Available online www.jocpr.com

Journal of Chemical and Pharmaceutical Research, 2013, 5(11):32-38

Research Article

ISSN : 0975-7384
CODEN(USA) : JCPRC5

Exponential stable control for a class of networked control systems with stochastic delays

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ABSTRACT

The problem of exponential stable control for networked systems with stochastic time delays is considered in this paper. A new approach is obtained to model the networked control systems with interval Bernoulli distribution t time delays. With linear matrix inequality approach, the mean-square exponential stability controller is designed by solving linear matrix inequalities. Moreover, a Lyapunov functional with some stack matrices is introduced in this paper. A numerical example is given to demonstrate the validity of the results.

Keywords*:* Networked control systems, Mean-square exponential stable, Stochastic delays, Linear matrix inequality (LMI)

INTRODUCTION

Networked control systems (NCSs) are systems where the feedback loop is closed via a communication network in which information, from various components such as sensors, controllers and actuators, is exchanged with limited bandwidth. NCSs have received increasing attentions in recent years due to their low cost simple installation and maintenance and high reliability[1-2].

However, the network itself is dynamic system that exhibits characteristics such as network-induced delays. The delays come from the time sharing of the communication medium as well as the computation time required for physical signal coding and communication processing. As is known, network-induced delays can degrade a system's performance and even cause system instability. Many researchers have studied stability analysis and controller design for NCSs[3-4]. It is quite common in practice that the time delays occur in a random way, rather than a deterministic way. Based on a similar Bernoulli stochastic model, [5-6] study NCSs with both sensor-to-controller and controller-to-actuator stochastic delays, and design the controller gain. One step stochastic delays or one stochastic packet dropout is considered in these papers.

The stabilization problem for a networked control system with Markov communication delays existing in both the system state and the mode signal is considered in[7]. The problem of the stabilization of NCSs with packet dropout is studied in[8]. A networked predictive control method for networked systems with stochastic delay and data dropout is proposed in [9-10] to compensate the networked-induced delay. This paper considers the problem of mean-square exponential stability control for a class of networked control systems with interval distribution time delay. A new approach is given to model the networked control systems. Based on the LIM approach and Lyapunov stability theorem, the mean-square exponential stability condition is given.

PROBLEM FORMULATION

Consider the following control system with delay

$$\dot{x}(t) = Ax(t) + A_d x(t-d) + Bu(t)$$
$$x(t) = \phi(t) \qquad t \in [-d, \; 0]$$

(1)

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the input vector, d is state delay $A, A_d \in R^{n\times n}$ are known real constant matrices, $B \in R^{n\times m}$ is input matrix, $\phi(t) \in R^n$ is the given initial state on$[-d, \; 0]$.

Throughout this note, we suppose that all the system's states are available for a state feedback control. In the presence of the control network, data transfers between the controller and the remote system, e.g., sensors and actuators in a distributed control system will induce network delay in addition to the controller proceeding delay. We introduce stochastic delay $\tau(t)$ to denote the network-induced delay. In this note we make the following assumptions:

Assumption 1: Sensor and controller are clock-driven;
Assumption 2: Actuator is event-driven.
We will design the state feedback controller

$$u(t) = Kx(t - \tau(t))$$

(2)

Inserting the controller (2) into system (1), we obtain the closed system:

$$\dot{x}(t) = Ax(t) + A_d x(t-d) + BKx(t - \tau(t))$$
$$x(t) = \psi(t) \qquad t \in [-\bar{d}, \; 0]$$

(3)

The initial condition of the state is supplemented as $x(t) = \psi(t)$, where $\psi(t)$ is a smooth function on $[-\bar{d}, \; 0]$, $\bar{d} = \max\{\tau, \; d\}$ .Therefore, there exists a positive constant $\bar{\psi}$ satisfying

$$\|\dot{\psi}(t)\| \le \bar{\psi} \qquad t \in [-\bar{d}, \; 0]$$

It is assumed that there exists a constant $\tau_1 \in [0, \; \tau]$ such that the probability of $\tau(t)$ taking values in $[0, \; \tau_1)$ and$[\tau_1, \; \tau]$ can be observed. In order to employ the information of the probability distribution of the delays, the following sets are proposed firstly

$$\Omega_1 = \{t : \tau(t) \in [0, \; \tau_1)\}, \quad \Omega_2 = \{t : \tau(t) \in [\tau_1, \; \tau]\}$$

