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Research Article

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Estimation of the number of incidence in the epidemic dynamics model with latent period

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ABSTRACT

In the dynamic model of infectious diseases, the use of traditional methods to estimate the number of incidence, there is often a theoretical analysis of high complexity, immediacy and poor. The nonlinear dynamic model of the observer in the incidence of infectious diseases estimation method can better solve the problem. This paper presents a nonlinear observer on the incidence of infectious disease dynamics model estimation method. System dynamics model for infectious diseases have incubation period, the structure of the corresponding nonlinear systems observer, the observer for nonlinear systems to predict the number of cases. The present invention is not only simple, can be estimated coefficient estimates and infectious disease incidence rate, but also on other parameters such as illness mortality and recovery rates can also be estimated and the adaptability and accuracy have certain advantage of the prevention and control of infectious diseases actually have some reference value.

Key words: Epidemic dynamics model; latent period; number of incidence; estimation

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INTRODUCTION

Infectious diseases by pathogenic microorganisms (viruses, rickettsia, bacteria, spirochetes, etc.) and parasites (protozoa or worms) produced by the body after infection infectious diseases. In recent years, with the advent of AIDS, SARS and influenza HINI and spread of new infectious diseases, pose a serious threat to human life. Therefore, the infectious disease prevention, treatment, control and study its propagation mechanism is increasingly important. Currently, the study of infectious diseases, there are four methods: descriptive study, analytical studies, experimental studies and theoretical research. In the theoretical study, based on disease occurrence, development and environmental changes, etc., to reflect its dynamics establish a mathematical model to predict the epidemic patterns and trends of the disease, the cause of the epidemic and key factor analysis, to find its Optimal control and prevention strategies. Therefore, the use of mathematical models for qualitative research on infectious diseases has been widespread attention [1-2]. Using kinetic models of infectious diseases, the number of new cases of infectious diseases prediction that the incidence of this disease, contribute to the development trend of infectious diseases to predict, and thus the prevention and control of infectious diseases has important guiding significance. In infectious disease dynamics, the solution to this problem, mainly through traditional methods of solving differential equations analysis to achieve. However, due to the complexity, and the actual reality of the immediacy of forecasting models have caused the traditional methods have significant limitations in the application. Based on the literature [3], based on the consideration of the SEIR epidemic model with incubation period, using a nonlinear observer approach to the incidence of infectious diseases SEIR model and the infection rate coefficient is estimated. The results show that compared with traditional methods, this method is adaptive in terms of practicality and have a greater advantage, the actual work on the prevention and control of infectious diseases have a certain reference value.

MATHEMATICAL MODELS OF GENETIC ALGORITHMS

In infectious disease dynamics, the mathematical model is mainly used for a long time the so-called compartment model, its basic idea is Kermark and Mckendrick first proposed in 1927, and established the famous SIR compartment model [4]. SIR model first crowd divided into three categories, namely, with S(t), I(t), R(t) denote the number of susceptible class time t, and out of those infected by class category. Let the time t of a number of infectious diseases can be transmitted by the number of susceptible susceptible environment within the total proportional scale factor β (called the infection rate coefficient), so in the unit time t is the time for all patients infected (the number of new cases) was βSI, also known as the disease incidence; number of infected persons removed from the time t and the number of infected persons is directly proportional to the ratio coefficient δ, called the recovery rate coefficient; another assumption is removed class R lifelong immunity will not be infected persons become infected again. In all the above conditions, a model diagram SIR infectious diseases:

Fig. 1: SIR epidemic model diagram

Infectious disease dynamics model diagram above corresponds to:

$$\begin{cases} \dot{S}(t) = -\beta SI \\ \dot{I}(t) = \beta SI - \delta I \\ \dot{R}(t) = \delta I \end{cases} \tag{1}$$

In the model (1), in order to predict the number of new-onset cases of infectious diseases, need to estimate βSI value. In reality, especially in the first few emerging infectious diseases onset infection rate coefficient β is not known, so for infectious disease dynamics model to estimate the incidence of disease problems, the traditional method is to use a method for solving differential equations and mathematical analysis first obtain the value of β, and then determine the value βSI of [1]. This method need to obtain the theoretical solution expression models: $I - I_0 = -(S - S_0) + \frac{\delta}{\beta} ln \frac{S}{S_0}$

When $t \to \infty$, we have $I(t) \to 0$, $S(t) \to S_\infty$, Solutions have: $\beta = \frac{\delta(ln S_0 - ln S_\infty)}{S_0 + I_0 - S_\infty}$.

