



Research Article

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**Kinematics analysis of beam pumping unit base on projection method**

**Feng Zi-Ming<sup>#</sup>, Li Qi<sup>#</sup>, Xu Ying<sup>@</sup> and Liu Bo-Wei<sup>\*</sup>**

<sup>#</sup>*School of Mechanical Science and Engineering, Northeast Petroleum University, Daqing, China*

<sup>\*</sup>*Daqing Oilfield Co. Ltd, Production Plant 2, Daqing, Heilongjiang, China*

<sup>@</sup>*Petrochina Daqing Petrochemical Company, Daqing, China*

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**ABSTRACT**

*The conventional beam unit movement equation is based on the vector method, the four bar linkage vector relation is expressed by complex mathematics, so it call people for more mathematical knowledge. Base on geometry relationship and trigonometric function relationship between the Four Bar Linkage, the beam unit movement equations can be established through the projection method. By programming and solving the movement equations, the movement curve accurately reflects the movement of the beam unit four bar linkage suspension point.*

**Keyword:** Projection Method; Four Bar Linkage; Movement Analysis; Beam Unit

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**INTRODUCTION**

Kinematics analysis of beam pumping units system was the basic to dynamic analysis and system optimization. Today, there are several kinematics analysis methods on beam pumping units, such as analytical method, graphical method and vector method. In 1981 year, Zhang[1] proposed plural method to elicit displacement and velocity and acceleration analytic formula of suspension center of horse head, and the plural method had been applied extensively.

In 1995, Liu Meng[2] use the analytic method to analyzed the movement of the conventional pumping unit, the method can be of any component parts on the pumping unit angular displacement, angular velocity and angular acceleration, and is suitable for the different speed for variables. Zhang Chunyi and Zhang Zhongxing [3] through analyzing the movement characteristics and load characteristic of the conventional beam pumping unit, according to the theory of analytic method and vector graphic, they present the research methods of residual force vector on the curve to express the size and direction of the carrying capacity, it can directly express the four connecting rod vector force of all nodes, the results show that; The friction and the increase of the quality of rod string, force vector on the curve shape and the magnitude of the force is very big change, the simulation result is under the condition of the friction force is small and no vibration. About the document of pumping unit four bar linkage motion analysis [4-5] is more, with the development of computer technology, also appeared a lot of polished rod motion simulation way, for example, using ADAMS and MATLAB and high-level language programming, they were also achieved a good result. In this paper, using projection method to analysis four bar linkage motion, the idea is simple and easy to understand, able to deduce the displacement velocity and acceleration of each link of mathematical functions, you can use simple programming to calculate, it also suitable for analysising other mechanical of four bar linkage of the motion law[6-9].

**PROJECTION METHOD ABOUT KINEMATICS ANALYSIS OF BEAM PUMPING UNITS**

In Figure 1, the positive angle of intersection angle between member bars is stipulated as follows;

- 1) The  $\theta$  about Torque angle is computing at the zero of clock, clockwise is positive and anticlockwise is negative.
- 2)  $\theta_2, \theta_3, \theta_4$  that are intersection angle between every bars are all computing at the base-bar,

3) Every intersection bar of four-bar linkage is stipulated as follows;

- $L_1$ —base bar length of beam pumping units;  
 $L_2$ —crank radius of beam pumping units;  
 $L_3$ —connecting rod length of beam pumping units;  
 $L_4$ —length of back-walking beam  
 $L_5$ —length of fore-walking beam

The equation set is established due to the geometrical relationship in the Figure 2;

$$\begin{cases} L_2 \cos(\theta_2) + L_3 \cos(\theta_3) - L_4 \cos(\theta_4) - L_1 \cos(\theta_1) = 0 \\ L_2 \sin(\theta_2) + L_3 \sin(\theta_3) - L_4 \sin(\theta_4) - L_1 \sin(\theta_1) = 0 \end{cases} \quad (1)$$

Because  $\theta_1$  is the intersection angle between base bar and base bar, so  $\theta_1 = 0$ , formula (1) can be simplified into the following equations;

$$\begin{cases} L_2 \cos(\theta_2) + L_3 \cos(\theta_3) - L_4 \cos(\theta_4) - L_1 = 0 \\ L_2 \sin(\theta_2) + L_3 \sin(\theta_3) - L_4 \sin(\theta_4) = 0 \end{cases} \quad (2)$$

It is very tedious to solve the system of equations (2) using algebraic method, the formula (3) are implicit function about  $\theta_3, \theta_4$ , it is not complex to program and compute it.

