



Research Article

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## Learning Methods of Radial Basis Function Neural Network

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### ABSTRACT

Radial Basis Function Neural Network (RBF) is widely used now; it can solve the following problems: the small sample, nonlinear, dimension and local minima. Firstly, this paper briefly introduces the basic principles of RBF, and then comprehensively accounts the RBF domestic and overseas study status, discusses the advantages and disadvantages of various methods, and compares the explanation, finally analyzes the existing problems and the development trend in the future.

**Key words:** Radial basis function; Feed forward neural network; Optimization

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### INTRODUCTION

RBF is a commonly used three layer feed forward neural network. Compared with BP network, RBF network not only has a physiological basis, but also the structure is simpler, the learning speed is faster. It not only has solid theoretical basis, a concise mathematical form and intuitive geometric, but also can solve the small sample, nonlinear, dimension and local minima, so it is widely used in the practice[1-4]. It is a very effective multilayer feed forward network after the multilayer perceptron.

At present, according to the characteristics of radial basis function neural network proposed many algorithms, this paper comprehensively describes the algorithm, analyzes the existing problems and the prospect of development trend.

### THE PRINCIPLE OF RADIAL BASIS FUNCTION NEURAL NETWORK

RBF neural network is a feed-forward neural network, generally divided into three layers of structure, if the output layer only has one node, consider as shown in Figure 1 of multi input single output of 3 layer RBF network design, the hidden layer radial basis function of Gaussian, the structure can be easily extended to multi output node.

The number and location of network hidden center affects the performance of radial basis function network, the core problem of RBF network RBF network learning is to determine the number of basis functions, the center vector and its shape.

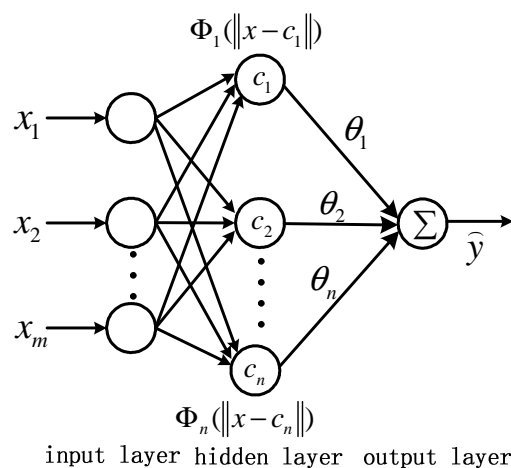


Fig.1 Structure of the RBF Neural Networks

Figure 1 is a RBF m-n-1 network structure, network with M input, n hidden nodes, and 1 output.

Assuming the base width parameters are identical and known value, then the output of RBF network model as shown in formula (1).

$$\hat{y}(x) = \sum_{i=1}^n \theta_i \exp(-\|x - c_i\|^2 / 2\rho^2) \quad (1)$$

In the  $x = [x_1, x_2, \dots, x_m]^T$  input vector for the network,  $\theta_i$  is the  $i$  connection weights of hidden nodes to the output,  $c_i = [c_{1,i}, c_{2,i}, \dots, c_{m,i}]^T$  as the data center vector of hidden layer of the  $i$  radial basis function,  $\rho$  base width parameter function,  $\|\cdot\|$  is the Euclidean norm.

The output layer RBF network is linear neurons, in the network hidden center vector  $c_i$  and base width parameter  $\rho$  is determined, using simple linear learning algorithm can derive the output weights of  $\theta_i$  network. So the key to set up RBF network model is to determine the parameters of hidden center vector, base width parameters and number of nodes, or to find the parameter  $\Xi = \{c_i, \rho, n\}$ .

### COMMON LEARNING ALGORITHM OF RADIAL BASIS FUNCTION NEURAL NETWORK

By formula(1) shows, given training samples, learning algorithm of RBF network should address the following issues: (1) structure design, that is how to determine the number of hidden nodes of network; (2) determine the hidden center vector  $c_i$  and the base width parameter  $\rho$  of radial basis function; (3)output weight correction. In general, if known network hidden nodes, data center and propagation constant, RBF net from the input to the output is a linear equation group, the weights learning can use the least squares method. Therefore, this need to solve is the first two problems.