The literature [5] based on actual data, using the method of bubonic plague in 1666 England village of Eyam Sheffield area were analyzed, the conclusion that the infection rate coefficient (1 / month). On this basis, the number of incidence is obtained βSI.

The above estimation of disease incidence and infection rates is inadequate in actual use. First estimation methods in the literature [5] given disease eventually need to know when the end of the number of susceptible, that the estimated β is at the end of epidemics, rather than in the course of disease transmission, so that the method in real-time there is a great shortage of. Meanwhile, in practice, the infection rate β depends not only on the type of disease, but also on the individual circumstances in which people and infected human social environment, so β has great complexity. In the dynamic model of infectious disease research, according to the actual factors, the establishment of the number of incidence in many forms, for example: bilinear incidence βSI, standardized incidence ratio βSI/N, et al proposed by the J.A.P. Heesterbeek has saturating contact rate The number of incidence, as well as recently proposed new rates βI(1+*f*(I))S and other forms [2]. Thus, the incidence of these diseases nonlinear, the corresponding differential model seeking theoretical solutions and qualitative analysis is more complex, so the use of the literature [5] estimation methods has great difficulty.

Nonlinear control theory controller design is an important research branch, its application in industrial control more widely, but the application of the kinetic model was relatively rare diseases. Using control theory nonlinear observer method, reasonable structure nonlinear controller for the number of incidence has a latency period of the SEIR model to estimate the dynamics of infectious diseases, a new way, this estimation method is simple and effective, and not dependent model can predict the type of disease incidence and estimates the number of the disease, the simulation results show that the method is effective.

NONLINEAR OBSERVER ESTIMATED INCIDENCE OF INFECTIOUS DISEASES

Research on nonlinear control system design has made great progress in recent years [6-8], but the results achieved in terms of nonlinear observer is not much, especially nonlinear observer in the control system parameter estimation applications are rare. In this paper, the literature [9] gives a parameter estimation method based on nonlinear observer, this nonlinear observer for multiple parameters of the system can only be estimated, but also can be carried out simultaneously on the status and parameters of the amount of output estimates. The process of establishing a nonlinear observation is as follows:

First, consider the following systems: $\dot{x}(t)=f(x,u,\theta)$, where x in the system state is output, u is the control input, and θ is a vector of parameters to be estimated. It is assumed that the state and the control vector is estimated variable parameter vectors.

According to the standard theory can be constructed as follows Nonlinear Observer [10]:

$$\hat{\theta}=-\phi(x)+z \tag{2}$$

$$\dot{z}=-\Phi(x)f(x,u,\hat{\theta}) \tag{3}$$

Which $\phi(x)$ is a nonlinear function, and $\Phi(x)$ is the Jacobian matrix of sheets: $\Phi(x)=\left[\frac{\partial\phi_i(x)}{\partial x_j}\right]$. $\hat{\theta}$ is the definition of the estimated value of the parameter θ. In addition, the observer error depends on:

$$\dot{e}=-\dot{\hat{\theta}}=-\Phi(x)\dot{x}-\dot{z}=-\Phi(x)\left[f(x,u,\theta)-f(x,u,\theta-e)\right] \tag{4}$$

The observer for parameter estimation of the structure shown in figure 2:

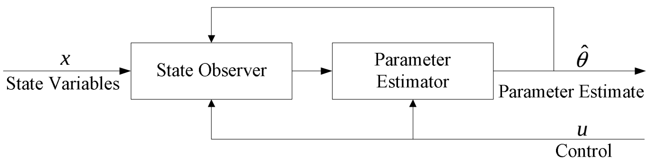

Fig. 2: Nonlinear parameter estimation observer schematic diagram

In the nonlinear observer of use, the non-linear function $\phi(x)$ is important, $L(t)=\Phi(x)F(x,u)$ is a positive definite matrix, where the matrix $F(x,u)=\left[\partial f(x,u,\theta)/\partial\theta_j\right]$.

