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H_{∞} Output feedback sliding mode control for a class of uncertain systems with time delay

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ABSTRACT

The problem of H_{∞} static output feedback sliding mode control for a class of nonlinear delay systems with normbounded uncertainties and external disturbance is considered in this paper. Based on Linear matrix inequality approach, a new approach is given to design the static output feedback sliding mode surface. Then, a sliding mode controller is obtained which make the systems states reach the sliding mode surface in finite time. All the conditions are expressed in terms of LMI. Finally, a numerical example is given to demonstrate the validity of the results.

Key words: Delay systems; H_{∞} control; Static output feedback; Sliding mode control

INTRODUCTION

Time delay is frequently encountered in various engineering, communication, and biological systems[1]. The characteristics of dynamic systems are significantly affected by the presence of time delays, even to the extent of instability in extreme situations. Therefore, the study of delay systems has received much attention, and various analysis and synthesis methods have been developed over the past years[2,3].

As is known, based on using of discontinuous control laws, the sliding mode control approach is known to be an efficient alternative way to tackle many challenging problems of robust stabilization. Li, et al. considered the problem of adaptive fuzzy sliding mode control for a class of nonlinear time delay systems[4]. Kown, et al.gave an improved delay-dependent condition to design robust controller for uncertain time-delay systems. Based on LMI approach, Chen, et al. considered the problem of exponential stability for uncertain stochastic systems with multiple delays[5]. Xia, et al. and Qu, et al. designed the robust sliding mode controller for uncertain systems with delays by using LMI approach[6,7]. The problem of discrete-time output feedback sliding mode control for time-delay systems with uncertainty is researched[8]. But the static output feedback sliding mode control for delay systems has never been presented.

This paper presents the problem of H_{∞} static output feedback sliding mode control for a class of nonlinear delay systems with norm-bounded uncertainties and external disturbance. Based on Linear matrix inequality approach, a new approach is given to design the static output feedback sliding mode surface. Then, a sliding mode controller is obtained which make the systems states reach the sliding mode surface in finite time.

PROBLEM FORMULATION

Consider the following nonlinear systems with delay

$$\mathcal{L}(t) = (A + \Delta A(t))x(t) + (A_d + \Delta A_d(t))x(t - d) + Bu(t) + B_\omega \omega(t)$$

$$y(t) = Cx(t)$$

$$x(t) = \psi(t) \qquad -d \le t \le 0$$
(1)

where $x(t) \in R^n$ is system state, $u(t) \in R^m$ systems control input, $y(t) \in R^p$ systems output, d is a system state delay. $\psi(t)$ is the given initial state on [-d,0]. $A \in R^{n \times n}$, $A_d \in R^{n \times n}$, $B \in R^{m \times n}$, $B \in R^{m \times n}$ and $C \in R^{p \times n}$ are known constant matrices, and B has full column rank. $\Delta A(t) \in R^{n \times n}$ and $\Delta A_d(t) \in R^{n \times n}$ are unknown matrices representing the uncertainties and satisfying

$$[\Delta A(t) \quad \Delta A_d(t)] = GD(t)[H \quad H_d] \tag{2}$$

where G, H and H_d are constant matrices with appropriate dimensions, $\mathcal{D}(t)$ is unknown matrix satisfying

$$D^{T}(t)D(t) \leq I$$

 $\omega(t)$ is external disturbance and satisfying

$$\|\omega(t)\| \le \rho(t)$$

where $\rho(t)$ is known function on [-d, 0].

With Singular Value Decomposition of B, $B = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Omega \\ 0 \end{bmatrix} V^T$, nonsingular transformation $T = \begin{bmatrix} U_2^T \\ U_1^T \end{bmatrix}$ is constructed for systems (1) to make $TB = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}$; $TB_\omega = \begin{bmatrix} B_{\omega 1} \\ B_{\omega 2} \end{bmatrix}$

