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Gap element method and its application on force analysis of tubing strings

Liu Weikai#, Song Mingxing*, XU Ziyi@ and Zhang Xuehong#

#Petroleum Engineering College, Northeast Petroleum University, Heilongjiang Daqing, China
**Daqing Drilling Engineering Institute of Technology, Heilongjiang Daqing, China*
@School of Petroleum Engineering, China University of Petroleum, Beijing Campus, Beijing, China

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ABSTRACT

A static force model is set up for analyzing the dual layer contact between tubing strings featuring its actual work conditions. Dual layer gap element theory describes the frictions when the tubing strings are contacted, with its real contact each node of tubing strings reflects, the overall balance equation and its requirements for solutions can be given. Based on the force analysis, a section of tubing string is added on the bottom of the original strings for reducing the frictions, which a pre-added force is put under the artificial bottom hole that could reduce the average force of the tubing strings and elongate the life span of tubing strings.

Keywords: Tubing String; Force Analysis; Gap Element; Contact; Life Span

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INTRODUCTION

Broken tubing strings and off-center wears are one of the common problems the pumping unit face and is difficult to address during the oil production. The factors influencing the forces of tubing and sucker rods are many but limited to the well geometry, the quality of produced liquid, the placement of stabilizer and variance of conditions under the hole, etc. The force of tubing strings is really complex, when it is in upward stroke, the extracted sucker rod gets contacted with the inner layer of sucker rod in bended wellbore, and the tubing is in contact with the inner layer of tubing under the reaction force of sucker rod; when in downward stroke, the bottom of sucker rod is bend and deformed under the force of pump, and even spirally deformed which gets contacted with the inner layer of tubing, thus the tubing would be contacted with the inner wall of casings. It is obviously difficult to understand the non-linear question of force among the sucker rod, tubing and casing.

Regarding to the force analysis of tubing strings under the well, researchers of nationalities have proposed many force models, such as classical differential equation method[1], crossbar bend continuous beam[2], weighted residual method[3], initial parameters[4,5] , finite differential method[6-8] , finite element method[9], while these theories may not solve the contact problem among the sucker rod, tubing string and casing. This paper takes the static force model and Dual layer gap element theory to analyze the force among the three.

1 METHOD AND THEORETICAL EQUATION

1.1 STATIC FORCE MODEL IS SET FOR ANALYZING THE DUAL LAYER CONTACT WITH SUCKER ROD, TUBING STRING AND CASING

A force analysis model is set, as shown in figure 1, through the analysis of the sucker rod, tubing string and casing under work conditions. When constructing the model of figure 1, the whole tubing string from the wellhead to the bottom is under the following hypothesis: (1) the string and accessories are elastic deformed; (2) the inner wall of wellbore and strings are stiff and round ; (3) the contact with the sucker rod, tubing string and casing is under random state, and its contact deformation is within the scope of elastic deformation and there are reaction force and frictions at the contact; (4) all dynamic factors are neglected; (5) the axis of wellbore is a 3-D curve in the space and the location of wellbore axis is determined by the well profile or surveyed data of well depth, deviated angle and azimuth that are calculated by the spline function.

There are some modifications on the requirements of boundaries: the wellhead is simplified with a build-in end; when the tubing string is anchored, the bottom is simplified by a fixed hinge setting and a free end when it is not. The boundary of free move of sucker rod, tubing string and casing are relatively simplified as boundary of frictions. The contact friction is determined by the balance state of Dual layer system of sucker rod, tubing and casing. The main load are the weight of sucker rod and tubing, the inertia force of sucker rod, the reaction force of wellhead, buoyancy of oil and normal reaction, friction of rod and tubing at the contact point , piston force at the bottom of sucker rod and differential liquid pressure inside and outside of rod and tubing.

