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Least square method-based table tennis robot motion planning research

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ABSTRACT

The paper takes table tennis robot stroking table tennis as research objects, it makes physical force analysis, and gets the physical force model existed drawbacks, to further optimize model, it proposes least square method motion trajectory model to recognize and predict table tennis motion trend. Finally by table tennis collision process, it deduces release speed, racket speed and others connections that plays positive roles in robot table tennis development.

Key words: table tennis robot, trajectory prediction model, least square method, identifying prediction

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INTRODUCTION

In recent years, with table tennis development, robot intellectualization and multilayer information network and others gradually enter into sports undertakings, from which table tennis robot belongs to one event of them, its development goes through a long journey, and it develops towards machinery reconfigurable structure and modularization directions, and so do virtual robot stroking table tennis. Many scholars have made researches on it, from which An Dan-Yang had ever analyzed table tennis picking machine structures and key techniques, and finally he got the requirements of using one-way or both way rotations for different volumes and power consumptions sizes, matched photo-electrical code and trigger to detect so that realized system close-cycle control.

The paper just on the basis of above statement, it makes prediction and analysis of table tennis stroking racket strategy and table tennis trajectory, opportunity estimation and speed as well as others' correlation analysis and then further makes contributions to table tennis robot development.

2 Table tennis motions' mechanical analysis

2.1 Predict trajectory

In order to let table tennis robot to be able to stroke table tennis, robot needs to move in advance and predict table tennis movement direction before it arriving at racket.

If table tennis during motion process, it conforms to air resistance and gravity influence, three directions in coordinate axis are mutual independent from each other, and table tennis speed and resistance are in direct proportion, then table tennis motion force is as following Figure 1 show:

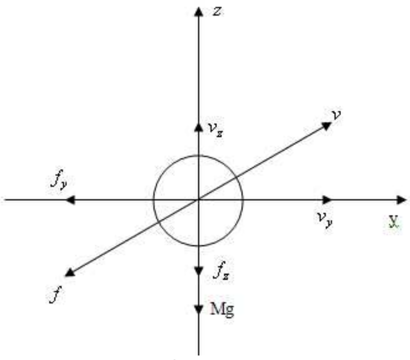

Figure 1: Table Tennis force analysis process diagram

By above Figure 1table tennis force, we can analyze it, in plumbed direction, it suffers air resistance and gravity that:

$$A_z = \frac{dV_z}{dt} = \frac{F_z}{m} = \frac{\mu}{m} V_z - g \tag{1}$$

In horizontal plane, it still suffers resistance effects, we take x as an example, that:

$$A_x = \frac{dV_x}{dt} = \frac{F_x}{m} = \frac{\mu}{m} V_x \tag{2}$$

Table tennis mass is using m to express, and viscous resistance coefficient is using μ to express.

Make analysis of table tennis motion process in horizontal plane:

In speed:

$$\begin{cases} V_x(t) = V_x e^{-kt} \\ V_y(t) = V_y e^{-kt} \\ V_z(t) = (V_z + \frac{g}{k}) e^{-kt} - \frac{g}{k} \end{cases} \tag{3}$$

In displacement:

$$\begin{cases} S_x(t) = \int_0^t V_x(\tau) d\tau = \frac{V_x}{k}(1 - e^{-kt}) \\ S_y(t) = \int_0^t V_y(\tau) d\tau = \frac{V_y}{k}(1 - e^{-kt}) \\ S_z(t) = \int_0^t V_z(\tau) d\tau = \frac{V_z + \frac{g}{k}}{k}(1 - e^{-kt}) - \frac{g}{k} t \end{cases} \tag{4}$$

In above formula, k needs to meet $k = \frac{\mu}{m}$.

Table tennis motion prediction is carrying on speed and displacement in the direction of x, y, z, but for practical status, considered speed and displacement effects on prediction are not quite obvious.

2.2 Apply least square method identifying table tennis operation status prediction research

By above force analysis model, we can see that due to model self constraints, it cannot correctly predict table tennis trajectory, so we introduces least square system identifying prediction model.