PREDICTION METHOD OF DISEASE INCIDENCE IN THE EPIDEMIC DYNAMICS MODEL WITH LATENT PERIOD

Consider the actual situation spread of infectious diseases, in order to make the model better get in line with the actual situation, the system (1) to make the following improvements: First, consider the impact of immunization, the introduction of vaccination items pS, where p is susceptible class S the proportional coefficient of immunization; while common infectious diseases are all incubation period, so on the basis of SIR epidemic model, the added latency of class E(t), where μ is the time t to the onset of the incubation period the proportion of patients with factor ; fatal disease with an assumption, a is a latency in patients E, I invectives mortality; disease occurrence rate β(t)SI, and other symbols are the same meaning and the system (1).

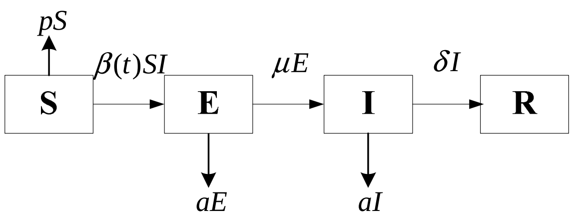

Fig. 3: The epidemic dynamics model with latent period diagram

Under the above assumptions, the corresponding infectious disease dynamics model:

$$\begin{cases}\dot{S}=-\beta(t)SI-pS\\ \dot{E}=\beta(t)SI-\mu E-aE\\ \dot{I}=\mu E-\delta I-aI\\ \dot{R}=\delta I\end{cases} \tag{5}$$

Since our main disease incidence β(t)SI estimated, so here only consider the system (5) consisting of the first three equations, and assuming that θ=β(t)SI, and the disease incidence β(t)SI parameters to be estimated. Further, since in practice, the immunization of susceptible can be controlled, so that the model can be seen as a factor immunization power control system, the system $\dot{x}(t) = f(x, u, \theta)$ that is u=pS. Therefore, the following simplified system:

$$\begin{cases} \dot{S} = -\theta - u \\ \dot{E} = \theta - \mu E - aE \end{cases} \tag{6}$$

By a system parameter estimation method for nonlinear observer (2), (3) given for the system (6), to establish the corresponding nonlinear systems Observer:

$$\begin{cases} \dot{z} = \phi_x(E)((\mu + a)E - \hat{\theta}) \\ \hat{\theta} = \phi(E) + z \end{cases} \tag{7}$$

Where z is the observer state variables, $\hat{\theta}$ is the estimated number of incidence, but also the output observer, $\phi(x)$ is a nonlinear function, and $\phi_x(I) = [\frac{\partial \phi(x)}{\partial x}]_{x=E}$.

Because the system (6) is only one of the parameters to be estimated θ, so for nonlinear observer (7), we take the function $\phi(x) = kx$. Because $\phi(x)$ corresponding Jacobian matrix is $\phi(x) = k$, so there is $L(t) = k$. Obviously, when $k > 0$, L is positive definite, parameter estimation methods to meet the conditions.

SIMULATION

Nonlinear observer on the incidence of infectious diseases SEIR model with vaccination simulation experiments. Figure 4, 5, 6 respectively, when k=0.5, k=2 and k=12, the comparative picture of disease incidence β(t)SI values of SIR model with nonlinear numerical simulation method to estimate the value of the observer, which system parameters were taken β(t) = 0.02, μ = 0.2, a = 0.05.

