.8~0.2 9.0~0.6 10~.0.1	8.4~0.15 9.5~0.5 9.2~0.25 9.4~0.1	9.0~0.1 9.2~0.1 9.4~0.6 9.7~0.2
Balance beam	8.7~0.1 8.9~0.2 9.1~0.6 9.9~0.1	8.4~0.1 8.8~0.2 9.0~0.6 10~0.1	8.8~0.05 9.2~0.05 9.8~0.5 10~0.4	8.4~0.1 8.8~0.1 9.2~0.6 9.8~0.2	8.1~0.1 9.1~0.5 9.3~0.3 9.5~0.1
Vaulting horse	8.5~0.1 8.7~0.1 8.9~0.5 9.1~0.3	8.3~0.1 8.7~0.1 8.9~0.6 9.3~0.2	8.7~0.1 8.9~0.2 9.1~0.6 9.9~0.1	8.4~0.1 8.8~0.2 9.0~0.6 10~0.1	8.2~0.1 9.2~0.5 9.4~0.3 9.6~0.1
Floor exercises	8.4~0.15 9.5~0.5 9.2~0.25 9.4~0.1	8.4~0.1 8.8~0.1 9.2~0.6 9.8~0.2	8.2~0.1 9.3~0.5 9.5~0.3 9.8~0.1	9.3~0.1 9.5~0.1 9.7~0.5 9.9~0.3	9.1~0.1 9.3~0.1 9.5~0.6 9.8~0.2

Standard normal curve N(0,1) is a kind of special normal distribution curve, and standard normal population value probability in any one interval (a,b) . Standard normal distribution is a kind of special normal distribution, standard normal distributed μ and σ^2 are 0and1,it often uses ξ (or Z) to show variable conforms to standard normal distribution, it records as $z \sim N(0,1)$.In general, normal distribution and standard normal distribution transformation : due to general normal population its image is not surely symmetric to y axis ,to any one normal population, the probability of its value less than x . Only need to use it to solve normal population probability in one specific interval.

Establish $0 \sim 1$ variable model

By consulting literature, it gets athlete each item score and probability distribution table general status, as Table 1.

When every player each single item score is estimated on the premise of best pessimistic status, solves best starting line-up, so actually it should solve on which line-up start the team total score can be highest. Firstly extract every athlete estimated performance in most pessimistic state, details refer to Table 2.

Table 2: Members score performance estimation under most pessimistic state

Item	1	2	3	4	5	6	7	8	9	10
High-low bars	8.4	9.3	8.4	8.1	8.4	9.4	9.5	8.4	8.4	9.0
Balance beam	8.4	8.4	8.1	8.7	9.0	8.7	8.4	8.8	8.4	8.1
Vaulting horse	9.1	8.4	8.4	9.0	8.3	8.5	8.3	8.7	8.4	8.2
Floor exercises	8.7	8.9	9.5	8.4	9.4	8.4	8.4	8.2	9.3	9.1

According to above analysis of the problem, establish model as following:

$i:1、2、\cdots 10$, represents ten athletes serial number;

$\mathrm{j}:1、2、3、4$, successively represent sports items high-low bars, balance beam , vaulting horse and floor exercises.

In the model, set $0 \sim 1$ variable x_{ij} and y_i to assist to establish model

$$x_{ij} = \begin{cases} 0 & \text{athlete i do not attend the jth sport} \\ 1 & \text{athlete i attends the jth sport} \end{cases}$$

In addition, in order to easy to solve model, the model needs to introduce the second $0 \sim 1$ integer variable y_i,

$$y_i = \left[\frac{1}{4} \sum_{j=1}^{4} x_{ij} \right]$$

In order to fix its value range in the range of $0 \sim 1$,may as well do effective processing, after solving integers, it can be expressed as

$$y_i = \begin{cases} 0 & \text{athlete i do not attend the all-around gymnastics} \\ 1 & \text{athlete i attend the all-around gymnastics} \end{cases}$$

Every item can have 6 players to participate, the best case is full of people,then: $\sum_{j=1}^{10} x_{ij} = 6$

Every team has four people to participate in all-round competition, then: $\sum_{i=1}^{10} y_i = 4$

Objective function is highest team total score , list formula as following: $\max \quad z = \sum_{i=1}^{10} \sum_{j=1}^{4} a_{ij} x_{ij}$

Among them, a_{ij} represents the i athlete score that participates in the j item.

Set in case of pessimistic , the i athlete participates in the j item score is b_{ij}, then $a_{ij} = b_{ij}$. By above conditions, through above and analysis process, it is clear that under most pessimistic status, the paper established $0 \sim 1$ integer programming model is as following:

$$\max \quad z = \sum_{i=1}^{10}\sum_{j=1}^{4} a_{ij}x_{ij}$$

$$s.t.\begin{cases} \sum_{i=1}^{10} x_{ij} = 6 \\ \sum_{i=1}^{10} y_i = 4 \\ (1-y_i)\bullet\sum_{j=1}^{4} x_{ij} \le 3 \\ x_{ij} = 0,1 \end{cases}$$

Model solution

By $lingo$ programming,it gets under most pessimistic status, the team starting line-up and each athlete score status can refer to Table 3.