With the transformation z(t) = Tx(t), the systems (1) can be rewritten

$$\begin{split} \mathbf{E}(t) &= \begin{bmatrix} \mathbf{E}_{1}(t) \\ \mathbf{E}_{2}(t) \end{bmatrix} = T(A + \Delta A(t))x(t) + T(A_{d} + \Delta A_{d}(t))x(t - d) + TBu(t) + TB_{\omega}\omega(t) \\ &= (TAT^{-1} + T\Delta A(t)T^{-1})z(t) + (TA_{d}T^{-1} + T\Delta A_{d}(t)T^{-1})z(t - d) \\ &+ TBu(t) + TB_{\omega}\omega(t) \end{split}$$

Inserting (2) into the above formulation, we obtain

$$\mathbf{E}(t) = (U_{2}^{T}AU_{2} + U_{2}^{T}GDHU_{2})z_{1}(t) + (U_{2}^{T}AU_{1} + U_{2}^{T}GDHU_{1})z_{2}(t) + (U_{2}^{T}A_{d}U_{2} + U_{2}^{T}GDH_{d}U_{2})z_{1}(t-d) + (U_{2}^{T}A_{d}U_{1} + U_{2}^{T}GDH_{d}U_{1})z_{2}(t-d) + B_{\omega 1}\omega(t)
\mathbf{E}(t) = (U_{1}^{T}AU_{2} + U_{1}^{T}GDHU_{2})z_{1}(t) + (U_{1}^{T}AU_{1} + U_{1}^{T}GDHU_{1})z_{2}(t) + (U_{1}^{T}A_{d}U_{2} + U_{1}^{T}GDH_{d}U_{2})z_{1}(t-d) + (U_{1}^{T}A_{d}U_{1} + U_{2}^{T}GDH_{d}U_{1})z_{2}(t-d) + B_{2}u(t) + B_{\omega 2}\omega(t)$$
(3)

For the systems (3), selecting the static output feedback sliding mode surface as following

$$\sigma(t) = Sy(t) \tag{4}$$

With

$$\sigma(t) = Sy(t) = SCT^{-1}z(t) = SC[U_2 \ U_1]z(t) = SCU_2z_1(t) + SCU_1z_2(t) = 0$$

by the assumption that SCU_1 is nonsingular, we obtain

$$z_2(t) = -(SCU_1)^{-1}SCU_2z_1(t) = -Fz_1(t)$$

Where $F = (SCU_1)^{-1}SCU_2$.

Inserting the above formulation into the systems (3), the sliding mode equation is obtained

$$\mathbf{R}(t) = \overline{A}z_1(t) + \overline{A}_d z_1(t - d) + B_{\omega 1}\omega(t) \tag{5}$$

where

$$\overline{A} = U_2^T A(U_2 - U_1 F) + U_2^T GDH(U_2 - U_1 F)$$

$$\overline{A}_d = U_2^T A_d(U_2 - U_1 F) + U_2^T GDH_d(U_2 - U_1 F)$$

RESULTS

Lemma1[2] For known constant $\varepsilon > 0$ and matrices D, E, F which satisfying $F^T F \leq I$, then the following matrix inequality is hold

$$DEF + E^{T}F^{T}D^{T} \le \varepsilon DD^{T} + \varepsilon^{-1}E^{T}E$$

Lemma2[9]The LMI

$$\begin{bmatrix} Y(x) & W(x) \\ * & R(x) \end{bmatrix} > 0$$

is equivalent to

$$R(x) > 0$$
, $Y(x) - W(x)R^{-1}(x)W^{T}(x) > 0$

where
$$Y(x) = Y^{T}(x)$$
, $R(x) = R^{T}(x)$ depend on x .

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & -B_{\omega 1} X^T & d N_1^{\omega} & \Sigma_{16} \\ * & \Sigma_{22} & \Sigma_{23} & -\rho_2 B_{\omega 1} X^T & d N_2^{\omega} & \Sigma_{26} \\ * & * & \Sigma_{33} & -\rho_3 B_{\omega 1} X^T & d N_3^{\omega} & 0 \\ * & * & * & -R & 0 & 0 \\ * & * & * & * & -d O & 0 \\ * & * & * & * & * & -\alpha I \end{bmatrix} < 0$$