According to the method of choosing the data center, the design method of the RBF network can be divided into two categories.

#### (1) Selected the data center from the sample input

This kind of method is relatively easy to implement, and determine the number of hidden nodes in the weight learning, and to ensure that the learning error is less than a given value. Algorithm of selecting the data center in the sample input, typical methods of the OLS algorithm, ROLS algorithm, ROLS+D-opt algorithm. These methods of data center will no longer change, and the numbers of hidden nodes have been identified in the beginning, or dynamic adjustment in the network learning process.

#### (2)Regulation method of dynamic data center

Data center in this method is dynamically adjusted during the learning process, such as a variety of clustering, the

most commonly used with K- clustering, C- clustering, Kohonen clustering method, gradient descent method and RAN. Clustering method is a heuristic algorithm process, not through optimization of an objective function are derived, which are very sensitive to the selection of the initial value. Advantages of clustering method is able to determine propagation constants of the hidden nodes according to the distance to each cluster center, defect is identified in the data center uses the information of input samples and did not use the output information, so that not only affects the pattern classification of network capacity, but also restrict the regression ability of network, at the same time clustering method is unable to determine the number of clustering.

#### **A COMPREHENSIVE DESCRIPTION OF THE LEARNING METHOD FOR RADIAL BASIS FUNCTION NEURAL NETWORK**

To realize RBF network has unique advantages and potential application prospects, in recent years many scholars to study based on network structure design of radial, appeared more abundant research results. Platt proposed a growth oriented resource allocation for RBF network model (Resource Allocation Network, RAN). In RAN, can according to the different processing objects increase network hidden layer neuron number, and deal with complex issues [5]. But in its operation, the network structure is not reduced, making the network redundancy, the network scale becomes large. Lu in Platt is proposed based on the research of RAN, designed a minimum resource network (Minimal Resource Allocation Network, MRAN ),MRAN avoids the drawbacks of RAN, network hidden layer neurons can increase and decrease, so as to obtain the appropriate network structure [6]. But in the process of designing MRAN is not on the parameters of structural adjustment adjustment [7], the speed of convergence is slow. Esposito proposed a growth of RBF neural network algorithm, by training the network hidden layer neurons to adjust, but affected by the initial value of the lot, may make the performance of the network is not stable, easy to fall into local optimum; Gonzalez using genetic algorithm to adjust the structure of RBF, but the calculation and the algorithm is difficult[8]; Guo proposed a combination of genetic algorithm and the hybrid learning algorithm ( HLA ) in two steps of the RBF network design, the hidden node centers and the base width parameter GA algorithm primaries in training samples in optimal use first, and then use HLA to adjust these parameters, through experimental verification, RBF network show that the method designed to effectively, structure of thrift, has good properties of [9]; Fu proposed an RBF neural network algorithm for deletion type, in the training of all the samples, then cut the hidden layer of network, this method is limited in use, not suitable for real time system [10]; Huang RBF neural network is presented a growth pruning type ( Generalized Growing and Pruning RBF, GGAP-RBF ), were calculated to determine the increase or decrease of neurons in the hidden layer through the importance of hidden layer neurons, but need to consult the overall sample data, and then set the initial values of the network, so it is not suitable for online learning[11]. Feng optimization of the structure of RBF neural network, the use of evolutionary computation in the particle swarm optimization algorithm, in the process of optimization, the particle swarm algorithm requires a global search, which makes the calculation complexity and computational time variable length [12]. Guerra proposed a learning algorithm of RBF networks, Adjust the Gauss RBF hidden layer function of this method by clustering center vector, and optimize the network center and base width parameter by using the PSO method, calculating the network output weights using Penrose-Moore pseudo inverse method[13]. Lian proposed a self-organizing RBF neural network (Self-Organizing, RBF, SORBF), by calculating the network approximation error to design a self-organizing network structure [14]. But they did not consider the connection weights within the network and set the parameters in the structural changes, the total training time is too long. Huang proposed a recursive orthogonal least squares algorithm (ROLSA) and particle swarm forward RBF neural network optimization algorithm (MVHC-ROLS-PSO ) [15], the algorithm parameter setting is more complex. Chen proposed an optimization algorithm of orthogonal selection based on (OFS) RBF neural network structure, to adjust the RBF neural network using Leave-One-Out, determine the network structure [16]. But the use of the global search algorithm reduces the overall learning speed. Qiao Junfei, Han Honggui put forward a kind of dynamic optimization design method, used to solve the structure design of radial basis function neural network problems, the sensitivity method ( Sensitivity, Analysis, SA ) analysis of the impact of the output weights of radial basis function neural network of neurons in the hidden layer to the output of the neural network, the decision to add or delete the hidden layer of network in neurons, it is confirmed that the RBF neural network structure is too large or too small problem, and prove its convergence; RBF network precision by using gradient descent parameter correction algorithm, the neural network structure and parameter self-tuning [17]. Two years later, Han Honggui, Qiao Junfei put forward the design method of neural network structure based on RBF information strength, the elastic design the structure of RBF neural network, the output information hidden layer neurons using radial basis function neural network and the analysis of the connection strength network hidden layer and output layer neuron interaction information between neurons, delete or add network hidden layer neurons, at the same time, adjust the network topology structure, to solve the structure of RBF neural network design problems[18]. In a typical non-linear function approximation, modeling of key water quality parameters in wastewater treatment process, the result proves its correctness. The hot issue in recent years is still the optimizing RBF neural network structure and the adjustment of the structure of RBF network.