For observed value as $\{u(k), y(k)\}, k = 1, 2, \cdots, N+n$ output and input N pieces of values, corresponding system can be expressed as:

$$Y = \eta\theta + e \tag{5}$$

In above formula, each parameter should meet:

$$\theta = [a_1, a_{2,} \cdots, a_n; b_1, b_2, \cdots, b_n]^T \tag{6}$$

$$Y = [y(n+1), y(n+2), \cdots, y(n+N)]^T \tag{7}$$

$$e = [e(n+1), e(n+2), \cdots, e(n+N)]^T \tag{8}$$

$$\eta = \begin{bmatrix} -y(n) & -y(n-1) & \cdots & -y(1) & u(n) & \cdots & u(1) \\ -y(n+1) & -y(n) & \cdots & -y(2) & u(n+1) & \cdots & u(2) \\ \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ -y(n+N) & -y(n+N+1) & \cdots & -y(N) & u(n+N) & \cdots & u(N) \end{bmatrix} \tag{9}$$

After that, according to least square method principle, we take positives, and then corresponding function is:

$$J(\theta) = \sum_{i=1}^{N} (Y - \eta\theta)^T (Y - \eta\theta) \tag{10}$$

So as to let $J(\theta)$ appear minimum value, then:

$$\frac{\partial J}{\partial \theta} = \frac{\partial}{\partial \theta}[(Y - \eta\theta)^T (Y - \eta\theta)] = -2\eta^T (Y - \eta\theta) = 0 \tag{11}$$

And corresponding estimated value is:

$$\theta = (\eta^T Y)^T \eta^T Y \tag{12}$$

Now table tennis motion trajectory that needs to meet is:

$$p = \theta \cdot t \tag{13}$$

In above formula, θ is parameter, and is also identified.
If regard coordinates axis direction as time function relative curve, then it has:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_x & v_x & p_x \\ a_y & v_y & p_y \\ a_z & v_z & p_z \end{bmatrix} \begin{bmatrix} t^2 \\ t \\ 1 \end{bmatrix} \tag{14}$$

In above formula, a_x, v_x, p_x, a_y, v_y, p_y, a_z, v_z, p_z property is the same as θ.
For randomly collected one point, we can express it as:

$$p_i = [x_i \ y_i \ z_i \ t_i] \tag{15}$$

In formula, time for the first measurement point time t_0 is using t_i to express, and measurement point position is using (x_i, y_i, z_i) to express.

In the following, collect two samples, and then matrix form is:

$$\begin{bmatrix} t_i^2 & t_i & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & t_i^2 & t_i & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & t_i^2 & t_i & 1 \\ t_{i+1}^2 & t_{i+1} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & t_{i+1}^2 & t_{i+1} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & t_{i+1}^2 & t_{i+1} & 1 \\ t_{i+2}^2 & t_{i+2} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & t_{i+2}^2 & t_{i+2} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & t_{i+2}^2 & t_{i+2} & 1 \end{bmatrix} \begin{bmatrix} a_x \\ v_x \\ p_x \\ a_y \\ v_y \\ p_y \\ a_z \\ v_z \\ p_z \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \\ z_i \\ x_{i+1} \\ y_{i+1} \\ z_{i+1} \\ x_{i+2} \\ y_{i+2} \\ z_{i+2} \end{bmatrix} \tag{16}$$

By utilizing $Ax = b$ form to solve, but it has larger errors, to reduce the errors, we optimize them, increase selected point's number $3j$, then it can get:

$$Ax = b \tag{17}$$

Among them, A、x、b can be expressed as:

$$A = \begin{bmatrix} t_i^2 & t_i & 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & t_i^2 & t_i & 1 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & t_{i+j-1}^2 & t_{i+j-1} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & t_{i+j-1}^2 & t_{i+j-1} & 1 \end{bmatrix}$$

$$x = [a_{x_i}\ v_{x_i}\ p_{x_i}\ a_{x_i+j-1}\ v_{x_i+j-1}\ p_{x_i+j-1}]^T$$

$$b = [x_i\ y_i\ z_i\ x_{i+j-1}\ y_{i+j-1}\ z_{i+j-1}]^T$$

And after solving identification parameter x, it is $3j \times 1$, A、b is also $3j \times 3j$ matrix.

2.3 Ball landing model

If ball after falling is a kind of elastic collision process, regarding energy loss, we can define z to be speed reverse direction lose, x、y positive direction lose, then total direction lose can be expressed as:

$$k = [k_x\ k_y\ -k_z] \tag{18}$$

Corresponding table tennis landing process is as following Figure 2 show:

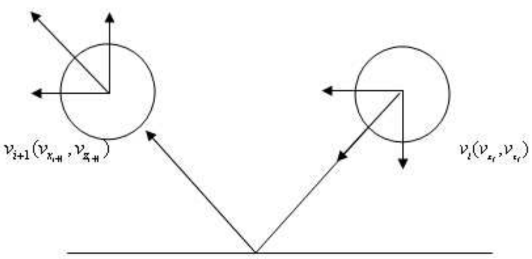

Figure 2: Schematic collision course with the desktop

In t_1 moment, predict ball release positions that can be expressed as p'_x, p'_y, p'_z forms, then after colliding matrix can be expressed as:

$$\theta' = \begin{bmatrix} a_x & k_x v_x & p'_x \\ a_y & k_y v_y & p'_y \\ a_z & -k_z v_z & p'_z \end{bmatrix} \tag{19}$$

Among them, t_1 solving is using following formula:

$$p_z = a_z t^2 + v_z t + p_z = 0 \tag{20}$$

In above formula, energy lose can be solved according to experiment.