$$(6)$$

where

$$\begin{split} & \Sigma_{11} = N_1^0 + N_1^T - U_2^T A(U_2 X^T - U_1 Z) - (U_2 X^T - U_1 Z)^T A^T U_2 + \alpha U_2^T G G^T U_2 \\ & \Sigma_{12} = N_2^T - N_1^0 - U_2^T A_d (U_2 X^T - U_1 Z) - \rho_2 (U_2 X^T - U_1 Z)^T A^T U_2 + \alpha \rho_2 U_2^T G G^T U_2 \\ & \Sigma_{13} = N^0 + N_3^T + X^T - \rho_3 (U_2 X^T - U_1 Z)^T A^T U_2 + \alpha \rho_3 U_2^T G G^T U_2 \\ & \Sigma_{16} = (U_2 X^T - U_1 Z)^T H^T \\ & \Sigma_{22} = -N_2^0 - N_2^T - \rho_2 U_2^T A_d (U_2 X^T - U_1 Z) - \rho_2 (U_2 X^T - U_1 Z)^T A_d^T U_2 + \alpha \rho_2^2 U_2^T G G^T U_2 \\ & \Sigma_{23} = -N_2^T + \rho_2 X^T - \rho_3 (U_2 X^T - U_1 Z)^T A_d^T U_2 + \alpha \rho_2 \rho_3 U_2^T G G^T U_2 \\ & \Sigma_{26} = (U_2 X^T - U_1 Z)^T H_d^T \\ & \Sigma_{33} = d O + \rho_3 X^T + \rho_3 X^T + \alpha \rho_3^2 U_2^T G G^T U_2 \end{split}$$

We can Design the sliding mode surface

$$\sigma(t) = Sy(t)$$

Where matrix S satisfying

$$SC(U_1F - U_2) = 0, F = ZX^{-T}$$

Proof. Selecting Lyapunov functional such as

$$V(t) = z_1^T(t)Pz_1(t) + \int_{-d}^{0} \int_{t+\theta}^{t} \frac{dt}{dt}(s)Qds d\theta$$

Where P, Q are positive-definite matrices of Theorem1.

Then, along the solution of system (5) we have

$$P_{t-d}^{\mathcal{R}}(t) = 2z_{1}^{T}(t)P_{t}^{\mathcal{R}}(t) + d\mathcal{R}_{1}^{\mathcal{T}}(t)Q_{t}^{\mathcal{R}}(t) - \int_{t-d}^{t}\mathcal{R}_{1}^{\mathcal{T}}(s)Q_{t}^{\mathcal{R}}(s)ds + 2(z_{1}^{T}(t)N_{1}) + z_{1}^{T}(t-d)N_{2} + z_{1}^{T}(t)N_{3})(z_{1}(t) - z_{1}(t-d) - \int_{t-d}^{t}\mathcal{R}_{1}(s)ds) + 2(z_{1}^{T}(t)M_{1} + z_{1}^{T}(t-d)M_{2} + \mathcal{R}_{1}^{\mathcal{T}}(t)M_{3})(-\overline{A}z_{1}(t) - \overline{A}_{d}z_{1}(t-d) + \mathcal{R}_{1}(t)) \\ \leq 2z_{1}^{T}(t)P_{t}^{\mathcal{R}}(t) + d\mathcal{R}_{1}^{\mathcal{T}}(t)Q_{t}^{\mathcal{R}}(t) - \int_{t-d}^{t}\mathcal{R}_{1}^{\mathcal{T}}(s)Q_{t}^{\mathcal{R}}(s)ds + 2(z_{1}^{T}(t)N_{1} + z_{1}^{T}(t-d)N_{2} + z_{1}^{T}(t)N_{3})(z_{1}(t) - z_{1}(t-d)) + 2(z_{1}^{T}(t)M_{1} + z_{1}^{T}(t-d)M_{2} + \mathcal{R}_{1}^{\mathcal{T}}(t)M_{3})(-\overline{A}z_{1}(t) - \overline{A}_{d}z_{1}(t-d) - B_{\omega_{1}}\omega(t) + \mathcal{R}_{1}^{\mathcal{T}}(t)) + d(z_{1}^{T}(t)N_{1} + z_{1}^{T}(t-d)N_{2} + z_{1}^{T}(t)N_{3})Q^{-1}(z_{1}^{T}(t)N_{1} + z_{1}^{T}(t)N_{1} + z_{1}^{T}(t)N_{2}(t)Q_{1}^{T}(t)N_{1} + z_{1}^{T}(t)N_{2}(t)Q_{2}^{T}(t)Q_{1}^{T}(t)N_{1} + z_{1}^{T}(t)N_{2}(t)Q_{2}^{T}(t)Q_{2}^{T}(t)Q_{2}^{T}(t)Q_{2}^{T}(t)Q_{2}^{T}(t)Q_{2}^{T}(t)Q_{2}^{T}(t)Q_{2}^{T}(t)Q_{2}^{T}(t)Q_{2}^{T}(t)Q_{2}^{T}(t)Q_{2}^{T}(t)Q_{2}^{T}(t)Q_{2$$