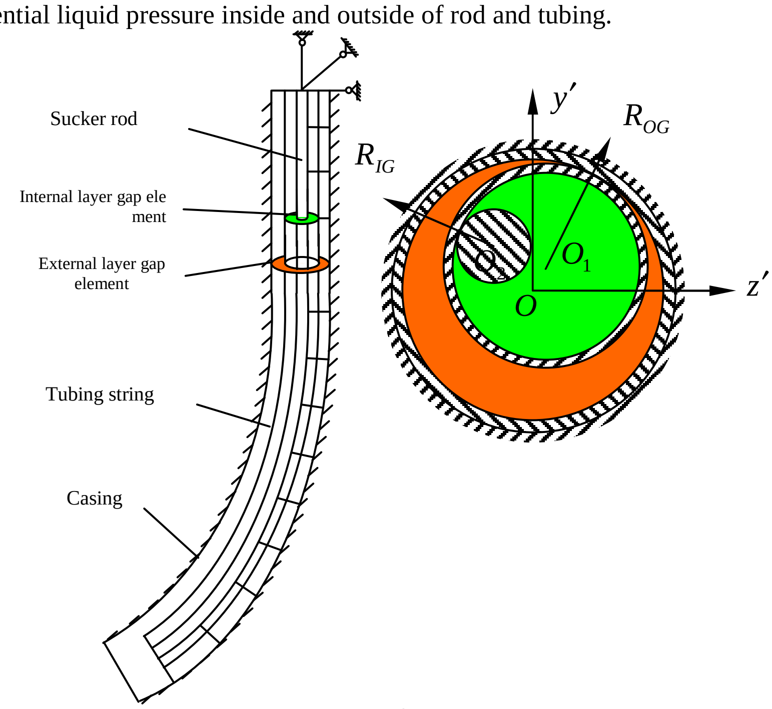

Figure 1. **Static force analysis model and gap element model of sucker rod, tubing string and double layer contact of strings**

2 DUAL LAYER CONTACT GAP ELEMENT ANALYSIS OF SUCKER ROD, TUBING STRING AND CASING

Dual layer finite element theory can be used to describe the contact friction between tubing strings according to to the work of tubing. And finite element is normally adopted to separate the rod and tubing based on the static force analysis model along the axis into several spatial beam units, the node of each unit is set as Dual layer contact friction gap element, the sucker rod, tubing and casing strings are united as a whole through this. The gap element of contact friction in varied angles is a virtual unit consisting liquid or gas shaped as a thick round circle, the external boundary of gap element contacts with the casing and inner boundary with the outer wall of tubing string; the unit that the outer boundary of the gap element of inner layer connects with the inner wall of tubing string and inner boundary connects with outer wall of sucker rod has following physical features: the inner layer between tubing strings is *not in contact would not inf*luence the move of tubing and its compressive stiffness reaches zero; when in contact, the gap element at nodes would be completely compressed , the inner and outer boundary of gap element is tangent at the contact point, its compressive stiffness equals a certain number or a number that is large enough that could prevent the intervention between tubing strings, which the slide of tubing strings is seen on the surface of sidewall with contact force, friction force, torque and drag are generated.

2.1 OVERALL BALANCED EQUATIONS

The overall coordinate of tubing string is set up and each spatial unit has partial coordinates.

The relation of nodal displacement vector between partial coordinates and overall coordinates is:

$$\{\delta'\}^e = [\boldsymbol{T}]\{\delta\}^e \,, \tag{1}$$

$\{\delta'\}^e$ is nodal displacement of partial coordinate ; $[\boldsymbol{T}]$ is the matrix of coordinate transformation; $\{\delta\}^e$ is nodal displacement of overall coordinate.

The coordinate transformation of nodal displacement vector between partial coordinate and overall coordinates is

$$\{F_i'\}^e = [\boldsymbol{T}]\{F_i\}^e \text{ ,} \tag{2}$$

$\{F_i'\}^e$ is nodal vector of a tubing string in partial coordinate; $\{F_i\}^e$ is nodal vector of a tubing string in overall coordinate.

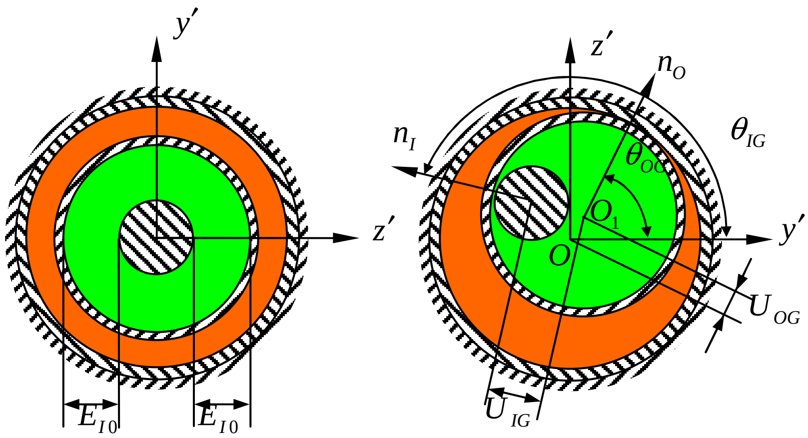

Figure 2. **The deformation of gap element with dual layer contact system chart**