Obviously, $\Omega_1 \cup \Omega_2 = R^+$ and $\Omega_1 \cap \Omega_2 = \Phi$

Then we define two functions as:

$$h_1(t) = \begin{cases} \tau(t) & t \in \Omega_1 \\ 0 & t \notin \Omega_1 \end{cases}, \quad h_2(t) = \begin{cases} \tau(t) & t \in \Omega_2 \\ \tau_1 & t \notin \Omega_2 \end{cases}$$

(4)

Corresponding to $\tau(t)$ taking values in different intervals, a stochastic variable $\beta(t)$ is defined

$$\beta(t) = \begin{cases} 1 & t \in \Omega_1 \\ 0 & t \in \Omega_2 \end{cases} \tag{5}$$

Where we suppose that $\beta(t)$ is a Bernoulli distributed sequence with

$$\mathrm{Prob}\{\beta(t) = 1\} = E\{\beta(t)\} = \beta$$

where $\beta \in [0,1]$ is a constant.

With (4) , (5) ,we know that the systems(3) is equivalent to

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-d) + \beta(t) BKx(t-h_1(t)) + (1-\beta(t))BKx(t-h_2(t)) \\ &= \bar{A}\xi(t) \\ x(t) &= \psi(t) \qquad\qquad t \in [-\bar{d}, \quad 0] \end{aligned} \tag{6}$$

where

$$\bar{A} = [A \quad A_d \quad \beta(t)BK \quad (1-\beta(t))BK]$$

$$\xi^T(t) = [x^T(t), \quad x^T(t-d), \quad x^T(t-h_1(t)), \quad x^T(t-h_2(t))]$$

RESULTS

Lemma 1[2] For any vectors a, b and matrices N, X, Y, Z with appropriate dimensions, if the following matrix inequality holds

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \geq 0$$

then we have

$$-2a^T Nb \leq \inf_{X,Y,Z} \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} X & Y-N \\ Y^T - N^T & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

Lemma 2[4] The LMI $\begin{bmatrix} Y(x) & W(x) \\ * & R(x) \end{bmatrix} > 0$ is equivalent to

$$R(x) > 0, Y(x) - W(x)R^{-1}(x)W^T(x) > 0$$

where $Y(x) = Y^T(x), R(x) = R^T(x)$ depend on x.

Theorem 1 For the given constants $\alpha > 0, 1 \geq \beta \geq 0$, if there exist positive-definite matrices $P, Q, R \in R^{n\times n}$ and matrices $K \in R^{m\times n}$, X, Y with appropriate dimensions, such that the following matrix inequality holds

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ * & \Theta_{22} \end{bmatrix} < 0 \tag{7}$$

Where

$$\Theta_{11} = \begin{bmatrix} PA + A^T P + Q + 2\alpha P + \tau_2 X_{11} + \tau_2 A^T RA & P\bar{A}_d + \tau_2 X_{12} + \tau_2 A^T RA_d \\ * & -e^{-2\alpha d} Q + \tau_2 X_{22} + \tau_2 A_d^T RA_d \end{bmatrix}$$

$$\Theta_{12}=\begin{bmatrix} P\beta BK+Y_1+\tau_2 X_{13}+\tau_2 A^T R\beta BK & P(1-\beta)BK+\tau_2 X_{14}-Y_1+\tau_2 A^T R(1-\beta)BK \\ Y_2+\tau_2 X_{23}+\tau_2 A_d^T R\beta BK & \tau X_{24}-Y_2+\tau_2 A_d^T R(1-\beta)BK \end{bmatrix}$$

$$\Theta_{22}=\begin{bmatrix} \tau_2 X_{33}+Y_3+Y_3^T+\tau_2 K^T B^T R\beta BK & -Y_3+Y_4^T+\tau_2 X_{34} \\ * & \tau_2 X_{44}-Y_4-Y_4^T+\tau_2 K^T B^T R(1-\beta)BK \end{bmatrix}$$

with the controller (2), the network control systems(6) is mean-square exponentially stable.