Table 3: Each team member starting and score status under most pessimistic status

Item	1	2	3	4	5	6	7	8	9	10
High-low bars		9.3			8.4	9.4	9.5		8.4	9.0
Balance beam		8.4		8.7	9.0	8.7		8.8	8.4	
Vaulting horse	9.1	8.4		9.0	8.3	8.5			8.4	
Floor exercises		8.9	9.5		9.4	8.4			9.3	9.1

According to above Table 3, it is easily known that under most pessimistic status, players that participate in all-round item are No. 2、5、6、9 athletes, other players participate in single item competition. In this case, it can let each player to get highest total score in scoring most pessimistic state, as Table 4.

Table 4: Best starting line-up

Item	First item player	Second item player	Third item player	Fourth item player	All-round item player
Player	7、10	4、8	1、4	3、10	2、5、6、9

Model establishment

Firstly analyze if according to previous information and recent each kind of information, winning team total score is no less than 236.2 points this time, because if items are 24 pieces at most and all participate total score is just 240 points, it must participate in whole 24 items if it wants to win this time. To get starting line-up meet conditions and win, then it needs probability that team total score to be no less than 236.2 points to be largest, then the problem can be converted into solution:

$$\text{Max} \quad P\left(\sum_{i=1}^{10}\sum_{j=1}^{4} a_{ij}x_{ij} \ge 236.2\right) \quad (i = 1,\cdots 10,\ j = 1,2\cdots 4)$$

Among them a_{ij} represents the i athlete participates in j item's score, x_{ij} represents whether the i athlete participates in the j item or not

Set team total score is: $S = \sum_{i=1}^{10}\sum_{j=1}^{4} a_{ij}x_{ij}$

Then score expectation is: $E(S) = \sum_{i=1}^{10}\sum_{j=1}^{4} c_{ij}x_{ij}$

Among them, c_{ij} is the i athlete participating in j item event's score average value.

Score variance is : $D(S) = \sum_{i=1}^{10}\sum_{j=1}^{4} D_{ij}x_{ij}$

Among them, D_{ij} is the i athlete participating in the j item event score variance (refer to Table 5).

By converting problems, it finds the problem conforms to normal distribution, from which
$\mu = E(S) \qquad \sigma = \sqrt{D(S)}$

Convert non-standard normal distribution into standard normal distribution $u = \dfrac{S - E(S)}{\sqrt{D(S)}}$

Among them, $u_0 = \dfrac{236.2 - \sum_{i=1}^{10}\sum_{j=1}^{4} c_{ij}x_{ij}}{\sqrt{\sum_{i=1}^{10}\sum_{j=1}^{4} D_{ij}x_{ij}}}$

Due to score probability conforms to normal distribution, probability that the team total score no less than 236.2 points is: $P(u \geq u_0) = 1 - \phi(u_0)$

To random variable Y that conforms to standard normal distribution,when $Y \leq x$, its score density function is increased single-valued function, so solve $1 - \phi(u_0)$ maximum value can be converted into solving u_0 minimum value that objective function, $\dfrac{236.2 - \sum_{i=1}^{10}\sum_{j=1}^{4} c_{ij}x_{ij}}{\sqrt{\sum_{i=1}^{10}\sum_{j=1}^{4} D_{ij}x_{ij}}}$ minimum value. The optimization model that meets conditions is :

$$\min \quad \frac{236.2 - \sum_{i=1}^{10}\sum_{j=1}^{4} c_{ij}x_{ij}}{\sqrt{\sum_{i=1}^{10}\sum_{j=1}^{4} D_{ij}x_{ij}}} \quad (i = 1,2\cdots 10,\ j = 1,2\cdots 4)$$

$$s.t.\begin{cases} \sum_{i=1}^{10} x_{ij} = 6 \\ \sum_{i=1}^{10} y_i = 4 \\ (1 - y_i)\sum_{j=1}^{4} x_{ij} \leq 3 \\ x_{ij} = 0,1 \end{cases}$$

Table 5: Participated athlete each item scoring variance

Item \ Athlete	1	2	3	4	5	6	7	8	9	10
High-low bars	0.1425	0.0180	0.1440	0.1280	0.1425	0.0180	0.0180	0.1440	0.1425	0.0380
Balance beam	0.1440	0.0800	0.1280	0.0880	0.0380	0.0880	0.1440	0.0840	0.1520	0.1280
Vaulting horse	0.0380	0.1440	0.1425	0.0380	0.0720	0.0320	0.0720	0.0880	0.1440	0.1280
Floor exercises	0.0880	0.0380	0.0180	0.1440	0.0180	0.1425	0.1520	0.1580	0.0320	0.0380

STANDARD NORMAL DISTRIBUTION MODEL SOLUTIONS

The same as 0-1 model,use lingo to program, solve and get Table 6.