The spatial beam unit of separated tubing string features tensile pressure, counter bending and resistance against torsional rigidity, elastic potential energy of each unit can be expressed as followed:

$$\prod_p = \int_L \frac{1}{2}\{\varepsilon\}^T[D]\{\varepsilon\}\mathrm{d}L - \int_L \{u\}^T\{q\}\mathrm{d}L - \{\delta'\}^{eT}\{F'\}^e \text{ ,} \tag{3}$$

$$\{\varepsilon\} = [\boldsymbol{B}]\{\delta'\}^e \text{ ,} \tag{4}$$

L is the unit length ; $\{\varepsilon\}$ is strain vector; $[\boldsymbol{D}]$ elastic matrix; $\{q\}$ is load vector; $\{u\}$ is nodal displacement vector; $[\boldsymbol{B}]$ is strain matrix, thus the stiffness matrix equation based on the minimum potential theory can be expressed as:

$$[\boldsymbol{K}'_{0i}]^e\{\delta'\}^e = \{F_i'\}^e \text{ ,} \tag{5}$$

$[\boldsymbol{K}'_{0i}]^e$ is the unit stiffness matrix of a tubing string in partial coordinate

$$[\boldsymbol{K}'_{0i}]^e = \int_L [\boldsymbol{B}]^T[\boldsymbol{D}]\ [\boldsymbol{B}]\ \mathrm{d}l \text{ .} \tag{6}$$

The transformation from the nodal displacement and nodal vector of partial coordinate to those of in the overall coordinate can calculate the unit stiffness matrix $[\boldsymbol{K}_{0i}]^e$ of tubing in overall coordinate and that of in partial coordinate:

$$[\boldsymbol{K}_{0i}]^e = [\boldsymbol{T}]^{-1}[\boldsymbol{K}'_{0i}]^e[\boldsymbol{T}] \text{,} \tag{7}$$

Obviously, the sucker rod and tubing string are not the same deformed objects; each of them takes on different load and changes respectively, and it is in the contact area that these two would be interacted and this paper takes more emphasis on that part using the gap element theory.

2.2 THE GAP ELEMENT OF DUAL LAYER FRICTION IN VARIED ANGLES

When it is taking the static analysis of sucker rod, tubing and casing dual layer system, the gap element dual layer system is set at the nodes of beam unit. Dual layer gap element $OX'Y'Z'$ and beam unit of the partial coordinate are in the same radius of the ring, as illustrated in figure 2. And displacement U_{IG} the sucker rod generated in the direction of n_I (the azimuth is θ_{IG}) in relation to the inner layer gap element of the tubing string, the initial gap element of inner gap element is E_{I0}, the gap element after deformation is E_{I1}, so

$$E_{I1} = E_{I0} - U_{IG} \tag{8}$$

$$\varepsilon_{IG} = \frac{U_{IG}}{E_{I0}} \tag{9}$$

ε_{IG} is strain of inner lay gap element, the stiffness rigidity of inner layer gap element is $E_{IG}A_I$, $C_I = E_{I0}E_{IG}A_I E_{I0}$, $B_{I1} = 1/E_{I0}$ thus the contact is $R_{n_I G}$, inner layer contact friction is R_{XIG} and elastic potential energy of inner layer gap element is:

$$R_{n_I G} = E_{IG}A_I U_{IG} \tag{10}$$

$$R_{XIG} = f_I R_{n_I G} \tag{11}$$

$$\prod_{IG} = \frac{1}{2}U_{IG}R_{n_I G} = \frac{1}{2}\varepsilon_{IG}C_I\varepsilon_{IG} \tag{12}$$

f_I is the friction coefficient of inner layer, using the C_I and B_{I1} with zero into a matrix $[C_I]$ and $[B_{I1}]$ that is in the same order of beam unit, the unit stiffness rigidity matrix equation of sucker rod beam unit of inner layer gap element is:

$$[K_R]^e = [K_0]^e + [B_{I1}]^T[C_I][B_{I1}] \tag{13}$$

As can be included from the above, the unit stiffness rigidity matrix equation of tubing string beam unit of external layer gap element is:

$$[K_T]^e = [K_0]^e + [B_{I1}]^T[C_I][B_{I1}] + [B_{O1}]^T[C_O][B_{O1}] \tag{14}$$

of which the compressive rigidity of tubing string in external layer gap element is $E_{OG}A_O$, $C_O = E_{O0}E_{OG}A_O E_{O0}$, $B_{O1} = 1/E_{O0}$, E_{O0} is the initial gap of external gap element, E_{O1} is the gap after it is deformed, U_{OG} is the displacement of the tubing string in the external layer gap element along a spherical direction n_O (azimuth is θ_{OG}).The friction of inner layer is f_O, the strain force of external layer $R_{n_O G}$, friction of inner layer R_{XOG} are :

$$R_{n_O G} = E_{OG}A_O U_{OG} \tag{15}$$

$$R_{XOG} = f_O R_{n_O G} \tag{16}$$

The overall balanced equation of sucker rod, tubing string and casing strings through the transformation of coordinates and superposition principle can be calculated by:

$$[\boldsymbol{K}]\{\delta\}=\{F\},\qquad (17)$$

$[\boldsymbol{K}],\{\delta\}$ and $\{F\}$ is the stiffness rigidity, nodal displacement and nodal vector of overall coordinates.

During the process of solving the equation of（17）， the contact condition of sucker rod, tubing string and casing through Dual layer contact system may be classified into 5 categories and 9 kinds：which are free mode （ $U_{IG}<E_{I0}$， $U_{OG}<E_{O0}$ ）、single layer contact mode（ $U_{IG}=E_{I0}$， $U_{OG}<E_{O0}$； or $U_{IG}<E_{I0}$， $U_{OG}=E_{O0}$ ）、Dual layer contact mode（ $U_{IG}=E_{I0}$， $U_{OG}=E_{O0}$ ）、single layer intrude mode（ $U_{IG}>E_{I0}$， $U_{OG}<E_{O0}$； or $U_{IG}>E_{I0}$， $U_{OG}=E_{O0}$； or $U_{IG}<E_{I0}$， $U_{OG}>E_{O0}$； or $U_{IG}=E_{I0}$ ， $U_{OG}>E_{O0}$； ）and Dual layer intrude mode （ $U_{IG}>E_{I0}$， $U_{OG}>E_{O0}$ ）. The real contact mode is classified into 3 categories and 4 kinds, which is free mode, single layer contact mode (2 kinds) and Dual layer contact mode. The contact mode of each node may be one of the above. The equations are:

Free mode：

$$\left.\begin{array}{l}\dfrac{U_{IG}}{E_{I0}}<1-m_{\varepsilon}\\ R_{n_I G}\le m_R\\ \dfrac{U_{OG}}{E_{O0}}<1-m_{\varepsilon}\\ R_{n_O G}\le m_R\end{array}\right\}\qquad (18)$$

m_{ε} m_R is the positive value. Single layer contact mode：

$$\left.\begin{array}{l}\left|\dfrac{U_{IG}}{E_{I0}}-1\right|\le m_{\varepsilon}\\ R_{n_I G}\ge m_R\\ \left|\theta_{IG_{i+1}}-\theta_{IG_i}\right|\le m_{\theta}\\ \dfrac{U_{OG}}{E_{O0}}<1-m_{\varepsilon}\\ R_{n_O G}\le m_R\end{array}\right\}\qquad (19)$$

m_{θ} is positive value, $\theta_{IG_{i+1}}$、θ_{IG_i} is the contact angle of inner layer gap element of i+1 and i through iterative computation.

$$\left.\begin{array}{l}\dfrac{U_{IG}}{E_{I0}}<1-m_{\varepsilon}\\ R_{n_I G}\le m_R\\ \left|\dfrac{U_{OG}}{E_{O0}}-1\right|\le m_{\varepsilon}\\ R_{n_O G}\ge m_R\\ \left|\theta_{OG_{i+1}}-\theta_{OG_i}\right|\le m_{\theta}\end{array}\right\}\qquad (20)$$

$\theta_{OG_{i+1}}$ 、 θ_{OG_i} is the contact angle of external layer gap element of i+1 and i through iterative computation.