where $N_1, N_2, N_3, M_1, M_2, M_3$ are constant matrices with appropriate dimensions to be confirmed.

$$\xi(t) = \begin{bmatrix} z_1^T(t) & z_1^T(t-d) & \mathbf{A}_1^T(t) & \omega^T(t) \end{bmatrix}^T$$

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & -M_1 B_{\omega 1} \\ * & \Xi_{22} & \Xi_{23} & -M_2 B_{\omega 1} \\ * & * & \Xi_{33} & -M_3 B_{\omega 1} \\ * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\Xi_{11} = N_1 + N_1^T - M_1 \overline{A} - \overline{A}^T M_1^T + dN_1 Q^{-1} N_1^T$$

$$\Xi_{12} = N_2^T - N_1 - \overline{A}^T M_2^T - M_1 \overline{A}_d + dN_1 Q^{-1} N_2^T$$

$$\Xi_{13} = P + N_3^T - \overline{A}^T M_3^T + M_1 + dN_1 Q^{-1} N_3^T$$

$$\Xi_{22} = -N_2 - N_2^T - M_2 \overline{A}_d - \overline{A}_d^T M_2^T + dN_2 Q^{-1} N_2^T$$

$$\Xi_{23} = -N_3^T - \overline{A}_d^T M_3^T + M_2 + dN_2 Q^{-1} N_3^T$$

$$\Xi_{33} = dQ + M_3 + M_3^T + dN_3 Q^{-1} N_3^T$$

The inequality

$$\Xi < 0 \tag{7}$$

is equivalent to

$$\Xi = \Theta + d \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ 0 \end{bmatrix} Q^{-1} \begin{bmatrix} N_1^T & N_2^T & N_3^T & 0 \end{bmatrix} - \begin{bmatrix} M_1 U_2^T G \\ M_2 U_2^T G \\ M_3 U_2^T G \\ 0 \end{bmatrix} D \begin{bmatrix} H(U_2 - U_1 F) & H_d(U_2 - U_1 F) \end{bmatrix}$$

$$0 \quad 0 \Big] - \Big[H(U_2 - U_1 F) \quad H_d(U_2 - U_1 F) \quad 0 \quad 0 \Big]^T D^T \begin{bmatrix} M_1 U_2^T G \\ M_2 U_2^T G \\ M_3 U_2^T G \\ 0 \end{bmatrix}^T < 0$$

where

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & -M_1 B_{\omega 1} \\ * & \Theta_{22} & \Theta_{23} & -M_2 B_{\omega 1} \\ * & * & \Theta_{33} & -M_3 B_{\omega 1} \\ * & * & * & -\gamma^2 I \end{bmatrix}$$

$$\Theta_{11} = N_1 + N_1^T - M_1 U_2^T A (U_2 - U_1 F)$$
$$- (U_2 - U_1 F)^T A^T U_2 M_1^T$$

$$\Theta_{12} = N_2^T - N_1 - M_1 U_2^T A_d (U_2 - U_1 F) - (U_2 - U_1 F)^T A^T U_2 M_2^T$$

$$\Theta_{13} = P + N_3^T + M_1 - (U_2 - U_1 F)^T A^T U_2 M_3^T$$

$$\Theta_{22} = -N_2 - N_2^T - M_2 U_2^T A_d (U_2 - U_1 F)$$
$$-(U_2 - U_1 F)^T A_d^T U_2 M_2^T$$