$$\begin{cases} \theta_3 = \arccos \left[ \frac{L_4^2 - L_1^2 - L_2^2 - L_3^2 - 2L_2L_3 \sin(\theta_2) \sin(\theta_3) + 2L_1L_2 \cos(\theta_2)}{2L_2L_3 \cos(\theta_2) - 2L_1L_3} \right] \\ \theta_4 = \arccos \left[ \frac{L_3^2 - L_1^2 - L_4^2 - L_2^2 + 2L_2L_4 \sin(\theta_2) \sin(\theta_4) + 2L_1L_2 \cos(\theta_2)}{2L_1L_4 - 2L_2L_4 \cos(\theta_2)} \right] \end{cases} \quad (3)$$

Angle velocity of every rod piece can be achieved by the geometrical relationship shown in figure 1,  $\omega_2 = -\omega$ ,  $\theta$  is angle velocity of crank,  $\alpha_2 = 0$ ,  $\alpha_2$  is angle acceleration of crank.

$$\begin{cases} \omega_3 = \frac{L_2 \omega_2 \sin(\theta_4 - \theta_2)}{L_2 \sin(\theta_3 - \theta_4)} \\ \omega_4 = \frac{L_2 \omega_2 \sin(\theta_2 - \theta_3)}{L_4 \sin(\theta_4 - \theta_3)} \end{cases} \quad (4)$$

Angle acceleration of every can be achieved with derivation of formula set (4);

$$\begin{cases} a_3 = \frac{-a_2 L_2 \sin(\theta_4 - \theta_2) + L_2 \omega_2^2 \cos(\theta_4 - \theta_2) + L_3 \omega_3^2 \cos(\theta_4 - \theta_3) - L_4 \omega_4^2}{L_3 \sin(\theta_3 - \theta_4)} \\ a_4 = \frac{a_2 L_2 \sin(\theta_3 - \theta_2) - L_2 \omega_2^2 \cos(\theta_3 - \theta_2) + L_4 \omega_4^2 \cos(\theta_3 - \theta_4) - L_3 \omega_3^2}{L_4 \sin(\theta_3 - \theta_4)} \end{cases} \quad (5)$$

Because the angle acceleration of crank is shown as  $a_2 = 0$ , formula (5) can be simplified as follow;

$$\begin{cases} a_3 = \frac{L_2 \omega_2^2 \cos(\theta_4 - \theta_2) + L_3 \omega_3^2 \cos(\theta_4 - \theta_3) - L_4 \omega_4^2}{L_3 \sin(\theta_3 - \theta_4)} \\ a_4 = \frac{-L_2 \omega_2^2 \cos(\theta_3 - \theta_2) + L_4 \omega_4^2 \cos(\theta_3 - \theta_4) - L_3 \omega_3^2}{L_4 \sin(\theta_3 - \theta_4)} \end{cases} \quad (6)$$

Line velocity of every bar piece is;

$$V_4 = V_2 \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \quad (7)$$

When the horse head suspension center is located at the center extreme of up and down dead position, the minimum and maximum intersection angles  $\psi_{\min}$ 、 $\psi_{\max}$  , between pumping unit base bar and back walking beam, is respectively;

$$\begin{cases} \psi_{\min} = \arccos\left(\frac{L_1^2 + L_4^2 - (L_3 - L_2)^2}{2L_1L_4}\right) \\ \psi_{\max} = \arccos\left(\frac{L_1^2 + L_4^2 - (L_3 + L_2)^2}{2L_1L_4}\right) \end{cases} \quad (8)$$

Stroke length  $S$  of Suspension center is;

$$S = (\psi_{\max} - \psi_{\min})L_5 \quad (9)$$

The zero suspension displacement is taken as down dead center, the up moving direction of polish rod is taken as the positive direction, so suspension center's instant displacement  $S_c$  is;

$$S_c = (\psi_{\max} - \psi)L_5 \quad (10)$$

$\psi$  – intersection angle of back walking beam and base bar at arbitrary time.