Dual layer contact mode：

$$\left.\begin{aligned}
&\left|\frac{U_{IG}}{E_{I0}}-1\right| \le m_\varepsilon \\
&R_{n_I G} \ge m_R \\
&\left|\theta_{IG_{i+1}}-\theta_{IG_i}\right| \le m_\theta \\
&\left|\frac{U_{OG}}{E_{O0}}-1\right| \le m_\varepsilon \\
&R_{n_O G} \ge m_R \\
&\left|\theta_{OG_{i+1}}-\theta_{OG_i}\right| \le m_\theta
\end{aligned}\right\} \tag{21}$$

The gap element is introduced into (17) and its answer must be calculated through iterative computations to make all nodes meet the requirements of equations from (18) ~ (20).

2.3 CALCULATION OF EQUIVALENT STRESS AND STRENGTH CHECKOF SUCKER ROD AND TUBING STRING

The generalized displacement of sucker rod and tubing string could be calculated through loop iteration of (17), and on which it is based to calculate the axial force N_R and N_T , flexural torque M_{R_y} 、 M_{R_z} 、 M_{T_y} and M_{T_x} , shearing force Q_{R_y} 、 Q_{R_z} 、 Q_{T_y} and Q_{T_z} on any section of sucker rod and tubing string, thus the maximum normal stress and shearing strength are :

sucker rod:

$$\left.\begin{aligned}
&\sigma_{R_x} = \frac{N_R}{A_R} \pm \frac{\sqrt{M_{R_y}^2 + M_{R_z}^2}}{W_R} \\
&\tau_R = \frac{M_{R_x}}{W_{R_n}}
\end{aligned}\right\} \tag{22}$$

tubing string:

$$\left.\begin{aligned}
&\sigma_{T_x} = \frac{N_T}{A_T} \pm \frac{\sqrt{M_{T_y}^2 + M_{T_z}^2}}{W_T} \\
&\tau_T = \frac{M_{T_x}}{W_{T_n}}
\end{aligned}\right\} \tag{23}$$

A_R 、 A_T 、 W_R 、 W_T 、 W_{R_n} and W_{T_n} are section area, modules of bending section and torsional section of sucker rod and tubing string, of which $\pm$ takes + when the axial force is positive and vice versa. The equation of shearing strength neglects the section shearing strengths and normal stress is the main stress on the section, thus the maximum absolute value of normal stress on the section is the danger section and point of maximum normal stress on danger section is the danger point, and it is on the outer edge of danger section.

In addition, the hoop stress and radial stress generated by the inner pressure p_1 and external pressure p_2 are:

sucker rod：

$$\left.\begin{aligned}
&\sigma_{R\theta} = -p_1 \\
&\sigma_{Rr} = -p_1
\end{aligned}\right\} \tag{24}$$

sucker rod：

$$\left.\begin{array}{l}\sigma_{T\theta}=\dfrac{2\alpha}{1-\alpha^2}p_1-\dfrac{1+\alpha^2}{1-\alpha^2}p_2\\ \sigma_{Tr}=-\max(|p_1|,|p_2|)\end{array}\right\}\qquad(25)$$

$\alpha=\dfrac{d_T}{D_T}$， is the ratio of inner and outer diameter of tubing string.

Two of principal stress at the section can be calculated from the principal stress equation:

$$\begin{array}{l}\sigma_{R\max}\\ \sigma_{R\min}\end{array}=\frac{\sigma_{R\theta}+\sigma_{Rx}}{2}\pm\sqrt{\left(\frac{\sigma_{R\theta}-\sigma_{Rx}}{2}\right)^2+\tau_R^2}\qquad(26)$$

$$\begin{array}{l}\sigma_{T\max}\\ \sigma_{T\min}\end{array}=\frac{\sigma_{T\theta}+\sigma_{Tx}}{2}\pm\sqrt{\left(\frac{\sigma_{T\theta}-\sigma_{Tx}}{2}\right)^2+\tau_T^2}\qquad(27)$$

Equivalent stress of sucker rod and tubing string are:

$$\sigma_{R4}=\sigma_{R1}-\sigma_{R3}=\max(\sigma_{Rr},\sigma_{R\max},\sigma_{R\min})-\min(\sigma_{Rr},\sigma_{R\max},\sigma_{R\min})\qquad(28)$$

$$\sigma_{T4}=\sigma_{T1}-\sigma_{T3}=\max(\sigma_{Tr},\sigma_{T\max},\sigma_{T\min})-\min(\sigma_{Tr},\sigma_{T\max},\sigma_{T\min})\qquad(29)$$

The strength check conditions of sucker rod and tubing string are:

$$\sigma_{R4}\le[\sigma]\qquad(30)$$

$$\sigma_{T4}\le[\sigma]\qquad(31)$$

3 CALCULATION OF REAL CASES

Tubing strings of 3 wells are calculated based on the above theories. The layer temperature of 3 wells is around 65 ℃, depth is 1450~1520 m, pump diameter is 38mm. To prove the accuracy , the calculation is in table 1, from which the maximum discrepancies of surveyed values compared to the calculated values of polished rod load is 13.33%, less than 15%, which means the model this paper hypothesis is right and can be used in field.