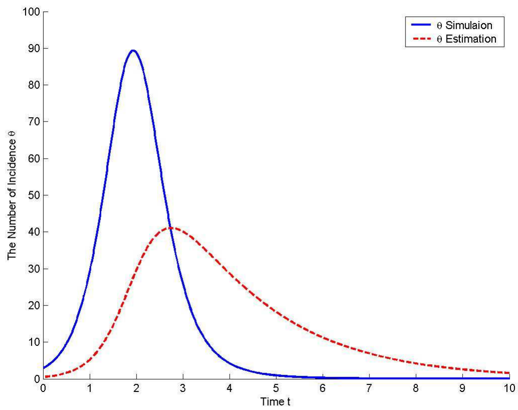

Fig. 4 When k = 0.5, the number of incidence of the epidemic model simulation diagram

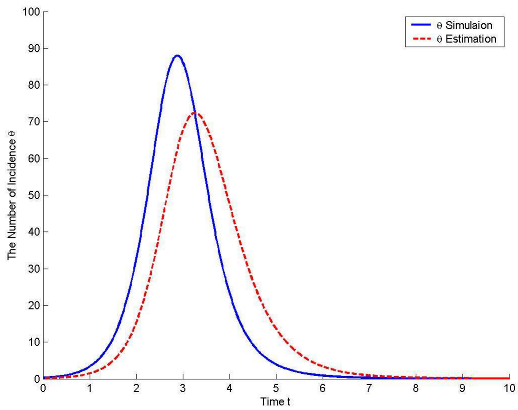

Fig. 5 When k =2, the number of incidence of the epidemic model simulation diagram

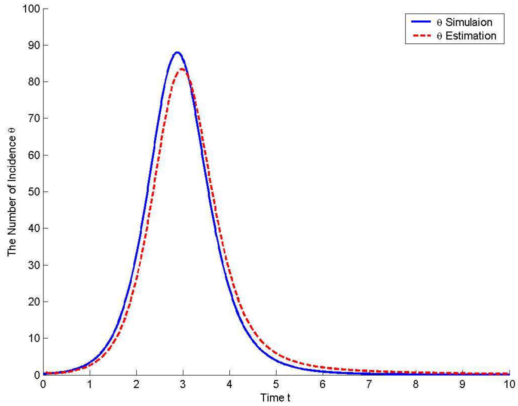

Fig. 6 When k = 12, the number of incidence of the epidemic model simulation diagram

In addition, the use of these diseases as well as estimates of the incidence of susceptible moment, the number of infected persons can obtain an estimate of the corresponding infection rate β, and Figure 7 is k = 12 during infection rate β simulation results, when t=20, estimated value of the infection rate β is close to the value of the model parameter β=0.02, the error is less than 10^{-5}.

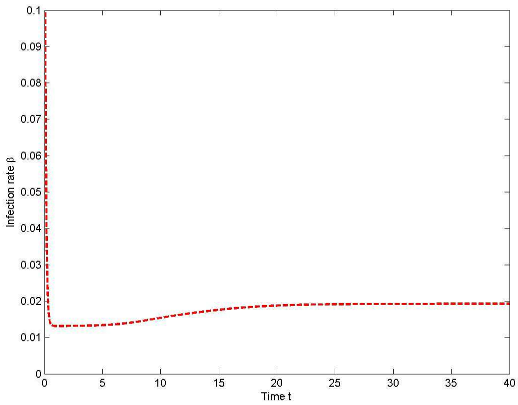

Fig. 7 When k = 12, infection rate of the epidemic model estimates

CONCLUSION

When the use of differential dynamic model of the propagation of infectious diseases, mainly relies on traditional methods for solving differential equations of the theory, but due to the complexity of the model and its theoretical solution is often difficult to draw, so its limitations. The use of non-linear controller to estimate the parameters of the model, such as the common type of disease incidence, this method is simple. Meanwhile, this method can not only estimate the coefficient estimates and infectious disease incidence rate and other parameters such as mortality and illness recovery rate can also be estimated, and have certain advantages in terms of adaptability and accuracy of prevention and control of infectious diseases has potential applications

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