$$\Theta_{23} = -N_3^T + M_2 - (U_2 - U_1 F)^T A_d^T U_2 M_3^T$$

$$\Theta_{33} = dQ + M_3 + M_3^T$$

With lemma 1, we know that the following inequality holds for given constant $\alpha > 0$

$$-\begin{bmatrix} M_{1}U_{2}^{T}G \\ M_{2}U_{2}^{T}G \\ M_{3}U_{2}^{T}G \\ 0 \end{bmatrix} D[H(U_{2}-U_{1}F) \quad H_{d}(U_{2}-U_{1}F) \quad 0 \quad 0] -[H(U_{2}-U_{1}F) \quad H_{d}(U_{2}-U_{1}F) \quad 0 \quad 0]^{T} D^{T} \begin{bmatrix} M_{1}U_{2}^{T}G \\ M_{2}U_{2}^{T}G \\ M_{3}U_{2}^{T}G \\ 0 \end{bmatrix}^{T} \\ \leq \alpha^{-1} [H(U_{2}-U_{1}F) \quad H_{d}(U_{2}-U_{1}F) \quad 0 \quad 0]^{T} [H(U_{2}-U_{1}F) \quad H_{d}(U_{2}-U_{1}F) \quad 0 \quad 0] + \alpha \begin{bmatrix} M_{1}U_{2}^{T}G \\ M_{2}U_{2}^{T}G \\ M_{2}U_{2}^{T}G \\ M_{3}U_{2}^{T}G \\ 0 \end{bmatrix}^{T} \\ M_{3}U_{2}^{T}G \\ M_{3}U_{2}^{T}G \\ 0 \end{bmatrix}$$

$$\leq \alpha^{-1} \left[H(U_2 - U_1 F) \quad H_d(U_2 - U_1 F) \quad 0 \quad 0 \right]^T \left[H(U_2 - U_1 F) \quad H_d(U_2 - U_1 F) \quad 0 \quad 0 \right] + \alpha \begin{bmatrix} M_1 U_2' G \\ M_2 U_2^T G \\ M_3 U_2^T G \\ 0 \end{bmatrix} \begin{bmatrix} M_1 U_2' G \\ M_2 U_2^T G \\ M_3 U_2^T G \\ 0 \end{bmatrix}$$

With lemma 2, we know that the inequality (7) is equivalent to

$$\Delta = \begin{bmatrix}
\Delta_{11} & \Delta_{12} & \Delta_{13} & -M_1 B_{\omega_1} & dN_1 & \Delta_{16} \\
* & \Delta_{22} & \Delta_{23} & -M_2 B_{\omega_1} & dN_1 & \Delta_{26} \\
* & * & \Delta_{33} & -M_3 B_{\omega_1} & dN_1 & 0 \\
* & * & * & -\gamma^2 I & 0 & 0 \\
* & * & * & * & -dQ & 0 \\
* & * & * & * & * & -\alpha I
\end{bmatrix} < 0 \tag{8}$$

$$\begin{split} & \Delta_{11} = N_1 + N_1^T - M_1 U_2^T A(U_2 - U_1 F) - (U_2 - U_1 F)^T A^T U_2 M_1^T + \alpha M_1 U_2^T G G^T U_2 M_1^T \\ & \Delta_{12} = N_2^T - N_1 - M_1 U_2^T A_d (U_2 - U_1 F) - (U_2 - U_1 F)^T A^T U_2 M_2^T + \alpha M_1 U_2^T G G^T U_2 M_2^T \\ & \Delta_{13} = P + N_3^T + M_1 - (U_2 - U_1 F)^T A^T U_2 M_3^T + \alpha M_1 U_2^T G G^T U_2 M_3^T \\ & \Delta_{16} = (U_2 - U_1 F)^T H^T \\ & \Delta_{22} = -N_2 - N_2^T - M_2 U_2^T A_d (U_2 - U_1 F) - (U_2 - U_1 F)^T A_d^T U_2 M_2^T + \alpha M_2 U_2^T G G^T U_2 M_2^T \\ & \Delta_{23} = -N_3^T + M_2 - (U_2 - U_1 F)^T A_d^T U_2 M_3^T + \alpha M_2 U_2^T G G^T U_2 M_3^T \\ & \Delta_{26} = (U_2 - U_1 F)^T H_d^T \end{split}$$