In the RBF network learning method, proposed by Chen Orthogonal Least Squares is a common RBF network design method, this method is based on the error OLS algorithm derived from the drop rate index, according to each of the orthogonal vector decline rate of contribution to the size of the error, according to the order of selection for the center vector network, automatically avoids numerical the ill posed problem caused by random selection center [19]. Many scholars have conducted further study, in 1996 Chen proposed Regularized Orthogonal Least Squares RBF network design method, the orthogonal forward regression and regularization method, through the Bayesian method to generate the regularization coefficient, avoid network over fitting in a sample of noise data, so as to improve the generalization ability of network [20]; In 2001 Hong and Harris OLS algorithm and A-Optimality Design combination, the regression model selection methods, the method is introduced into the composite cost function, by using the approximation capability of OLS optimization method, the improved model performance of A- optimal design, simulation results show that, the RBF network design with the conventional OLS method has better performance ; based on the same principle, in 2002 Hong and Harris proposed OLS algorithm and D-Optimality Experimental Design combination selection method based on regression model. Optimal approximation ability model in the use of the OLS method at the same time, the design matrix by D- optimal design to maximize the selected sub model to improve the model robust performance of [21]; the 2003 Chen, Hong and Harris combined with ROLS algorithm and D- optimal experiment design method, by introducing D- optimal costs to the price function, automatic design of RBF network structure frugal at the same time, improve the performance of regression model selection, improve the parameter robust regression model[22]; In 2003, Hong used a significant regression factor optimization method put forward regression and Predicted REsidual Sums of Squares statistics, the characteristic of this method is that: in the prior to the orthogonalization process framework, calculation of PRESS value to reduce the amount of calculation, economical model structure, Gou Jianyong has good generalization performance model[23]; Hong systematic review three kinds of learning methods, ROLS combined with A-opt method, ROLS combined with D-opt method, ROLS and PRESS statistics method, the three methods are extensions to the OLS method or the ROLS method, which can improve the computational efficiency also can improve the performance of the regression models[24]. Due to the structure characteristics of RBF network and the multilayer perceptron is different, the key RBF network design is the hidden layer centers position selection, and selection of network center position can be attributed to the selection problem in linear regression model of the neutron, so the methods can be effectively applied to the design of RBF network.

## CONCLUSION

A key problem in RBF network design is the design meets the accuracy requirements of the minimum structure of neural network, in order to ensure the generalization ability of the network. In recent years, the theory and application of RBF algorithm has made considerable progress, but in the process a large number of training data in the practical application, there are still problems such as computing speed and storage capacity. With the continuous development and improvement of the theory, RBF in pattern recognition, regression analysis, technology, medicine and other fields of biological information, will get the more extensive application.

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