The tilt angle between ground rocking bar and base bar is

$$\begin{cases} \varphi_{\max} = \arccos\left(\frac{L_3^2 + L_1^2 - (L_4 + L_2)^2}{2L_3L_1}\right) \\ \varphi_{\min} = \arccos\left(\frac{L_3^2 + L_1^2 - (L_4 - L_2)^2}{2L_3L_1}\right) \end{cases} \quad (11)$$

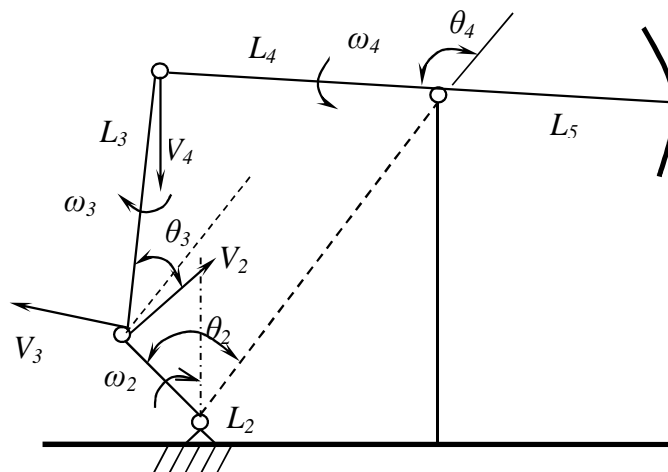


Figure 1 structural diagram of four-bar linkage of beam pumping units

## CASE COMPUTATION

Four bar linkage kinematic characteristic above this paper is taken as theoretical basis, the traditional walking beam pumping unit kinematic simulation can be achieved with iterative computations program. CYJ3-1.5-6.5HB type pumping unit is taken as case, all the length of four bar linkage can be achieved by referring the relative handbook, then the kinematic function, about walking beam pumping unit, is founded. The figure 2 to figure 5 are showing the displacement curve and velocity curve and acceleration curve against to crank angle using the above formula.

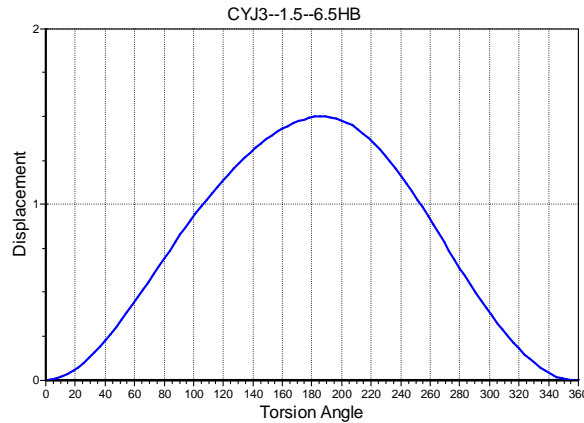


Figure 2 Displacement curve

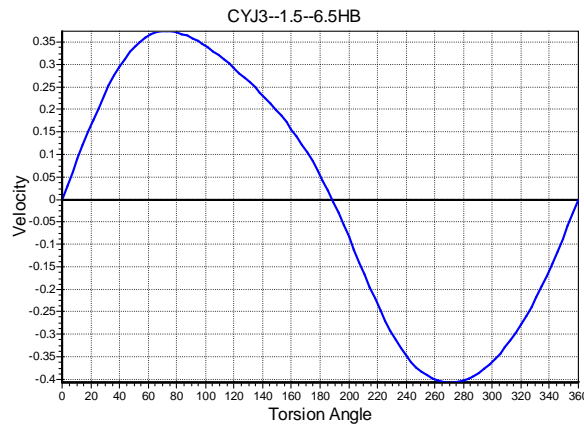


Figure 3 Velocity curve

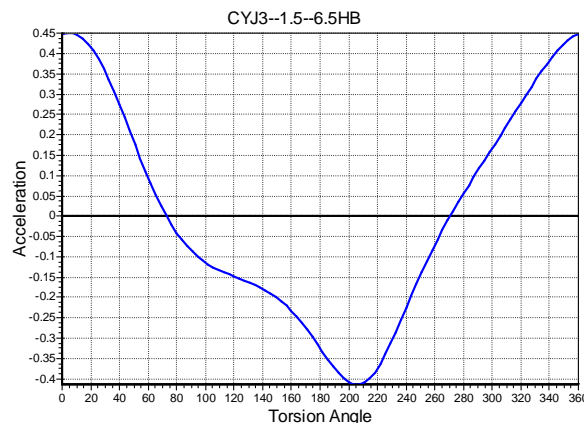


Figure 4 Acceleration curve

## CONCLUSION

The kinematic equation of displacement, velocity and acceleration of walking beam unit is achieved by geometric projection method. The derived suspension center kinematic function in this paper is taken as simulation mathematic model, and the pumping unit kinematic simulation software is programed with VB6.0. At last, the CYJ3-1.5-6.5HB type pumping unit is selected as computed case, and the suspension displacement, velocity and acceleration of

pumping unit are computed and form relative curve. Comparing to the other kinematic analysis methods of four bar linkage, the geometric projection method in this paper has achieved the same conclusion.

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