Proof： Choose a Lyapunov functional candidate for the system (6) as follows：

$$V(t)=x^T(t)Px(t)+\int_{t-d}^{t} x^T(s)Qe^{2\alpha(s-t)}x(s)ds+\int_{-\tau_2}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Re^{2\alpha(s-t)}\dot{x}(s)dsd\theta$$

where P, Q, R positive-definite matrices in theorem1.

Then, along the solution of system (6) we have

$$\begin{aligned}\dot{V}(t)+2\alpha V(t)&=2x^T(t)P\dot{x}(t)+x^T(t)Qx(t)-x^T(t-d)Qe^{-2\alpha d}x(t-d)\\&\quad+\tau\dot{x}^T(t)R\dot{x}(t)+2\alpha x^T(t)Px(t)-\int_{t-\tau_2}^{t}\dot{x}^T(s)Re^{2\alpha(s-t)}\dot{x}(s)ds\end{aligned}$$

（8）

With

$$x(t-h_1(t))-x(t-h_2(t))-\int_{t-h_2(t)}^{t-h_1(t)}\dot{x}(s)ds=0$$

For any $4n\times n$ matrix

$$N=\begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix}$$

We know

$$0=\xi^T(t)N[x(t-h_1(t))-x(t-h_2(t))-\int_{t-h_2(t)}^{t-h_1(t)}\dot{x}(s)ds]$$

（9）

With lemma1 and (9), we obtain

$$\begin{aligned}0&\le 2\xi^T(t)N[x(t-h_1(t))-x(t-h_2(t))]+\int_{t-h_2(t)}^{t-h_1(t)}\begin{bmatrix}\xi(t)\\ \dot{x}(s)\end{bmatrix}^T\begin{bmatrix} X & Y-N \\ Y^T-N^T & Re^{2\alpha(s-t)}\end{bmatrix}\begin{bmatrix}\xi(t)\\ \dot{x}(s)\end{bmatrix}ds\\&\le 2\xi^T(t)Y[x(t-h_1(t))-x(t-h_2(t))]+\tau_2\xi^T(t)X\xi(t)+\int_{t-\tau_2}^{t}\dot{x}^T(s)Re^{2\alpha(s-t)}\dot{x}(s)ds\end{aligned}$$

（10）

Inserting（10）into（8）, we have：

$$\begin{aligned}\dot{V}(t)+2\alpha V(t)\le{}& x^T(t)[PA+AP+Q+2\alpha P]x(t)\\&+2x^T(t)PA_d x(t-d)+2x^T(t)P\beta(t)BKx(t-h_1(t))\\&+2x^T(t)P(1-\beta(t))BKx(t-h_2(t))\\&-x^T(t-d)Qe^{-2\alpha d}x(t-d)+2\xi^T(t)Y\begin{bmatrix}0 & 0 & I & -I\end{bmatrix}\xi(t)\\&+\tau_2\xi^T(t)X\xi(t)+\tau_2\dot{x}^T(t)R\dot{x}(t)\end{aligned}$$

（11）

Where

$$\dot{x}^T(t)R\dot{x}(t)$$

$$= \tau_2\xi^T(t)\begin{bmatrix} A^TRA & A^TRA_d & A^TR\beta(t)BK & A^TR(1-\beta(t))BK \\ * & A_d^TRA_d & A_d^TR\beta(t)BK & A_d^TR(1-\beta(t))BK \\ * & * & \beta^2(t)K^TB^TRB\ K & \beta(t)(1-\beta(t))K^TB^TRBK \\ * & * & * & (1-\beta(t))^2K^TB^TRB \end{bmatrix}\xi(t)$$