Table 6: Winning starting line-up

Participated Item	Participated first item player	Participated second item player	Participated third item player	Participated fourth item player	Participated all-round item player
Participated member	6、7	1、8	1、4	6、8	3、5、9、10

Then solve score expectation E =224.6, $(u_0)_{\min} = 7.918$, $P(u \geq u_0) = 1 - \phi(u_0) \approx 0$, Because solved winning probability is nearly equal to 0, and the probability is arranged line-up maximum value that meets the conditions,so start with the line-up , winning probability is almost zero, and nearly cannot win. By Matlab,it draws score normal distribution Figure 1,it can verify results.

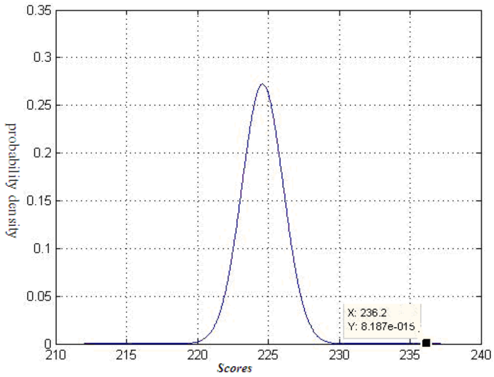

Figure 1: Score normal distribution chart that meets conditions

If it wants to have 90% confident to combat opponents, then it should have $P(S > w) \geq 90\%$ hat conforms to normal distribution, so it has : $1-\phi\left(\frac{w-E(S)}{\sqrt{D(S)}}\right) \geq 90\%$

Search normal distribution table and can get: $\frac{w-E(S)}{\sqrt{D(S)}} \leq -1.28$

Through deformation simplifying, sort and get: $w \leq \sum_{i=1}^{10}\sum_{j=1}^{4} c_{ij}x_{ij} - 1.28\sqrt{\sum_{i=1}^{10}\sum_{j=1}^{4} D_{ij}x_{ij}}$

Only need to solve subsequent maximum value that can solve u value. Solved optimization model is :

$$\max \sum_{i=1}^{10}\sum_{j=1}^{4} c_{ij}x_{ij} - 1.28\sqrt{\sum_{i=1}^{10}\sum_{j=1}^{4} D_{ij}x_{ij}} \quad (i = 1,\cdots 10,\ j = 1,2\cdots 4)$$

$$s.t.\begin{cases} \sum_{i=1}^{10} x_{ij} = 6 \\ \sum_{i=1}^{10} y_i = 4 \\ (1-y_i)\bullet\sum_{j=1}^{4} x_{ij} \leq 3 \\ x_{ij} = 0,1 \end{cases}$$

Apply lingo into programming, solve maximum value as 223.3301, and further get $u \leq 223.3301$
Then best starting line-up is Table 7.

Table 7: Best starting line-up

Item	First item player	Second item player	Third item player	Fourth item player	All-round item player
Player	2、7	5、8	1、4	2、5	3、6、9、10

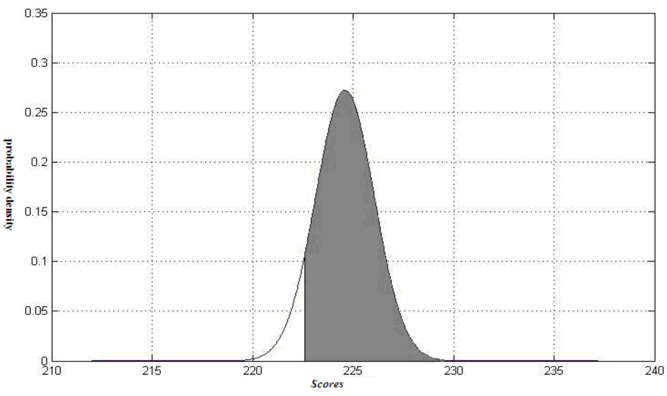

Figure 2: Score normal distribution chart that meets conditions

To sum up, it can get 90% confident to defeat player that is not bigger than 223.3301points.
By Matlab,it draws scores normal distribution Figure 2, it can verify results.

CONCLUSION

The model comprehensive considers each player score status and probability, it provides reasonable $0\sim1$ integer programming mathematical model. The model selects complex line-up selective problems from permutation and combination lots of data, applies $0\sim1$ integer programming, and simplifies problems into simple optimization problems. And meanwhile apply probability theory to combine estimated problems with optimization model, computation is simple, thought is clear, it is easy to understand. Not only gives each player maximum value into play, but also makes reasonable estimation on winning prospect and scoring prospect.

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