Pre- and Post-multiplying the inequality (8) by

 $\Delta_{33} = dQ + M_3 + M_3^T + \alpha M_3 U_2^T G G^T U_2 M_2^T$

$$\begin{aligned} &\textit{diag}\{M_0^{-1}, M_0^{-1}, M_0^{-1}, M_0^{-1}, M_0^{-1}, M_0^{-1}, I\} \quad \text{and} \quad &\textit{diag}\{M_0^{-T}, M_0^{-T}, M_0^{-T}, M_0^{-T}, M_0^{-T}, I\} \text{ , by giving some transformations } M_1 = M_0, M_2 = \rho_2 M_0, M_3 = \rho_3 M_0, X = M_0^{-1}, Z = FX^T, \text{ for } XPX^T, \text{ on }$$

 $R = \gamma^2 X X^T$, where ρ_2, ρ_3 are constants to be obtained, we know that the inequality (8) is equivalent to (6).

From the inequality (6), we obtain

$$P^{\mathbf{g}}(t) \le -\xi^{T}(t)\Xi\xi(t) + \gamma^{2}\omega^{T}(t)\omega(t)$$

therefore

$$V(t) - V(t_0) \le -\int_{t_0}^{t} \xi^{T}(s) \Xi \xi(s) ds + \int_{t_0}^{t} \gamma^2 \omega^{T}(s) \omega(s) ds$$

If $t \rightarrow 0$, with the initial condition, we obtain

$$-\lambda_{\min}(\Xi) \int_{t_0}^{t} z_1^{T}(s) z_1(s) ds \le -\lambda_{\min}(\Xi) \int_{t_0}^{t} \xi^{T}(s) \xi(s) ds \le \gamma^2 \int_{t_0}^{t} \omega^{T}(s) \omega(s) ds$$

Therefore

$$\|z_1(t)\|_2 \le \frac{\gamma}{\sqrt{-\lambda_{\min}(\Xi)}} \|\omega(t)\|_2$$
.

If $\omega(t) = 0$, we can obtain $\mathcal{P}(t) < 0$, the sliding mode equation is stable.

Theorem 2 For the nonlinear delay systems (1), with the controller

$$u(t) = -(SCB)^{-1}[SCAx(t) + SCA_{d}x(t-d) + \frac{\|SC\|\sigma(t)}{\|\sigma(t)\|}(\|GH\| + \|GH_{d}\| + \|B_{\omega}\|\rho(t)) + k\sigma(t) + \varepsilon sign\sigma(t)]$$
(9)

Where k, ε are constants satisfying $k > 0, \varepsilon > 0$, then the systems states will reach the sliding mode surface (4) in finite time.

Proof. Along the solution of system (1) we have

$$\sigma^{T}(t)\mathcal{S}C(A+\Delta A)x(t) + \sigma^{T}(t)SC(A_{d}+\Delta A_{d})x(t-d) + \sigma^{T}(t)SCB_{\omega}\omega(t)$$

$$-\sigma^{T}(t)SCAx(t) - \sigma^{T}(t)SCA_{d}x(t-d) - \frac{\sigma^{T}(t)\|SC\|\sigma(t)}{\|\sigma(t)\|}(\|GH\| + \|GH_{d}\|$$

$$+ \|B_{\omega}\|\rho(t)) - \sigma^{T}(t)k\sigma(t) - \sigma^{T}(t)\varepsilon sign\sigma(t)$$

$$\leq -\sigma^{T}(t)k\sigma(t) - \sigma^{T}(t)\varepsilon sign\sigma(t) < 0$$
(10)

With the controller (9) and the above equation (10), we know that the reaching condition is satisfied.

CONCLUSION

This paper considers the problem of H_{∞} static output feedback sliding mode control for a class of nonlinear delay systems with norm-bounded uncertainties and external disturbance. A static output feedback sliding mode surface is designed by using linear matrix inequality approach. Then the sliding mode controller is designed to make the states reach sliding mode surface in finite time.

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