3 Table tennis speed control model

If table tennis robot under cooperating of robotic arms, during table tennis and racket colliding process, speed is $\vec{v}_k$, projectile speed is $\vec{v}_f$, and incident speed is $\vec{v}_0$, as following Figure 3 show:

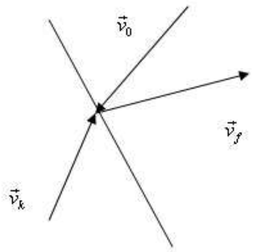

Figure 3: The impact of table tennis ball and racket

According to above figure, it can solve that on racket tangent line, it has:

$$^{T}v_{f\tau} = {}^{T}v_{\tau} + ({}^{T}v_{h\tau} - {}^{T}v_{\tau})\alpha \tag{21}$$

And friction coefficient is using α to express, inelastic collision process condition that needs to meet is:

$$^{T}v_{fn} = {}^{T}v_{hn} - {}^{T}v_{n}\beta \tag{22}$$

Rebound coefficient is using β to express, using ^{T}R to represent transformation matrix, then it has:

$$^{T}v = {}^{T}R \cdot \ v \tag{23}$$

From the perspective of racket, under its coordinate system, normal direction and collision direction are different, then it has:

$$\begin{cases} ({}^{T}Rv_f)_x = ({}^{T}Rv\ )_x + ({}^{T}Rv_h - {}^{T}Rv\ )_x \alpha \\ ({}^{T}Rv_f)_y = ({}^{T}Rv_h)_y - ({}^{T}Rv\ )_y \beta \\ ({}^{T}Rv_f)_z = ({}^{T}Rv\ )_z + ({}^{T}Rv_h - {}^{T}Rv\ )_z \alpha \end{cases} \tag{24}$$

After sorting above formula, it can get:

$$\begin{cases} ({}^{T}Rv_f)_x = ({}^{T}Rv_h)_x \partial + ({}^{T}Rv\ )_x (1-\alpha) \\ ({}^{T}Rv_f)_y = ({}^{T}Rv_h)_y - ({}^{T}Rv\ )_y \beta \\ ({}^{T}Rv_f)_z = ({}^{T}Rv_h)_z \alpha + ({}^{T}Rv\ )_z (1-\alpha) \end{cases} \tag{25}$$

If $^{T}R = (r_x\ r_y\ r_z)^T$, then it can write as:

$$\begin{cases} r_x v_f = r_x[v_h\alpha + v(1-\alpha)] \\ r_y v_f = r_y v_h - r_y v\beta \\ r_z v_f = r_z[v_h\alpha + v(1-\alpha)] \end{cases} \tag{26}$$

By simplifying, it can get:

$$\begin{cases} r_x[v_f - (1-\alpha)v - \alpha v_h] = 0 \\ r_y(v_f + \beta v - v_h) = 0 \\ r_z[v_f - (1-\alpha)v - \alpha v_h] = 0 \end{cases} \tag{27}$$

Matrix after rotating, it can get three coordinate systems projection as r_x, r_y, r_z, by above formula, we can get racket motion form. In practical sports process, plumbed direction doesn't affect racket stroking ball, if it has z coordinate projected racket, then corresponding formula is:

$$r_y' = v_n = \frac{(v_{nx}\ v_{ny}\ v_{nz}\ )}{|(v_{nx}\ v_{ny}\ v_{nz}\ )|} = (v_{nx}\ v_{ny}\ v_{nz}\ ) \tag{28}$$

$$r_z' = (0\ 0\ 1) - \frac{(0\ 0\ 1)\cdot(v_{nx}\ v_{ny}\ v_{nz}\ )}{|(v_{nx}\ v_{ny}\ v_{nz}\ )|}(v_{nx}\ v_{ny}\ v_{nz}\ ) = (-v_{nx}v_{nz}\ -v_{ny}v_{nz}\ \ 1-v_{nz}{}^2\ ) \tag{29}$$

$$r_x' = (R_T)_y \times (R_T)_z = \begin{vmatrix} i & j & k \\ v_{nx} & v_{ny} & v_{nz} \\ -v_{nx}v_{nz} & -v_{ny}v_{nz} & 1-v_{nx}{}^2 \end{vmatrix} = (v_{ny}\ -v_{nx}\ \ 0\ ) \tag{30}$$