(12)

Obviously

$$2\xi^T(t)\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{bmatrix}[0 \quad 0 \quad I \quad -I]\xi(t) = \xi^T(t)\begin{bmatrix} 0 & 0 & Y_1 & -Y_1 \\ * & 0 & Y_2 & -Y_2 \\ * & * & Y_3+Y_3^T & -Y_3+Y_4^T \\ * & * & * & -Y_4-Y_4^T \end{bmatrix}\xi(t)$$

(13)

Inserting (12-13) into (11), we obtain

$$E\{\dot{V}(t)+2\alpha V(t)\} \le \sum_i^r \sum_j^r \mu_i(z(t))\mu_j(z(t))\xi^T(t)\Theta\xi(t)$$

With matrix inequality (7), we know

$$E\{\dot{V}(t)\} < -2\alpha E\{V(t)\}$$

Therefore

$$E\{V(t)\} < E\{V(0)\}e^{-2\alpha t} \le [\lambda_{\max}(P) + d\lambda_{\max}(Q) + \tau\lambda_{\max}(R)\bar{\psi}^2]E\{\|\psi(t)\|^2\}e^{-2\alpha t}$$

(14)

Obviously

$$E\{V(t)\} \ge \lambda_{\min}(P)E\{\|x(t)\|^2\}$$

(15)

From (14-15), we obtain

$$E\{\|x(t)\|\} < \sqrt{\frac{\lambda_{\max}(P) + d\lambda_{\max}(Q) + \tau\lambda_{\max}(R)\bar{\psi}^2}{\lambda_{\min}(P)}}E\{\|\psi(t)\|\}e^{-\alpha t}$$

With the Lyapunov stability theorem and the above inequality, we know that the system (6) is mean-square exponentially stable.

Theorem 2 For the given constants $\alpha > 0, 1 \ge \beta \ge 0$, if there exist positive-definite matrices $\bar{P}, \bar{Q}, \bar{R} \in R^{n\times n}$ and matrix $\bar{K} \in R^{m\times n}$, $\bar{X}, \bar{Y}$ with appropriate dimensions, such that the following linear matrix inequality holds

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} \\ * & \Xi_{22} \end{bmatrix} < 0$$

(16)

Where

$$\Xi_{11}=\begin{bmatrix} A\bar{P}+\bar{P}A^T+\bar{Q} \\ +2\alpha\bar{P}+\tau_2\bar{X}_{11} & A_d\bar{P}+\tau_2\bar{X}_{12} & \beta B\bar{K}+\tau_2\bar{X}_{13}+\bar{Y}_1 \\ * & -e^{-2\alpha d}\bar{Q}+\tau_2\bar{X}_{22} & \tau_2\bar{X}_{23}+\bar{Y}_2 \\ * & * & \tau_2\bar{X}_{33}+\bar{Y}_3+\bar{Y}_3^T \end{bmatrix}$$

$$\Xi_{12}=\begin{bmatrix} (1-\beta)B\bar{K}-\bar{Y}_1+\tau_2\bar{X}_{14} & \tau_2\beta\bar{P}A^T & \tau_2(1-\beta)\bar{P}A^T \\ \tau_2\bar{X}_{24}-\bar{Y} & \tau_2\beta\bar{P}A_d^T & \tau_2(1-\beta)\bar{P}A_d^T \\ \tau_2\bar{X}_{34}+\bar{Y}_4^T-\bar{Y}_3 & \tau_2\beta\bar{K}^T B^T & 0 \end{bmatrix}$$

$$\Xi_{22}=\begin{bmatrix} \tau_2\bar{X}_{44}-\bar{Y}_4-\bar{Y}_4^T & 0 & \tau_2(1-\beta)\bar{K}^T B^T \\ & -\tau_2\beta\bar{R} & 0 \\ & & -\tau_2(1-\beta)\bar{R} \end{bmatrix}$$

with the controller $u(t)=\bar{K}\bar{P}^{-1}x(t)$, the systems(6) is mean-square exponentially stable.