Table 1 Comparison of Calculation Results of Polished Rod Load of 3 Wells

Well	Well 1		Well 2		Well 3	
Stroke	upwards	downwards	upwards	downwards	upwards	downwards
Surveyed load at kN	43.0	14.6	62.0	24.4	45.0	21.6
Polished rod load，kN	47.75	14.21	56.84	22.79	49.83	18.72
Relative error，%	6.40	-2.67	-8.32	-6.60	10.73	-13.33

Regarding to the string breaking, the best method is to improve the deformation condition of strings and the key is to reduce the stress amplitude and maximum mean stress of strings. Currently it is not possible to reduce the maximum stress amplitude and maximum mean stress due to the special work condition sucker rod has. While to the tubing string, a section of string can be added at the bottom of tubing string for putting pre-stress on the artificial bottom to reduce the maximum mean stress of tubing string by means of the calculation, and prevent the relative displacement of strings at bottom end from moving, reduce the stroke loss and improve oil pumping efficiency. The calculation of deformed condition after changes are made to the strings is analyzed in Table 2.

From the Table 2, the maximum mean stress before the change is 150.57MPa, maximum stress amplitude is 11.17MPa; the maximum mean stress after the change is 125.90MPa, which is reduced by 16.38%, and maximum stress amplitude is increased by 26.50% to 14.13MPa. It should be noted that the maximum stress amplitude is increased but the stress influence acts from the vicinity of well hole before the change to bottom of well hole after the change, and its mean stress of the maximum stress amplitude on the section is 20% of the maximum mean stress after changes are made, which greatly helps reduce the fatigue stress; the maximum equivalent stress dropped to 130.63MPa from 156.27MPa, lowered by 16.41%, and is almost the same as the maximum stress at the danger point after upward and downward stroke have taken place, and dynamic stress has seen dropped drastically compared with that of before the change, this paper takes the measures that could improve the deformed condition of tubing strings and ways of lasting the life span of tubing string.

Table 2 Deformation of Tubing Strings of 3 Wells before Change

Well No.	Structure	Stroke	Pre-compression kN	Maximum bending stress MPa	Maximum equivalent stress MPa	Maximum stress amplitude MPa	Maximum mean stress MPa
Well 1	Before the change	upwards	0	170.7	169.8	14.4	159.6
		downwards		174.0	173.2		
	after the change	upwards	30	145.1	144.2	17.2	133.7
		downwards		144.3	143.5		
Well 2	before the change	upwards	0	142.0	141.6	9.5	145.1
		downwards		151.3	150.9		
	after the change	upwards	30	121.0	120.6	12.2	119.6
		downwards		120.8	120.5		
Well 3	before the change	upwards	0	147.3	147.1	9.6	147.0
		downwards		155.2	155.0		
	after the change	upwards	30	127.9	127.8	13.0	124.4
		downwards		127.3	127.2		
Average	before the change			156.75	156.27	11.17	150.57
	after the change			131.07	130.63	14.13	125.90
	Reduced by			25.68	25.64	-2.96	24.67
	Percent			16.38%	16.41%	-26.50%	16.38%

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CONCLUSION

(1)A gap element model of dual layer contact friction in varied angles through the force analysis of sucker rod, tubing string and casing is set up, the calculation of stress of sucker rod and tubing string while taking into account of the three is given in this paper.

(2)The maximum discrepancy of surveyed values compared to the calculated values of polished rod load is 13.33%, minimum is 2.67%, its mean discrepancy is 8.01%, less than 15%, which means the model this paper hypothesis is right and can be used in field.

(3)While to the tubing string, a section of string can be added at the bottom of tubing string for putting pre-stress on the artificial bottom to reduce the maximum mean stress of tubing string by means of the calculation, which could reduce the maximum mean stress of tubing strings and increases the life span of tubing string.

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