We make normalization processing with r_x', r_y', r_z', and then it can get:

$${}^T R = \begin{pmatrix} \frac{v_{ny}}{\sqrt{v_{nx}{}^2 + v_{ny}{}^2}} & -\frac{v_{nx}}{\sqrt{v_{nx}{}^2 + v_{ny}{}^2}} & 0 \\ -\frac{v_{nx}v_{nz}}{\sqrt{1-v_{nz}{}^2}} & -\frac{v_{ny}v_{nz}}{\sqrt{1-v_{nz}{}^2}} & \frac{v_{nz}}{\sqrt{1-v_{nz}{}^2}} \end{pmatrix} \tag{31}$$

$$R_T = ({}^T R)^T = \begin{bmatrix} \frac{v_{ny}}{\sqrt{v_{nx}{}^2 + v_{ny}{}^2}} & v_{nx} & -\frac{v_{nx}v_{nz}}{\sqrt{1-v_{nz}{}^2}} \\ -\frac{v_{nx}}{\sqrt{v_{nx}{}^2 + v_{ny}{}^2}} & v_{ny} & -\frac{v_{ny}v_{nz}}{\sqrt{1-v_{nz}{}^2}} \\ 0 & v_{nz} & \sqrt{1-v_{nz}{}^2} \end{bmatrix} \tag{32}$$

By R_T , it deduces reference coordinate system RPY angle contained relation is:

$$\begin{cases} \beta = a\tan 2(-r_{31}, \sqrt{r_{11}^{\ 2} + r_{11}^{\ 2}}) = a\tan 2(0, \sqrt{v_{nx}^{\ 2} + v_{ny}^{\ 2}}) = 0 \\ \alpha = a\tan 2(r_{21}, r_{11}) = a\tan 2(--\frac{v_{nx}}{\sqrt{v_{nx}^{\ 2} + v_{ny}^{\ 2}}}, \frac{v_{ny}}{\sqrt{v_{nx}^{\ 2} + v_{ny}^{\ 2}}}) \\ \gamma = a\tan 2(r_{32}, r_{33}) = a\tan 2(v_{nz}, \sqrt{1 - v_{nz}^{\ 2}}) \end{cases} \tag{33}$$

So, ball and racket can be solved according to above formula.
From the perspective of racket, its force has:

$$v_\tau = v_f + \beta v - v_h \tag{34}$$

v_n and v_τ are a kind of orthogonal way, then;

$$v_n^{\ T} \cdot v_\tau = 0 \tag{35}$$

Expand above formula, speed relationship has:

$$\begin{aligned} v_n^{\ T} \cdot v_\tau = & [v_{fx} - (1-\alpha)v_x - \alpha v_{hx}](v_{fx} + \beta_{vx} - v_{hx}) + [v_{fy} - (1-\alpha)v_y - \alpha v_{hy}](v_{fy} + \beta_{vy} - v_{hy}) + \\ & [v_{fz} - (1-\alpha)v_z - \alpha v_{hz}](v_{fz} + \beta_{vz} - v_{hz}) \\ = & 0 \end{aligned}$$

By above formula, we can get table tennis robot relative stroking process each kind of parameters.

4 Robot stroking planning

In table tennis motion process, it should meet that robot works in the space, and no collision happens, by practical investigating and measuring, we regulate robot motion system coordinate range is nearly:

$$\begin{cases} -400 \le x \le 400 \\ -1470 \le y \le -1570 \\ 110 \le z \le 450 \end{cases} \tag{36}$$

In robot table tennis playing process, if table tennis within above motion ranges, then it is safety region. Respectively input x、y、z into above formula, then it has:

$$\begin{cases} -400 \le x \le 400 \\ -1470 \le y \le -1570 \\ 110 \le z \le 450 \end{cases} \tag{37}$$

By calculating, it has:

$$\begin{cases} t_x \in [t_{x_1}, t_{x_2}] \\ t_y \in [t_{y_1}, t_{y_2}] \\ t_z \in [t_{z_1}, t_{z_2}] \end{cases} \tag{38}$$

By above formula solved union set, after that select intermediate point $t = \frac{t_1 + t_2}{2}$, it can get speed $v = (v_x, v_y, v_z, t)$ and trajectory coordinate $p\ (x, y, z, t)$, racket stroking speed is using $v_h\ (v_{h_x}, v_{h_y}, v_{h_z})$ to express and self posture is using (α, β, γ) to express, t represents stroking time, $v_h\ (v_{h_x}, v_{h_y}, v_{h_z})$ represents racket stroking speed.

CONCLUSION

The paper uses kinematics to analyze table tennis stroking process, it solves which speed and what kind of postures that robot strokes table tennis should have and makes analysis, uses system trajectory and physical trajectory model to predict, in addition, it also researches on robot stroking table tennis collision model, therefore it fulfills table tennis trajectory planning problems, and the model has widely utilities.

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