Proof

$$\Theta=\Theta_0+\alpha_1^T\frac{1}{\tau_2\beta}R^{-1}\alpha_1+\alpha_2^T\frac{1}{\tau_2(1-\beta)}R^{-1}\alpha_2$$

Where

$$\Theta_0=\begin{bmatrix} PA+A^TP+Q+2\alpha P \\ +\tau_2X_{11} & PA_d+\tau_2X_{12} & P\beta BK+Y_1+\tau_2X_{13} & P(1-\beta)BK+\tau_2X_{14}-Y_1 \\ * & -e^{-2\alpha d}Q+\tau_2X_{22} & Y_2+\tau_2X_{23} & \tau_2X_{24}-Y_2 \\ * & * & \tau_2X_{33}+Y_3+Y_3^T & -Y_3+Y_4^T+\tau_2X_{34} \\ * & * & * & \tau_2X_{44}-Y_4-Y_4^T \end{bmatrix}$$

$$\alpha_1=\begin{bmatrix}\tau_2\beta RA & \tau_2\beta RA_d & \tau_2\beta RBK & 0\end{bmatrix}$$

$$\alpha_2=\begin{bmatrix}\tau_2(1-\beta)RA & \tau_2(1-\beta)RA_d & 0 & \tau_2(1-\beta)RBK\end{bmatrix}$$

With lemma2, we know that inequality $\Theta<0$ is equivalent to

$$\Sigma=\begin{bmatrix} PA+A^TP+Q+2\alpha P+\tau_2X_{11} & PA_d+\tau_2X_{12} & P\beta BK+Y_1+\tau_2X_{13} & P(1-\beta)BK+\tau_2X_{14}-Y_1 & \tau_2\beta A^TR & \tau_2(1-\beta)A^TR \\ * & -e^{-2\alpha d}Q+\tau_2X_{22} & \tau_2X_{23}+Y_2 & \tau_2X_{24}-Y_2 & \tau_2\beta A_d^TR & \tau_2(1-\beta)A_d^TR \\ * & * & \tau_2X_{33}+Y_{13}+Y_{13}^T & \tau_2X_{34}+Y_4^T-Y_3 & \tau_2\beta K^TB^TR & 0 \\ * & * & * & \tau_2X_{44}-Y_4-Y_4^T & 0 & \tau_2(1-\beta)K^TB^TR \\ * & * & * & * & -\tau_2\beta R & 0 \\ * & * & * & * & * & -\tau_2(1-\beta)R \end{bmatrix}<0$$

(17)

Pre- and Post-multiplying the inequality (17) by,

$$diag\{P^{-1} \quad P^{-1} \quad P^{-1} \quad P^{-1} \quad R^{-1} \quad R^{-1}\}$$

and giving some transformations :

$$\overline{P} = P^{-1} \quad \overline{Q} = P^{-1} Q P^{-1} \quad \overline{K} = K P^{-1} \quad \overline{X} = P^{-1} X P^{-1} \quad \overline{Y} = P^{-1} Y P^{-1} \quad \overline{R} = R^{-1}$$

we know that inequality $\Sigma < 0$ is equivalent to（16）. Therefore, the linear matrix inequality (16) is equivalent to（7）. With Theorem 1, the systems (6) is mean-square exponentially stable.

CONCLUSION

This paper considers the problem of mean-square exponential stability control for a class of networked control systems with interval distribution time delay. Based on the Lyapunov stability theorem, a sufficient condition and the controller design approach are given in term of LMI.

Acknowledgments
The authors would like to thank the associate editor and the anonymous reviewers for their constructive comments and suggestions to improve the quality and the presentation of the paper. This work is supported by National Nature Science Foundation of China under Grant 61073065; Nature Science Foundation of Henan Province under Grant 092300410145; The Education Department of Henan Province Key Foundation under Grant 13A110023, Anyang Normal University Innovation Foundation Project under Grant ASCX/2013-Z29.

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